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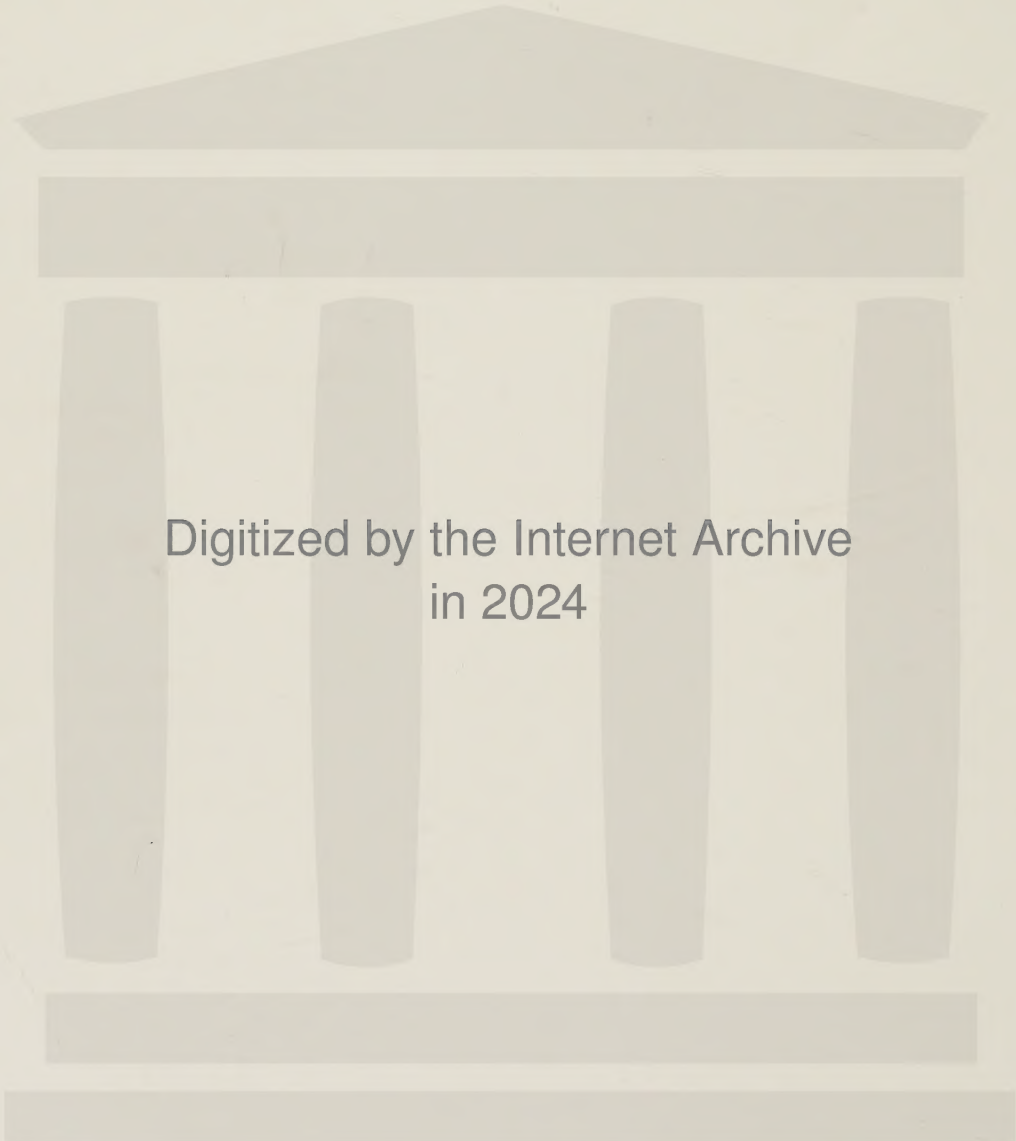


INTRODUCTORY ALGEBRA

AN APPLIED APPROACH

Fifth Edition

with MATH STUDY SKILLS WORKBOOK by Paul Nolting


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Table of Properties

Properties of Real Numbers

The Associative Property of Addition
If a, b, and c are real numbers, then
$(a + b) + c = a + (b + c)$.

The Commutative Property of Addition
If a and b are real numbers, then
$a + b = b + a$.

The Addition Property of Zero
If a is a real number, then
$a + 0 = 0 + a = a$.

The Multiplication Property of Zero
If a is a real number, then
$a \cdot 0 = 0 \cdot a = 0$.

The Inverse Property of Addition
If a is a real number, then
$a + (-a) = (-a) + a = 0$.

The Associative Property of Multiplication
If a, b, and c are real numbers, then
$(ab)c = a(bc)$.

The Commutative Property of Multiplication
If a and b are real numbers, then
$ab = ba$.

The Multiplication Property of One
If a is a real number, then
$a \cdot 1 = 1 \cdot a = a$.

The Inverse Property of Multiplication
If a is a real number and $a \neq 0$, then
$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$.

Distributive Property
If a, b, and c are real numbers, then
$a(b + c) = ab + ac$ or $(b + c)a = ba + ca$.

Properties of Equations

Addition Property of Equations
If $a = b$, then $a + c = b + c$.

Multiplication Property of Equations
If $a = b$ and $c \neq 0$, then $ac = bc$.

Properties of Exponents

If m and n are integers, then $x^m \cdot x^n = x^{m+n}$.

If m and n are integers, then $(x^m)^n = x^{mn}$.

If $x \neq 0$, then $x^0 = 1$.

If m and n are integers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

If m, n, and p are integers, then $(x^m y^n)^p = x^{mp} y^{np}$.

If n is a positive integer and $x \neq 0$, then
$x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

If m, n, and p are integers and $y \neq 0$, Then, $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

Properties of Inequalities

Addition Property of Inequalities
If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

Multiplication Property of Inequalities
If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.
If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

Principle of Zero Products

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

Properties of Radical Expressions

If a and b are positive real numbers, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

If a and b are positive real numbers, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Property of Squaring Both Sides of an Equation

If a and b are real numbers and $a = b$, then $a^2 = b^2$.

Table of Equations and Formulas

Slope of a straight line

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1},\ x_1 \neq x_2$$

Slope-intercept form of a straight line

$$y = mx + b$$

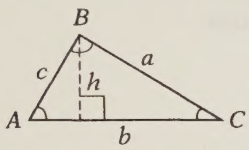
Point–Slope formula

$$y - y_1 = m(x - x_1)$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Perimeter and Area of a Triangle, and Sum of the Measures of the Angles

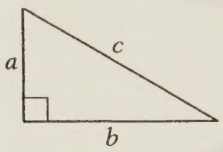


$$P = a + b + c$$

$$A = \frac{1}{2}bh$$

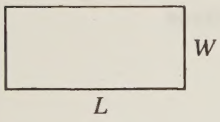
$$A + B + C = 180°$$

Pythagorean Theorem



$$a^2 + b^2 = c^2$$

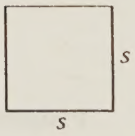
Perimeter and Area of a Rectangle



$$P = 2L + 2W$$

$$A = LW$$

Perimeter and Area of a Square



$$P = 4s$$

$$A = s^2$$

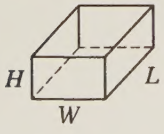
Circumference and Area of a Circle



$$C = 2\pi r \quad \text{or} \quad C = \pi d$$

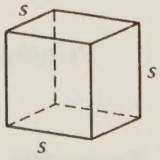
$$A = \pi r^2$$

Volume of a Rectangular Solid



$$V = LWH$$

Volume of a Cube



$$V = s^3$$

Volume of a Sphere



$$V = \frac{4}{3}\pi r^3$$

Volume of a Right Circular Cylinder



$$V = \pi r^2 h$$

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AN APPLIED APPROACH

Fifth Edition

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with

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Preface

The fifth edition of *Introductory Algebra: An Applied Approach* provides mathematically sound and comprehensive coverage of the topics considered essential in an introductory algebra course. The text has been designed not only to meet the needs of the traditional college student but also to serve the needs of returning students whose mathematical proficiency may have declined during years away from formal education.

In this new edition of *Introductory Algebra: An Applied Approach,* we have continued to integrate some of the approaches suggested by AMATYC. Each chapter begins with a mathematical vignette in which there may be a historical note, an application, or a curiosity related to mathematics. At the end of each section there are "Applying the Concepts" exercises, which include writing, synthesis, critical thinking, and challenge problems. Each chapter ends with a "Focus on Problem Solving," which introduces students to problem-solving strategies. This is followed by "Projects and Group Activities," which can be used for cooperative learning activities.

INSTRUCTIONAL FEATURES

Interactive Approach

Introductory Algebra: An Applied Approach uses an interactive style that provides a student with an opportunity to try a skill as it is presented. Each section is divided into objectives, and every objective contains one or more sets of matched-pair examples. The first example in each set is worked out; the second example, called "You Try It," is for the student to work. By solving this problem, the student practices concepts as they are presented in the text. There are complete worked-out solutions to these examples in an appendix at the end of the book. By comparing their solution to the solution in the appendix, students are able to obtain immediate feedback on and reinforcement of the concept.

Emphasis on Problem-Solving Strategies

Introductory Algebra: An Applied Approach features a carefully developed approach to problem solving that emphasizes developing strategies to solve problems. Students are encouraged to develop their own strategies, to draw diagrams, and to write strategies as part of their solution to a problem. In each case, model strategies are presented as guides for students to follow as they attempt the "You Try It" problem. Having students provide strategies is a natural way to incorporate writing into the math curriculum.

Emphasis on Applications

The traditional approach to teaching algebra covers only the straightforward manipulation of numbers and variables and thereby fails to teach students the practical value of algebra. By contrast, *Introductory Algebra: An Applied Approach* contains an extensive collection of contemporary application problems. Wherever appropriate, the last objective of a section presents applications that require the student to use the skills covered in that section to solve practical problems. This carefully integrated applied approach generates student awareness of the value of algebra as a real-life tool.

Completely Integrated Learning System Organized by Objectives

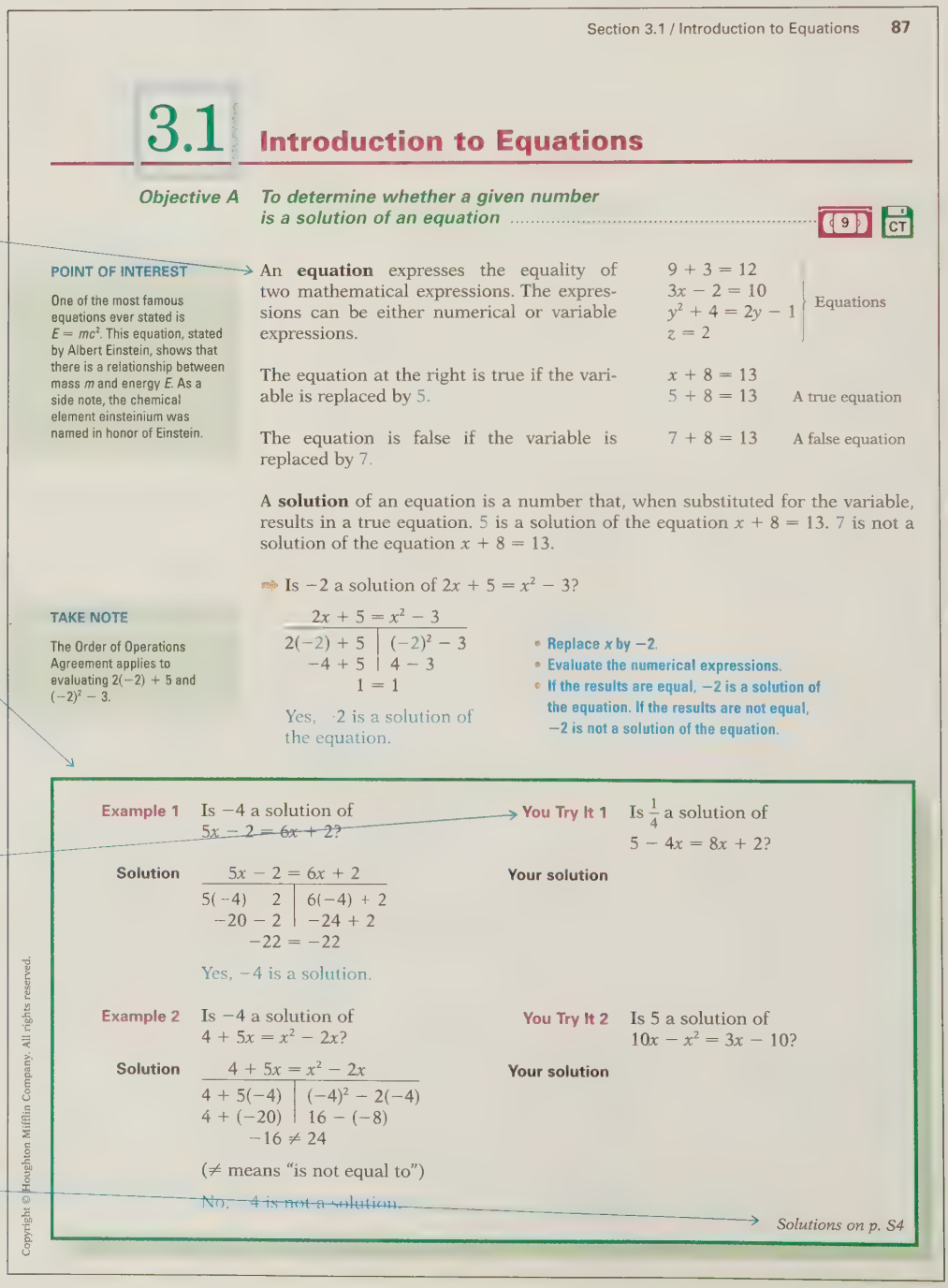
Each chapter begins with a list of the learning objectives included within that chapter. Each of the objectives is then restated in the chapter to remind the student of the current topic of discussion. The same objectives that organize the text are also used as the structure for exercises, testing programs, and the Computer

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Tutor. For each objective in the text, there is a corresponding computer tutorial and a corresponding set of test questions.

AN INTERACTIVE APPROACH

Instructors have long realized the need for a text that requires students to use a skill as it is being taught. *Introductory Algebra: An Applied Approach* uses an interactive technique that meets this need. Every objective, including the one shown below, contains at least one pair of examples. One of the examples is worked. The second example in the pair (You Try It) is not worked so that students may "interact" with the text by solving it. To provide immediate feedback, a complete worked-out solution to this example is provided in the Solutions Section at the end of the book. The benefit of this interactive style is that students can immediately determine whether a new skill has been learned before attempting a homework assignment or moving on to the next skill.



Section 3.1 / Introduction to Equations 87

3.1 Introduction to Equations

Objective A *To determine whether a given number is a solution of an equation*

POINT OF INTEREST

One of the most famous equations ever stated is $E = mc^2$. This equation, stated by Albert Einstein, shows that there is a relationship between mass m and energy E. As a side note, the chemical element einsteinium was named in honor of Einstein.

An **equation** expresses the equality of two mathematical expressions. The expressions can be either numerical or variable expressions.

$9 + 3 = 12$
$3x - 2 = 10$
$y^2 + 4 = 2y - 1$
$z = 2$
} Equations

The equation at the right is true if the variable is replaced by 5.

$x + 8 = 13$
$5 + 8 = 13$ A true equation

The equation is false if the variable is replaced by 7.

$7 + 8 = 13$ A false equation

A **solution** of an equation is a number that, when substituted for the variable, results in a true equation. 5 is a solution of the equation $x + 8 = 13$. 7 is not a solution of the equation $x + 8 = 13$.

Is -2 a solution of $2x + 5 = x^2 - 3$?

TAKE NOTE

The Order of Operations Agreement applies to evaluating $2(-2) + 5$ and $(-2)^2 - 3$.

$$\begin{array}{r|l} 2x + 5 = x^2 - 3 & \\ \hline 2(-2) + 5 & (-2)^2 - 3 \\ -4 + 5 & 4 - 3 \\ 1 = 1 & \end{array}$$

- Replace x by -2.
- Evaluate the numerical expressions.
- If the results are equal, -2 is a solution of the equation. If the results are not equal, -2 is not a solution of the equation.

Yes, -2 is a solution of the equation.

Example 1 Is -4 a solution of $5x - 2 = 6x + 2$?

Solution

$$\begin{array}{r|l} 5x - 2 = 6x + 2 & \\ \hline 5(-4) \quad 2 & 6(-4) + 2 \\ -20 - 2 & -24 + 2 \\ -22 = -22 & \end{array}$$

Yes, -4 is a solution.

You Try It 1 Is $\frac{1}{4}$ a solution of $5 - 4x = 8x + 2$?

Your solution

Example 2 Is -4 a solution of $4 + 5x = x^2 - 2x$?

Solution

$$\begin{array}{r|l} 4 + 5x = x^2 - 2x & \\ \hline 4 + 5(-4) & (-4)^2 - 2(-4) \\ 4 + (-20) & 16 - (-8) \\ -16 \neq 24 & \end{array}$$

($\neq$ means "is not equal to")

No, -4 is not a solution.

You Try It 2 Is 5 a solution of $10x - x^2 = 3x - 10$?

Your solution

Solutions on p. S4

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An explanatory passage begins each objective.

Paired examples follow the explanatory passage.

The interactive key is the You Try It in each pair. It has not been worked so that the student may practice the skill, referring to the worked example at the left if necessary.

Reference to the Solutions Section allows the student to check full solutions immediately.

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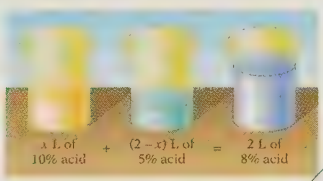
AN EMPHASIS ON APPLICATIONS

The traditional teaching approach neglects the difficulties that students have in making the transition from arithmetic to algebra. One of the most troublesome and uncomfortable transitions for the student is from concrete arithmetic to symbolic algebra. *Introductory Algebra: An Applied Approach* recognizes the formidable task the student faces by introducing variables in a very natural way—through applications of mathematics. A secondary benefit of this approach is that the student becomes aware of the value of algebra as a real-life tool.

The solution of an application problem in *Introductory Algebra: An Applied Approach* is always accompanied by two parts: **Strategy** and **Solution**. The strategy is a written description of the steps that are necessary to solve the problem; the solution is the implementation of the strategy. This format provides students with a structure for problem solving. It also encourages students to write strategies for solving problems, which, in turn, fosters organizing problem-solving strategies in a logical way.

Example 2
A chemist wishes to make 2 L of an 8% acid solution by mixing a 10% acid solution and a 5% acid solution. How many liters of each solution should the chemist use?

Strategy



Liters of 10% solution: x
Liters of 5% solution: $2 - x$

	Amount	*Percent*	*Quantity*
10% solution	x	0.10	$0.10x$
5% solution	$2 - x$	0.05	$0.05(2 - x)$
8% solution	2	0.08	$0.08(2)$

The sum of the quantities before mixing is equal to the quantity after mixing.

Solution

$$0.10x + 0.05(2 - x) = 0.08(2)$$
$$0.10x + 0.10 - 0.05x = 0.16$$
$$0.05x + 0.10 = 0.16$$
$$0.05x = 0.06$$
$$x = 1.2$$

$2 - x = 2 - 1.2 = 0.8$

The chemist needs 1.2 L of the 10% solution and 0.8 L of the 5% solution.

You Try It 2
A pharmacist dilutes 5 L of a 12% solution with a 6% solution. How many liters of the 6% solution are added to make an 8% solution?

Your strategy

Your solution

Solution on p. S8

A strategy, which the student may use in solving an application problem, is stated.

The strategy is used in the solution of the worked example.

Students are encouraged to write a strategy for the application problems they solve.

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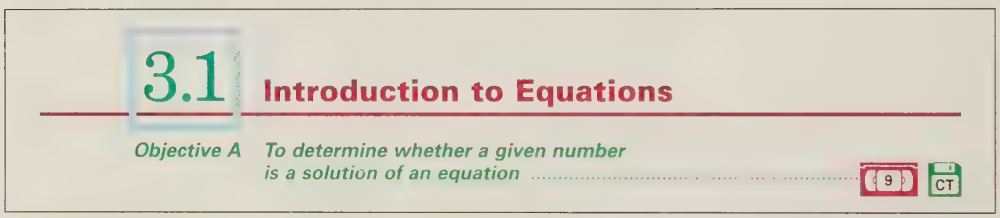
OBJECTIVE-SPECIFIC APPROACH

Many mathematics texts are not organized in a manner that facilitates management of learning. Typically, students are left to wander through a maze of apparently unrelated lessons, exercise sets, and tests. *Introductory Algebra: An Applied Approach* solves this problem by organizing all lessons, exercise sets, computer tutorials, and tests around a carefully constructed hierarchy of objectives. The advantage of this objective-by-objective organization is that it enables the student who is uncertain at any step in the learning process to refer easily to the original presentation and review that material.

The Objective-Specific Approach also gives the instructor greater control over the management of student progress. The Computerized Test Generator and the printed Test Bank are organized by the same objectives as the text. These references are provided with the answers to the test items, thereby allowing the instructor to quickly determine those objectives for which a student may need additional instruction.

The Computer Tutor is also organized around the objectives of the text. As a result, supplemental instruction is available for any objectives that are troublesome for a student.

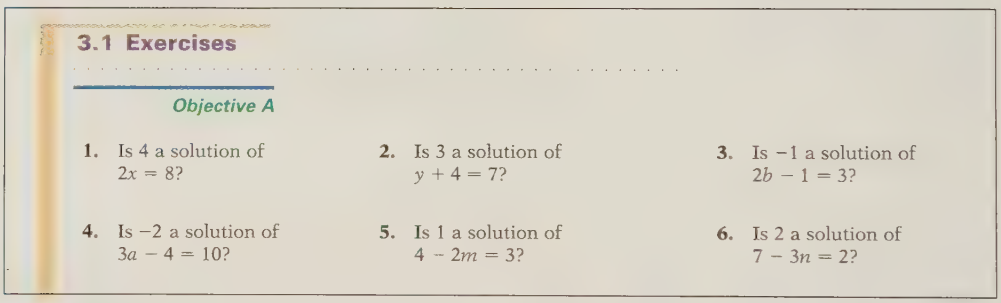
An objective statement names the topic of each lesson.



3.1 Introduction to Equations

Objective A *To determine whether a given number is a solution of an equation* ..... 9 CT

The exercise sets correspond to the objectives in the text.

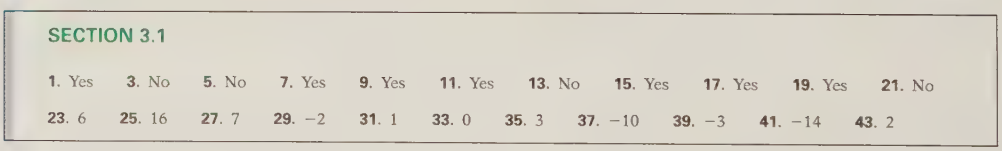


3.1 Exercises

Objective A

1. Is 4 a solution of $2x = 8$?
2. Is 3 a solution of $y + 4 = 7$?
3. Is -1 a solution of $2b - 1 = 3$?
4. Is -2 a solution of $3a - 4 = 10$?
5. Is 1 a solution of $4 - 2m = 3$?
6. Is 2 a solution of $7 - 3n = 2$?

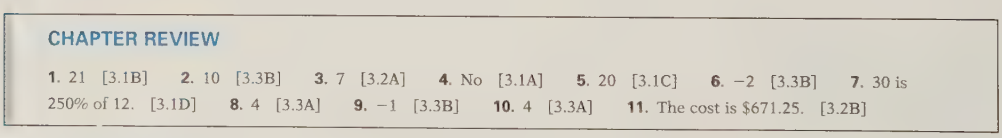
The answers to the odd-numbered exercises are provided in the Answer Section.



SECTION 3.1

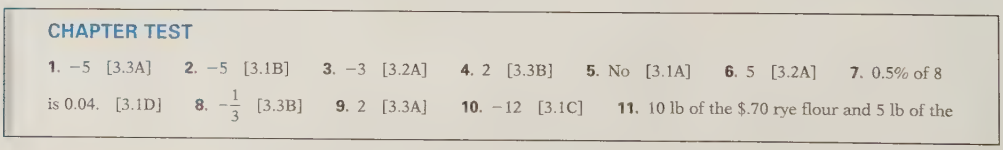
1. Yes 3. No 5. No 7. Yes 9. Yes 11. Yes 13. No 15. Yes 17. Yes 19. Yes 21. No
23. 6 25. 16 27. 7 29. −2 31. 1 33. 0 35. 3 37. −10 39. −3 41. −14 43. 2

The answers to the Chapter Review Exercises, the Chapter Test, and the Cumulative Review Exercises show the objective to study if the student incorrectly answers the exercise.



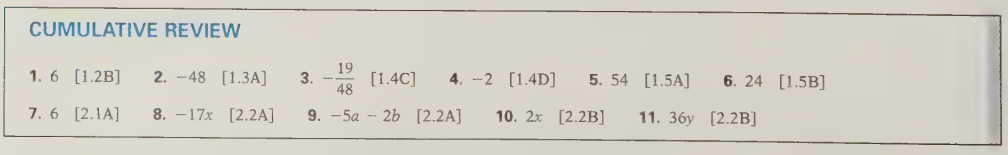
CHAPTER REVIEW

1. 21 [3.1B] 2. 10 [3.3B] 3. 7 [3.2A] 4. No [3.1A] 5. 20 [3.1C] 6. −2 [3.3B] 7. 30 is 250% of 12. [3.1D] 8. 4 [3.3A] 9. −1 [3.3B] 10. 4 [3.3A] 11. The cost is $671.25. [3.2B]



CHAPTER TEST

1. −5 [3.3A] 2. −5 [3.1B] 3. −3 [3.2A] 4. 2 [3.3B] 5. No [3.1A] 6. 5 [3.2A] 7. 0.5% of 8 is 0.04. [3.1D] 8. $-\frac{1}{3}$ [3.3B] 9. 2 [3.3A] 10. −12 [3.1C] 11. 10 lb of the $.70 rye flour and 5 lb of the



CUMULATIVE REVIEW

1. 6 [1.2B] 2. −48 [1.3A] 3. $-\frac{19}{48}$ [1.4C] 4. −2 [1.4D] 5. 54 [1.5A] 6. 24 [1.5B]
7. 6 [2.1A] 8. $-17x$ [2.2A] 9. $-5a - 2b$ [2.2A] 10. $2x$ [2.2B] 11. $36y$ [2.2B]

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ADDITIONAL LEARNING AIDS

Chapter Opener

The Chapter Opener relates a historical, contemporary, or interesting note about mathematics or its application.

Focus on Problem Solving

At the end of each chapter there is a Focus on Problem Solving, the purpose of which is to introduce the student to various successful problem-solving strategies. Each Focus consists of a problem and an appropriate strategy to solve the problem. Strategies such as guessing, trying to solve a simpler but similar problem, drawing a diagram, and looking for patterns are some of the techniques that are demonstrated.

Projects and Group Activities

The Projects and Group Activities feature can be used as extra credit or cooperative learning activities. The projects cover various aspects of mathematics including use of calculators, extended applications, and additional problem-solving strategies.

Chapter Summaries

At the end of each chapter there is a Chapter Summary that includes Key Words and Essential Rules that were covered in the chapter. These chapter summaries provide a single point of reference as the student prepares for a test.

Study Skills

The To the Student preface provides suggestions for using this text and approaches to creating good study habits. Students are referred to this preface at appropriate places in the text.

Computer Tutor

This state-of-the-art Tutor is a networkable, interactive, algorithmically driven software package. Features include full-color graphics, a glossary, extensive hints, animated solution steps, and a comprehensive class management system. Written by the authors, the Tutor and the text are in the same voice.

Glossary

A Glossary at the end of the book includes definitions of terms used in the text.

Margin Notes

There are three types of margin notes in the student text. *Point of Interest* notes feature interesting sidelights of the topic being discussed. The *Take Note* feature warns students that a procedure may be particularly involved or reminds students that there are certain checks of their work that should be performed. *Calculator Notes* provide suggestions for using a calculator in certain situations. In addition, there are *Instructor Notes* that are printed only in the Instructor's Annotated Edition. These notes provide suggestions for presenting the material or related material that can be used in class.

Index of Applications

The Index of Applications illustrates the power and scope of mathematics and its application. This may help some students see the benefits of mathematics as a tool that is used in everyday experiences.

EXERCISES

End-of-Section Exercises

Introductory Algebra: An Applied Approach contains more than 6000 exercises. At the end of each section there are exercise sets that are keyed to the corresponding

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learning objectives. The exercises are carefully developed to ensure that students can apply the concepts in the section to a variety of problem situations. Data Analysis exercises are identified by  . Calculator exercises are identified by  .

Applying the Concepts Exercises

The End-of-Section Exercises are followed by Applying the Concepts Exercises. These sections contain a variety of exercise types, including:

- challenge problems
- problems that require the student to determine if a statement is always true, sometimes true, or never true
- problems that ask students to determine incorrect procedures

Writing Exercises

Within the "Applying the Concepts Exercises," there are Writing Exercises denoted by  . These exercises ask students to write about a topic in the section or to research and report on a related topic.

Chapter Review Exercises

Review Exercises are found at the end of each chapter. These exercises are selected to help the student integrate all of the topics presented in the chapter. The answers to all Review Exercises are given in the answer section at the end of the book. Along with the answer, there is a reference to the objective that pertains to each exercise.

Chapter Test Exercises

The Chapter Test Exercises are designed to simulate a possible test of the material in the chapter. The answers to all Chapter Test Exercises are given in the answer section at the end of the book. Along with the answer, there is a reference to the objective that pertains to each exercise.

Cumulative Review Exercises

Cumulative Review Exercises, which appear at the end of each chapter (beginning with Chapter 2), help students maintain skills learned in previous chapters. The answers to all Cumulative Review Exercises are given in the answer section. Along with the answer, there is a reference to the objective that pertains to each exercise.

NEW TO THIS EDITION

The major change for this edition has been the integration of the application problems from Chapter 4 of the previous edition throughout the text. Chapter 3, Solving Equations, now contains a significant number of application problems that allow students to apply their equation-solving skills as they learn them. This helps connect the mathematics they are learning with its real-world applications.

Similar triangles and applications of similar triangles have been added to Chapter 6, Rational Expressions.

Problems on investment have been moved to Chapter 8, Systems of Equations.

We have added some problems that have too much data, thereby requiring the student to select the information needed to solve the problem.

For some exercises, not enough information is given to reach a single answer. Thus, there is more than one answer that satisfies the conditions of the problem.

The skill development exercises were thoroughly reviewed to ensure that there was adequate representation of various problem types. As a result of this review,

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we have changed or replaced some drill exercises to include problem types that were missing.

Approximately one-third of all the application problems were changed to reflect current data and trends. New application problems were added to demonstrate to students the variety of problems that require mathematical analysis.

Career notes were added to the chapter opener pages to illustrate to students the diverse ways in which mathematics is used in the workplace.

We have more than doubled the number of projects and group activities. Some of these projects have suggested Internet sites so that the student may continue to explore a topic.

In response to suggestions by users, the Chapter Review Exercises are no longer categorized by section. Thus there are no organizational clues to students as to the type of skill needed to solve an exercise. The answers to all Chapter Review Exercises are in the answer section. Along with the answer, there is a reference to the objective that pertains to each exercise.

SUPPLEMENTS FOR THE INSTRUCTOR

Instructor's Annotated Edition

The Instructor's Annotated Edition is an exact replica of the student text except that answers to all exercises are given in the text. Also, there are Instructor Notes in the margin that offer suggestions for presenting the material in that objective.

Instructor's Resource Manual with Chapter Tests

The Instructor's Resource Manual contains the printed Chapter Tests, which are the first of three sources of testing material. Eight printed tests—four free-response and four multiple-choice—are provided for each chapter. In addition, there are Cumulative Tests after Chapters 3, 6, 9, 11, and a Final Exam. The Instructor's Resource Manual also includes suggestions for course sequencing and outlines for the answers to the Writing Exercises.

Computerized Test Generator

The Computerized Test Generator is the second source of testing material. The database contains more than 2000 test items. The Test Generator is designed to provide an unlimited number of tests for each chapter, cumulative chapter tests, and a final exam. It is available for Windows and the Macintosh. Both versions also provide **on-line testing** and **gradebook** functions.

Printed Test Bank

The printed Test Bank, the third component of the testing material, is a printout of all items in the Computerized Test Generator. Instructors who do not have access to a computer can use the Test Bank to select items to include on a test being prepared by hand.

Solutions Manual

The Solutions Manual contains worked-out solutions for all end-of-section exercises, Chapter Review Exercises, Chapter Test Exercises, Cumulative Review Exercises, and the Final Exam.

SUPPLEMENTS FOR THE STUDENT

Student Solutions Manual

The Student Solutions Manual contains the complete solutions to all odd-numbered exercises in the text.

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Computer Tutor

The content of this interactive, state-of-the-art tutorial software was written by the authors and is in the same voice as the text. The Tutor supports every topic in the text. Problems are algorithmically generated; solution steps are animated; lessons and problems are presented in a colorful, lively manner, and an integrated classroom management system tracks and reports student performance. Features include:

- assessment
- free-response problems
- ability to repeat same problem type
- ability to print problems
- syllabus customization
- ability to bookmark

The Computer Tutor can be used in several ways: (1) to cover material the student missed because of absence; (2) to reinforce instruction on a concept that the student has not yet mastered; (3) to review material in preparation for exams.

The networkable Tutor is available on CD-ROM or floppy disk for Windows. There is also a version available for the Macintosh. The Tutor is free to any school upon adoption of this text; however, students can purchase a copy of the Tutor on floppy disk for home use.

Within each section of the book, a computer icon 

appears next to each objective. The icon serves as a reminder that there is a Computer Tutor lesson corresponding to that objective.

Videotapes

Within each section of the text, a videotape icon  appears next to an objective for which there is a corresponding video. The icon contains the reference number of the appropriate video. Each video topic is motivated through an application, and the necessary mathematics to solve that problem are then presented.

ACKNOWLEDGMENTS

We sincerely wish to thank the following reviewers, who reviewed the manuscript in various stages of development, for their valuable contributions:

Thomas Blackburn, *Northeastern Illinois University;* **Kim Brown,** *Tarrant County Junior College, NE;* **Floyd Fisher,** *Hillyer College, University of Hartford, CT;* **Ann L. Flamm,** *Reading Area Community College, PA;* **Mary Ellen Gallegos,** *San Francisco Community College, CA;* **Laura Hoye,** *Trident Technical College, SC;* **Michael A. Jones; Randy Leifson,** *Pierce College, WA;* **David Longshore,** *Victor Valley College, CA;* **Dr. Glenn Neiner,** *Red Rocks Community College, CO;* **Thomas M. O'Keefe,** *Bucks County Community College, PA;* **Leah C. Pierce,** *Chaffey College, CA;* **Bernard J. Piña,** *DABCC—New Mexico State University;* **Deana J. Richmond; Betty Jo Slozak,** *Wayne Community College, NC;* **Trudy Streilein,** *Harford Community College, MD.*

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To the Student

Take an active role in the learning process.

Many students feel that they will never understand math, while others appear to do very well with little effort. Oftentimes what makes the difference is that successful students take an active role in the learning process.

Do the homework.

Attend class regularly.

Participate in class.

Learning mathematics requires your *active* participation. Although doing homework is one way you can actively participate, it is not the only way. First, you must attend class regularly and become an active participant. Second, you must become actively involved with the textbook.

Introductory Algebra: An Applied Approach was written and designed with you in mind as a participant. Here are some suggestions on how to use the features of this textbook.

Use the features of the text.

There are 11 chapters in this text. Each chapter is divided into sections, and each section is subdivided into learning objectives. Each learning objective is labeled with a letter from A to D.

Read the objective statement.

Read the objective material.

Study the in-text examples.

First, read each objective statement carefully so you will understand the learning goal that is being presented. Next, read the objective material carefully, being sure to note each bold word. These words indicate important concepts that you should familiarize yourself with. Study carefully each in-text example (denoted by an orange arrow), noting the techniques and strategies used to solve the example.

Use the boxed examples.

1. **Study the example on the left.**
2. **Solve the You Try It example.**
3. **Check your work against the solution in the back of the book.**

You will then come to the key learning feature of this text, the *boxed examples*. These examples have been designed to assist you in a very specific way. Notice that in each example box, the example on the left is completely worked out and the "You Try It" example on the right is not. *You* are expected to work the right-hand example (in the space provided) in order to immediately test your understanding of the material you have just studied.

You should study the worked-out example carefully by working through each step presented. This allows you to focus on each step and reinforces the technique for solving that type of problem. You can then use the worked-out example as a model for solving similar problems.

Next, try to solve the "You Try It" example using the problem-solving techniques that you have just studied. When you have completed your solution, check your work by turning to the page in the Appendix where the complete solution can be found. The page number on which the solution appears is printed at the bottom of the example box in the right-hand corner. By checking your solution, you will know immediately whether or not you fully understand the skill you just studied.

Do the exercises.

Check your answers to the odd-numbered exercises.

When you have completed studying an objective, do the exercises in the exercise set that corresponds to that objective. The exercises are labeled with the same letter as the objective. Math is a subject that needs to be learned in small sections and practiced continually in order to be mastered. Doing all of the exercises in each exercise set will help you to master the problem-solving techniques necessary for success. As you work through the exercises for an objective, check your answers to the odd-numbered exercises with those in the back of the book.

Read the Chapter Summary.

Do the Chapter Review exercises.

After completing a chapter, read the Chapter Summary. This summary highlights the important topics covered in the chapter. Following the Chapter Summary are a Chapter Review, a Chapter Test, and a Cumulative Review (beginning with Chapter 2). Doing the review exercises is an important way of testing your understanding of the chapter. The answer to each review exercise is given at the back of the book. Each answer to the Chapter Test and Cumulative Review is followed

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Check your answers.

Restudy objectives you missed.

Do the Chapter Test.

Check your answers.

Restudy objectives you missed.

by a reference that tells which objective that exercise was taken from. For example, (4.2B) means Section 4.2, Objective B. After checking your answers, restudy any objective that you missed. It may be very helpful to retry some of the exercises for that objective to reinforce your problem-solving techniques.

The Chapter Test should be used to prepare for an exam. We suggest that you try the Chapter Test a few days before your actual exam. Take the test in a quiet place and try to complete the test in the same amount of time you will be allowed for your exam. When taking the Chapter Test, practice the strategies of successful test takers: 1) scan the entire test to get a feel for the questions; 2) read the directions carefully; 3) work the problems that are easiest for you first; and, perhaps most importantly, 4) try to stay calm.

When you have completed the Chapter Test, check your answers. If you missed a question, review the material in that objective and rework some of the exercises from that objective. This will strengthen your ability to perform the skills in that objective.

The Cumulative Review allows you to refresh the skills you have learned in previous chapters. This is very important in mathematics. By consistently reviewing previous material, you will retain the skills already learned as you build new ones.

Remember, to be successful: attend class regularly; read the textbook carefully; actively participate in class; work with your textbook using the "You Try It" examples for immediate feedback and reinforcement of each skill; do all of the homework assignments; review constantly; and work carefully.

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CHAPTER

1 Real Numbers



County agricultural agents work with farmers to help them produce better crops. Many agents are specialists in soils. They teach farmers about the physical and chemical characteristics of the soil, helping them to make the most productive use of their land. Agricultural agents must have a good understanding of percents in order to analyze the composition of nutrients in farmland. Their goal is to assist farmers in improving the quality and quantity of crop yields and in controlling diseases and pests.

Objectives

Section 1.1

To use inequality symbols with integers
To use opposites and absolute value

Section 1.2

To add integers
To subtract integers
To solve application problems

Section 1.3

To multiply integers
To divide integers
To solve application problems

Section 1.4

To write a rational number in simplest form and as a decimal
To convert between percents, fractions, and decimals
To add or subtract rational numbers
To multiply or divide rational numbers
To solve application problems

Section 1.5

To evaluate exponential expressions
To use the Order of Operations Agreement to simplify expressions



Early Egyptian Fractions

One of the earliest written documents of mathematics is the Rhind Papyrus. This tablet was found in Egypt in 1858, but it is estimated that the writings date back to 1650 B.C.

The Rhind Papyrus contains over 80 problems. A study of these problems has enabled mathematicians and scientists to understand some of the ways the early Egyptians used mathematics.

Evidence gained from the Papyrus shows that the Egyptian method of calculating with fractions was much different from the methods used today. All fractions were represented in terms of what are called "unit fractions." A unit fraction is a fraction in which the numerator is 1. This fraction was symbolized (using modern numbers) with a bar over the number. For example,

$$\overline{3} = \frac{1}{3} \qquad \overline{15} = \frac{1}{15}$$

The early Egyptians also tended to deal with powers of 2 (2, 4, 8, 16, ...). As a result, representing fractions with 2 in the numerator in terms of unit fractions was an important matter. The Rhind Papyrus has a table giving the equivalent unit fractions for all fractions with odd denominators from 5 to 101 and 2 as the numerator. Some of these are listed below.

$$\frac{2}{5} = \overline{3}\ \overline{15} \quad \left(\frac{2}{5} = \frac{1}{3} + \frac{1}{15}\right)$$

$$\frac{2}{7} = \overline{4}\ \overline{28}$$

$$\frac{2}{11} = \overline{6}\ \overline{66}$$

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1.1 Introduction to Integers

Objective A To use inequality symbols with integers

TAKE NOTE

We suggest you read "To the Student" on page xix. It provides an explanation of the organization of the chapters and lessons of this text.

It seems to be a human characteristic to group similar items. For instance, a biologist places similar animals in groups called *species*. Nutritionists classify foods according to *food groups*; for example, pasta, crackers, and rice are among the foods in the bread group.

Mathematicians place objects with similar properties in groups called *sets*. A **set** is a collection of objects. The objects in a set are called the **elements** of the set.

The **roster method** of writing sets encloses a list of the elements in braces. The set of sections within an orchestra is written {brass, percussion, string, woodwind}.

The numbers that we use to count objects, such as the number of students in a classroom or the number of horses on a ranch, are the *natural numbers*.

$$\textbf{Natural numbers} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$$

The three dots mean that the list of natural numbers continues on and on and that there is no largest natural number.

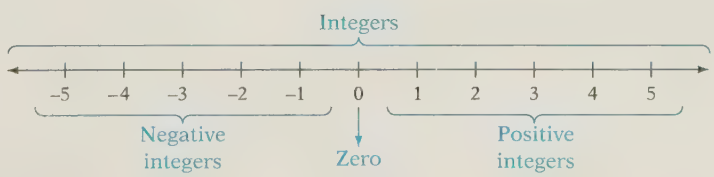
POINT OF INTEREST

The Alexandrian astronomer Ptolemy began using *omicron*, *o*, the first letter of the Greek word that means "nothing," as the symbol for zero in 150 A.D. It was not until the 13th century, however, that Fibonacci introduced 0 to the Western world as a placeholder so that we could distinguish, for example, 45 from 405.

The natural numbers alone do not provide all the numbers that are useful in applications. For instance, a meteorologist also needs the number zero and numbers below zero.

$$\textbf{Integers} = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$$

Each integer can be shown on a number line. The integers to the left of zero on the number line are called **negative integers.** The integers to the right of zero are called **positive integers,** or natural numbers. Zero is neither a positive nor a negative integer.



The **graph** of an integer is shown by placing a heavy dot on the number line directly above the number. The graphs of −3 and 4 are shown on the number line below.



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Consider the sentences below.

> The quarterback threw the football and the receiver caught *it*.
> A student purchased a computer and used *it* to write history papers.

In the first sentence, *it* is used to mean the football; in the second sentence, *it* means the computer. In language, the word *it* can stand for many different objects. Similarly, in mathematics, a letter of the alphabet can be used to stand for a number. Such a letter is called a **variable.** Variables are used in the following definition of inequality symbols.

POINT OF INTEREST

The symbols for "is less than" and "is greater than" were introduced by Thomas Harriot around 1630. Before that, ⌐ and ¬ were used for $>$ and $<$, respectively.

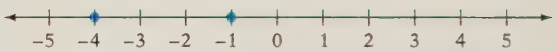
Inequality Symbols

If a and b are two numbers and a is to the left of b on the number line, then a **is less than** b. This is written $a < b$.

If a and b are two numbers and a is to the right of b on the number line, then a **is greater than** b. This is written $a > b$.

Negative 4 is less than negative 1.

$-4 < -1$



5 is greater than 0.

$5 > 0$



There are also inequality symbols for **is less than or equal to** ($\leq$) or **is greater than or equal to** ($\geq$).

$7 \leq 15$ 7 is less than or equal to 15.
This is true because $7 < 15$.

$6 \leq 6$ 6 is less than or equal to 6.
This is true because $6 = 6$.

POINT OF INTEREST

The Greek influence is again felt with the symbol $\in$ for "is an element of." Giuseppe Peano used this symbol in 1889 as an abbreviation for the Greek word for "is."

The symbol $\in$ means "is an element of." $2 \in B$ is read "2 is an element of set B."

Given $C = \{3, 5, 9\}$, then $3 \in C$, $5 \in C$, and $9 \in C$. $7 \notin C$ is read "7 is not an element of set C."

Example 1

Use the roster method to write the set of negative integers greater than or equal to -4.

Solution

$A = \{-4, -3, -2, -1\}$ • A set is designated by a capital letter.

You Try It 1

Use the roster method to write the set of positive integers less than 7.

Your solution

Solution on p. S1

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Example 2

Given $A = \{-6, -2, 0\}$, which elements of set A are less than or equal to -2?

Solution

Find the order relation between each element of set A and -2.

$-6 < -2$
$-2 = -2$
$0 > -2$

The elements -6 and -2 are less than or equal to -2.

You Try It 2

Given $B = \{-5, -1, 5\}$, which elements of set B are greater than -1?

Your solution

Solution on p. S1

Objective B To use opposites and absolute value

Two numbers that are the same distance from zero on the number line but are on opposite sides of zero are **opposite numbers,** or **opposites.** The opposite of a number is also called its **additive inverse.**

The opposite of 5 is -5.



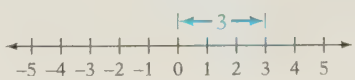
The opposite of -5 is 5.

The negative sign can be read "the opposite of."

$-(2) = -2$ The opposite of 2 is -2.
$-(-2) = 2$ The opposite of -2 is 2.

The **absolute value** of a number is its distance from zero on the number line. Therefore, the absolute value of a number is a positive number or zero. The symbol for absolute value is two vertical bars, $| \ |$.

The distance from 0 to 3 is 3. Therefore, the absolute value of 3 is 3.



$|3| = 3$

The distance from 0 to -3 is 3. Therefore, the absolute value of -3 is 3.



$|-3| = 3$

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POINT OF INTEREST

The definition of *absolute value* given in the box is written in what is called rhetorical style. That is, it is written without the use of variables. This is how *all* mathematics was written prior to the Renaissance. During that period from the 14th to the 16th century, the idea of expressing a variable symbolically was developed. In terms of that symbolism, the definition of absolute value is

$$|x| = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$$

Absolute Value

The absolute value of a positive number is the number itself. For example, $|9| = 9$.

The absolute value of zero is zero. $|0| = 0$

The absolute value of a negative number is the opposite of the negative number. For example, $|-7| = 7$.

➡ Evaluate: $-|-12|$

$-|-12| = -12$ • The absolute value sign does not affect the negative sign in front of the absolute value sign.

Example 3

Evaluate $|-4|$ and $-|-10|$.

Solution

$|-4| = 4$

$-|-10| = -10$

You Try It 3

Evaluate $|-5|$ and $-|-23|$.

Your solution

Example 4

Given $A = \{-12, 0, 4\}$, find the additive inverse of each element of set A.

Solution

$-(-12) = 12$

$-0 = 0$ • Zero is neither positive nor negative.

$-(4) = -4$

You Try It 4

Given $B = \{-11, 0, 8\}$, find the additive inverse of each element of set B.

Your solution

Example 5

Given $C = \{-17, 0, 14\}$, find the absolute value of each element of set C.

Solution

$|-17| = 17$

$|0| = 0$

$|14| = 14$

You Try It 5

Given $D = \{-37, 0, 29\}$, find the absolute value of each element of set D.

Your solution

Solutions on p. S1

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1.1 Exercises

Objective A

Place the correct symbol, $<$ or $>$, between the two numbers.

1. 8 -6 **2.** -14 16 **3.** -12 1 **4.** 35 28 **5.** 42 19

6. -42 27 **7.** 0 -31 **8.** -17 0 **9.** 53 -46 **10.** -27 -38

Answer True or False.

11. $-13 > 0$ **12.** $-20 > 3$ **13.** $12 > -31$ **14.** $9 > 7$ **15.** $-5 > -2$

16. $-44 > -21$ **17.** $-4 > -120$ **18.** $0 > -8$ **19.** $-1 \geq -1$ **20.** $-10 \leq -10$

For Exercises 21–26, use the roster method to write the set.

21. the natural numbers less than 9

22. the natural numbers less than or equal to 6

23. the positive integers less than or equal to 8

24. the positive integers less than 4

25. the negative integers greater than -7

26. the negative integers greater than or equal to -5

27. Given $A = \{-7, 0, 2, 5\}$, which elements of set A are greater than 2?

28. Given $B = \{-8, 0, 7, 15\}$, which elements of set B are greater than 7?

29. Given $D = \{-23, -18, -8, 0\}$, which elements of set D are less than -8?

30. Given $C = \{-33, -24, -10, 0\}$, which elements of set C are less than -10?

31. Given $E = \{-35, -13, 21, 37\}$, which elements of set E are greater than -10?

32. Given $F = \{-27, -14, 14, 27\}$, which elements of set F are greater than -15?

33. Given $B = \{-52, -46, 0, 39, 58\}$, which elements of set B are less than or equal to 0?

34. Given $A = \{-12, -9, 0, 12, 34\}$, which elements of set A are greater than or equal to 0?

35. Given $C = \{-23, -17, 0, 4, 29\}$, which elements of set C are greater than or equal to -17?

36. Given $D = \{-31, -12, 0, 11, 45\}$, which elements of set D are less than or equal to -12?

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37. Given that set A is the positive integers less than 10, which elements of set A are greater than or equal to 5?

38. Given that set B is the positive integers less than or equal to 12, which elements of set B are greater than 6?

39. Given that set D is the negative integers greater than or equal to -10, which elements of set D are less than -4?

40. Given that set C is the negative integers greater than -8, which elements of set C are less than or equal to -3?

Objective B

Find the additive inverse.

41. 4 **42.** 8 **43.** -9 **44.** -28 **45.** -36

Evaluate.

46. $-(-14)$ **47.** $-(-40)$ **48.** $-(77)$ **49.** $-(39)$ **50.** $-(-13)$

51. $|-74|$ **52.** $|-96|$ **53.** $-|-82|$ **54.** $-|-53|$ **55.** $-|81|$

Place the correct symbol, $<$ or $>$, between the values of the two numbers.

56. $|-83|$ $|58|$ **57.** $|22|$ $|-19|$ **58.** $|43|$ $|-52|$ **59.** $|-71|$ $|-92|$

60. $|-68|$ $|-42|$ **61.** $|12|$ $|-31|$ **62.** $|-45|$ $|-61|$ **63.** $|-28|$ $|43|$

64. Given $A = \{-8, -5, -2, 1, 3\}$ find
a. the opposite of each element of set A
b. the absolute value of each element of set A

65. Given $B = \{-11, -7, -3, 1, 5\}$, find
a. the opposite of each element of set B
b. the absolute value of each element of set B

APPLYING THE CONCEPTS

66. If x represents a negative integer, then $-x$ represents a _____ integer.

67. If x is an integer, is the inequality $|x| < -3$ always true, sometimes true, or never true?

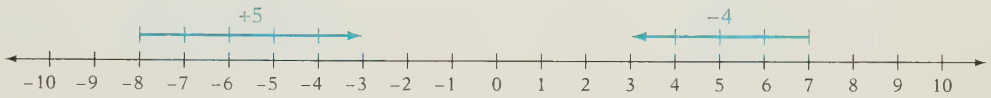
68. In your own words, explain the meaning of the absolute value of a number and the additive inverse of a number.

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1.2 Addition and Subtraction of Integers

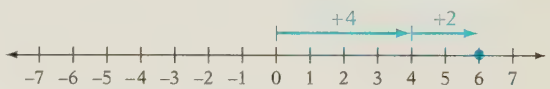
Objective A To add integers

A number can be represented anywhere along the number line by an arrow. A positive number is represented by an arrow pointing to the right, and a negative number is represented by an arrow pointing to the left. The size of the number is represented by the length of the arrow.

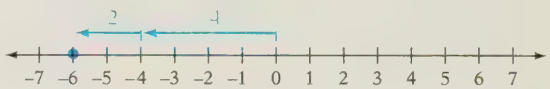


Addition is the process of finding the total of two numbers. The numbers being added are called **addends.** The total is called the **sum.** Addition of integers can be shown on the number line. To add integers, start at zero and draw an arrow representing the first number. At the tip of the first arrow, draw a second arrow representing the second number. The sum is below the tip of the second arrow.

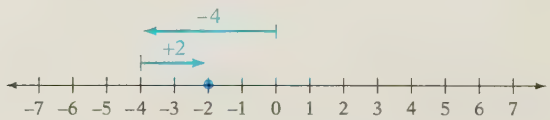
$4 + 2 = 6$



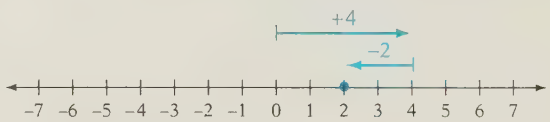
$-4 + (-2) = -6$



$-4 + 2 = -2$



$4 + (-2) = 2$



The pattern for addition shown on the number lines above is summarized in the following rules for adding integers.

Addition of Integers

To add two numbers with the same sign, add the absolute values of the numbers. Then attach the sign of the addends.

To add two numbers with different signs, find the absolute value of each number. Subtract the smaller of the two numbers from the larger. Then attach the sign of the number with the larger absolute value.

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➡ Add: $-12 + (-26)$

$-12 + (-26) = -38$

- The signs are the same. Add the absolute values of the numbers (12 + 26). Attach the sign of the addends.

➡ Add: $-19 + 8$

$|-19| = 19$

$|8| = 8$

- The signs are different. Find the absolute value of each number.

$19 - 8 = 11$

- Subtract the smaller number from the larger.

$-19 + 8 = -11$

- Attach the sign of the number with the larger absolute value. Because $|-19| > |8|$, use the sign of -19.

➡ Add: $-23 + 47 + (-18) + (-5)$

$-23 + 47 + (-18) + (-5)$
$= 24 + (-18) + (-5)$
$= 6 + (-5)$
$= 1$

- To add more than two numbers, add the first two numbers. Then add the sum to the third number. Continue until all the numbers are added.

TAKE NOTE

"To the Student" on page xix explains how best to use the boxed examples in this text, such as the one below.

Example 1
Add: $-52 + (-39)$

Solution
$-52 + (-39) = -91$

You Try It 1
Add: $100 + (-43)$

Your solution

Example 2
Add: $37 + (-52) + (-21) + (-7)$

Solution
$37 + (-52) + (-21) + (-7)$
$= -15 + (-21) + (-7)$
$= -36 + (-7)$
$= -43$

You Try It 2
Add: $-51 + 42 + 17 + (-102)$

Your solution

Solutions on p. S1

Objective B To subtract integers

Look at the two expressions below and note that each expression equals the same number.

$8 - 3 = 5$ 8 minus 3 is 5.
$8 + (-3) = 5$ 8 plus the opposite of 3 is 5.

This example suggests that to subtract two numbers, we add the opposite of the second number to the first number.

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First number	−	second number	=	First number	+	the opposite of the second number	
40	−	60	=	40	+	(−60)	= −20
−40	−	60	=	−40	+	(−60)	= −100
−40	−	(−60)	=	−40	+	60	= 20
40	−	(−60)	=	40	+	60	= 100

➡ Subtract: $-21 - (-40)$

Change this sign to plus.

$$-21 - (-40) = -21 + 40 = 19$$

Change −40 to the opposite of −40.

• Rewrite each subtraction as addition of the opposite. Then add.

➡ Subtract: $15 - 51$

Change this sign to plus.

$$15 - 51 = 15 + (-51) = -36$$

Change 51 to the opposite of 51.

• Rewrite each subtraction as addition of the opposite. Then add.

➡ Subtract: $-12 - (-21) - 15$

$$\begin{aligned} -12 - (-21) - 15 &= -12 + 21 + (-15) \\ &= 9 + (-15) \\ &= -6 \end{aligned}$$

• Rewrite each subtraction as addition of the opposite. Then add.

Example 3 Subtract: $-11 - 15$

Solution
$$\begin{aligned} -11 - 15 &= -11 + (-15) \\ &= -26 \end{aligned}$$

You Try It 3 Subtract: $19 - (-32)$

Your solution

Example 4 Subtract: $-14 - 18 - (-21) - 4$

Solution
$$\begin{aligned} &-14 - 18 - (-21) - 4 \\ &= -14 + (-18) + 21 + (-4) \\ &= -32 + 21 + (-4) \\ &= -11 + (-4) \\ &= -15 \end{aligned}$$

You Try It 4 Subtract: $-9 - (-12) - 17 - 4$

Your solution

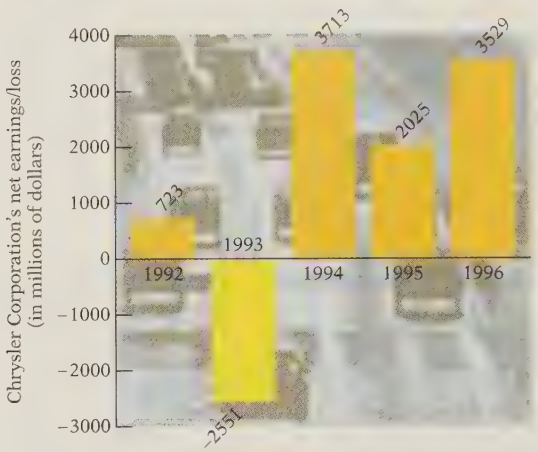
Solutions on p. S1

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Objective C To solve application problems

Positive and negative numbers are used to express the profitability of a company. A profit is recorded as a positive number; a loss is recorded as a negative number.

The bar graph below shows the net earnings or loss for Chrysler Corporation for the years 1992 to 1996. Calculate the total net earnings for Chrysler Corporation for the years 1992 to 1994.



Source: Chrysler Corporation's 1996 Report to Shareholders

Strategy

To determine the total net earnings, add the profits and losses for the years 1992, 1993, and 1994.

Solution

$$723 + (-2551) + 3713 = -1828 + 3713$$
$$= 1885$$

The total net earnings for 1992 to 1994 were \$1885 million.

Example 5

The average temperature on Mercury's sunlit side is 950°F. The average temperature on Mercury's dark side is −346°F. Find the difference between these two average temperatures.

Strategy

To find the difference, subtract the average temperature on the dark side (−346) from the average temperature on the sunlit side (950).

Solution

$$950 - (-346) = 950 + 346$$
$$= 1296$$

The difference between these average temperatures is 1296°F.

You Try It 5

Mar's average daytime temperature is −17°F. Mar's average nighttime temperature is −130°F. Find the difference between these two average temperatures.

Your strategy

Your solution

Solution on p. S1

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1.2 Exercises

TAKE NOTE

"To the Student" on page xix discusses the exercise sets in this textbook.

Objective A

Add.

1. $-3 + (-8)$ **2.** $-6 + (-9)$ **3.** $-8 + 3$ **4.** $-9 + 2$

5. $-3 + (-80)$ **6.** $-12 + (-1)$ **7.** $-23 + (-23)$ **8.** $-12 + (-12)$

9. $16 + (-16)$ **10.** $-17 + 17$ **11.** $48 + (-53)$ **12.** $19 + (-41)$

13. $-17 + (-3) + 29$ **14.** $13 + 62 + (-38)$ **15.** $-3 + (-8) + 12$ **16.** $-27 + (-42) + (-18)$

17. $13 + (-22) + 4 + (-5)$ **18.** $-14 + (-3) + 7 + (-21)$

19. $-22 + 20 + 2 + (-18)$ **20.** $-6 + (-8) + 14 + (-4)$

21. $-16 + (-17) + (-18) + 10$ **22.** $-25 + (-31) + 24 + 19$

23. $26 + (-15) + (-11) + (-12)$ **24.** $-32 + 40 + (-8) + (-19)$

25. $-17 + (-18) + 45 + (-10)$ **26.** $23 + (-15) + 9 + (-15)$

27. $46 + (-17) + (-13) + (-50)$ **28.** $-37 + (-17) + (-12) + (-15)$

29. $-14 + (-15) + (-11) + 40$ **30.** $28 + (-19) + (-8) + (-1)$

31. $-23 + (-22) + (-21) + 5$ **32.** $-31 + 9 + (-16) + (-15)$

33. $72 + (-22) + (-14) + (-9)$ **34.** $-22 + (-17) + 58 + 29$

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Objective B

Subtract.

35. $16 - 8$

36. $12 - 3$

37. $7 - 14$

38. $6 - 9$

39. $-7 - 2$

40. $-9 - 4$

41. $7 - (-2)$

42. $3 - (-4)$

43. $-6 - (-3)$

44. $-4 - (-2)$

45. $6 - (-12)$

46. $-12 - 16$

47. $-4 - 3 - 2$

48. $4 - 5 - 12$

49. $12 - (-7) - 8$

50. $-12 - (-3) - (-15)$

51. $-19 - (-19) - 18$

52. $-8 - (-8) - 14$

53. $-17 - (-8) - (-9)$

54. $7 - 8 - (-1)$

55. $-30 - (-65) - 29 - 4$

56. $42 - (-82) - 65 - 7$

57. $-16 - 47 - 63 - 12$

58. $42 - (-30) - 65 - (-11)$

59. $-47 - (-67) - 13 - 15$

60. $-18 - 49 - (-84) - 27$

61. $-19 - 17 - (-36) - 12$

62. $48 - 19 - 29 - 51$

63. $21 - (-14) - 43 - 12$

64. $17 - (-17) - 14 - 21$

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Objective C Application Problems

65. The temperature at which mercury boils is 360°C. Mercury freezes at −39°C. Find the difference between the temperature at which mercury boils and the temperature at which it freezes.

66. The temperature at which radon boils is −62°C. Radon freezes at −71°C. Find the difference between the temperature at which radon boils and the temperature at which it freezes.

The elevation, or height, of places on the earth is measured in relation to sea level, or the average level of the ocean's surface. The table below shows height above sea level as a positive number and depth below sea level as a negative number. Use the table for Exercises 67 to 70.

Continent	*Highest Elevation (in meters)*		*Lowest Elevation (in meters)*	
Africa	Mt. Kilimanjaro	5895	Qattara Depression	−133
Asia	Mt. Everest	8848	Dead Sea	−400
Europe	Mt. Elbrus	5634	Caspian Sea	−28
America	Mt. Aconcagua	6960	Death Valley	−86

67. Find the difference in elevation between Mt. Aconcagua and Death Valley.

68. What is the difference in elevation between Mt. Kilimanjaro and the Qattara Depression?

69. For which continent shown is the difference between the highest and lowest elevations greatest?

70. For which continent shown is the difference between the highest and lowest elevations smallest?

The table at the right shows the boiling point and the melting point in degrees Celsius of three chemical elements. Use this table for Exercises 71 and 72.

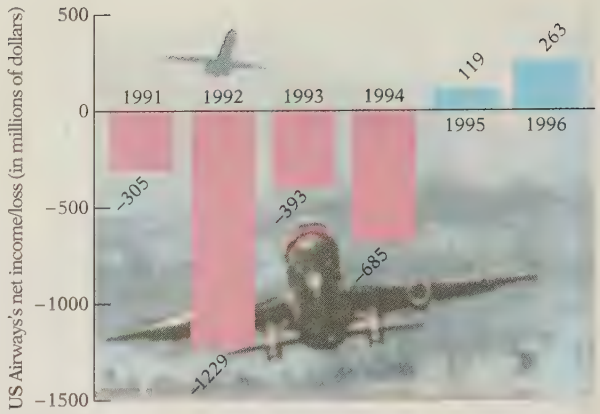
Chemical Element	*Boiling Point*	*Melting Point*
Carbon	4827	−355
Radon	−62	−71
Xenon	−107	−112

71. Find the difference between the boiling point and the melting point of carbon.

72. Find the difference between the boiling point and the melting point of xenon.

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The graph at the right shows the net income or loss for US Airways for the years 1991 to 1996. Use this graph for Exercises 73 to 75.



Sources: Transportation Department, US Airways

73. Determine the total net loss for US Airways for the years 1994 to 1996.

74. Calculate the total net loss for US Airways for the years 1991 to 1996.

75. Find the difference between the net losses in 1994 and in 1992.

A meteorologist may report a wind-chill temperature. This is the equivalent temperature, including the effects of wind and temperature, that a person would feel in calm air conditions. The table below gives the wind-chill temperature for various wind speeds and temperature. For instance, when the temperature is 5°F and the wind is blowing at 15 mph, the wind-chill temperature is −25°F. Use this table for Exercises 76 and 77.

Wind-Chill Factors

Wind Speed (mph)	*Thermometer Reading (degrees Fahrenheit)*																
	35	**30**	**25**	**20**	**15**	**10**	**5**	**0**	**−5**	**−10**	**−15**	**−20**	**−25**	**−30**	**−35**	**−40**	**−45**
5	33	27	21	19	12	7	0	−5	−10	−15	−21	−26	−31	−36	−42	−47	−52
10	22	16	10	3	−3	−9	−15	−22	−27	−34	−40	−46	−52	−58	−64	−71	−77
15	16	9	2	−5	−11	−18	−25	−31	−38	−45	−51	−58	−65	−72	−78	−85	−92
20	12	4	−3	−10	−17	−24	−31	−39	−46	−53	−60	−67	−74	−81	−88	−95	−103
25	8	1	−7	−15	−22	−29	−36	−44	−51	−59	−66	−74	−81	−88	−96	−103	−110
30	6	−2	−10	−18	−25	−33	−41	−49	−56	−64	−71	−79	−86	−93	−101	−109	−116
35	4	−4	−12	−20	−27	−35	−43	−52	−58	−67	−74	−82	−89	−97	−105	−113	−120
40	3	−5	−13	−21	−29	−37	−45	−53	−60	−69	−76	−84	−92	−100	−107	−115	−123
45	2	−6	−14	−22	−30	−38	−46	−54	−62	−70	−78	−85	−93	−102	−109	−117	−125

76. When the thermometer reading is −5°F, what is the difference between the wind-chill factor when the wind is blowing 10 mph and when the wind is blowing 30 mph?

77. When the thermometer reading is −20°F, what is the difference between the wind-chill factor when the wind is blowing 15 mph and when the wind is blowing 25 mph?

APPLYING THE CONCEPTS

78. If a and b are integers, is the expression $|a + b| = |a| + |b|$ always true, sometimes true, or never true?

79. Is the difference between two integers always smaller than either one of the numbers in the difference? If not, give an example for which the difference between two integers is greater than either integer.

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1.3 Multiplication and Division of Integers

Objective A To multiply integers

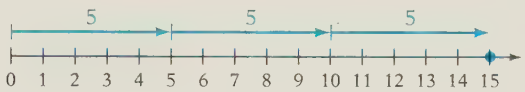
POINT OF INTEREST

The cross × was first used as a symbol for multiplication in 1631 in a book titled *The Key to Mathematics*. Also in that year, another book, *Practice of the Analytical Art*, advocated the use of a dot to indicate multiplication.

Several different symbols are used to indicate multiplication. The numbers being multiplied are called **factors;** for instance, 3 and 2 are factors in each of the examples at the right. The result is called the **product.** Note that when parentheses are used and there is no arithmetic symbol, the operation is multiplication.

$3 \times 2 = 6$
$3 \cdot 2 = 6$
$(3)(2) = 6$
$3(2) = 6$
$(3)2 = 6$

Multiplication is repeated addition of the same number. The product 3×5 is shown on the number line below.



5 is added 3 times.

$$3 \times 5 = 5 + 5 + 5 = 15$$

Now consider the product of a positive and a negative number.

−5 is added 3 times.

$$3(-5) = (-5) + (-5) + (-5) = -15$$

This suggests that the product of a positive number and a negative number is negative. Here are a few more examples.

$4(-7) = -28$ $\quad -6 \cdot 7 = -42$ $\quad (-8)7 = -56$

To find the product of two negative numbers, look at the pattern at the right. As −5 multiplies a sequence of decreasing integers, the products increase by 5.

The pattern can be continued by requiring that the product of two negative numbers be positive.

These numbers decrease by 1. ↓ ↓ These numbers increase by 5.

$-5 \times 3 = -15$
$-5 \times 2 = -10$
$-5 \times 1 = -5$
$-5 \times 0 = 0$
$-5 \times (-1) = 5$
$-5 \times (-2) = 10$
$-5 \times (-3) = 15$

Multiplication of Integers

To multiply two numbers with the same sign, multiply the absolute values of the numbers. The product is positive.

To multiply two numbers with different signs, multiply the absolute values of the number. The product is negative.

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➡ Multiply: $-2(5)(-7)(-4)$

$$-2(5)(-7)(-4) = -10(-7)(-4)$$
$$= 70(-4) = -280$$

• To multiply more than two numbers, multiply the first two. Then multiply the product by the third number. Continue until all the numbers are multiplied.

Consider the products shown at the right. Note that when there is an even number of negative factors, the product is positive. When there is an odd number of negative factors, the product is negative.

$$(-3)(-5) = 15$$
$$(-2)(-5)(-6) = -60$$
$$(-4)(-3)(-5)(-7) = 420$$
$$(-3)(-3)(-5)(-4)(-5) = -900$$
$$(-6)(-3)(-4)(-2)(-10)(-5) = 7200$$

This idea can be summarized by the following useful rule: **The product of an even number of negative factors is positive; the product of an odd number of negative factors is negative.**

Example 1 Multiply: $(-3)4(-5)$

Solution $(-3)4(-5) = (-12)(-5) = 60$

You Try It 1 Multiply: $8(-9)10$

Your solution

Example 2 Multiply: $12(-4)(-3)(-5)$

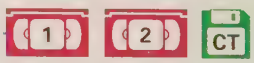
Solution $12(-4)(-3)(-5) = (-48)(-3)(-5)$
$= 144(-5) = -720$

You Try It 2 Multiply: $(-2)3(-8)7$

Your solution

Solutions on p. S1

Objective B To divide integers



For every division problem there is a related multiplication problem.

$$\underbrace{\frac{8}{2} = 4}_{\text{division}} \quad \text{because} \quad \underbrace{4 \cdot 2 = 8.}_{\text{related multiplication}}$$

This fact and the rules for multiplying integers can be used to illustrate the rules for dividing integers.

Note in the following examples that the quotient of two numbers with the same sign is positive.

$$\frac{12}{3} = 4 \text{ because } 4 \cdot 3 = 12. \qquad \frac{-12}{-3} = 4 \text{ because } 4(-3) = -12.$$

The next two examples illustrate that the quotient of two numbers with different signs is negative.

$$\frac{12}{-3} = -4 \text{ because } (-4)(-3) = 12. \qquad \frac{-12}{3} = -4 \text{ because } (-4)3 = -12.$$

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Division of Integers

To divide two numbers with the same sign, divide the absolute values of the numbers. The quotient is positive.

To divide two numbers with different signs, divide the absolute values of the numbers. The quotient is negative.

➡ Simplify: $-\frac{-56}{7}$

$$-\frac{-56}{7} = -\left(\frac{-56}{7}\right) = -(-8) = 8$$

The properties of division are stated below. In these statements, the symbol $\neq$ is read "is not equal to."

Properties of Zero and One in Division

If $a \neq 0$, $\frac{0}{a} = 0$.	Zero divided by any number other than zero is zero.
If $a \neq 0$, $\frac{a}{a} = 1$.	Any number other than zero divided by itself is one.
$\frac{a}{1} = a$.	A number divided by one is the number.
$\frac{a}{0}$ is undefined.	Division by zero is not defined.

The fact that $\frac{-12}{3} = -4$, $\frac{12}{-3} = -4$, and $-\frac{12}{3} = -4$ suggests the following rule.

If a and b are integers, and $b \neq 0$, then $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.

Example 3 Divide: $(-120) \div (-8)$

Solution $(-120) \div (-8) = 15$

You Try It 3 Divide: $(-135) \div (-9)$

Your solution

Example 4 Divide: $\frac{95}{-5}$

Solution $\frac{95}{-5} = -19$

You Try It 4 Divide: $\frac{-72}{4}$

Your solution

Example 5 Simplify: $-\frac{-81}{3}$

Solution $-\frac{-81}{3} = -(-27) = 27$

You Try It 5 Simplify: $-\frac{36}{-12}$

Your solution

Solutions on p. S1

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Objective C To solve application problems

In many courses, your course grade depends on the *average* of all your test scores. You compute the average by calculating the sum of all your test scores and then dividing that result by the number of tests. Statisticians call this average an **arithmetic mean.** Besides its application to finding the average of your test scores, the arithmetic mean is used in many other situations.

Stock market analysts calculate the **moving average** of a stock. This is the arithmetic mean of the changes in the value of a stock for a given number of days. To illustrate the procedure, we will calculate the 5-day moving average of a stock. In actual practice, a stock market analyst may use 15 days, 30 days, or some other number.

The table below shows the amount of increase or decrease, in cents, in the closing price of a stock for a 10-day period.

Day 1	*Day 2*	*Day 3*	*Day 4*	*Day 5*	*Day 6*	*Day 7*	*Day 8*	*Day 9*	*Day 10*
+50	−175	+225	0	−275	−75	−50	+50	−475	−50

To calculate the 5-day moving average of this stock, determine the average of the stock for days 1 through 5, days 2 through 6, days 3 through 7, and so on.

Days 1–5	*Days 2–6*	*Days 3–7*	*Days 4–8*	*Days 5–9*	*Days 6–10*
+50	175	+225	0	−275	−75
175	+225	0	−275	−75	−50
+225	0	−275	−75	−50	+50
0	−275	−75	−50	+50	−475
−275	−75	−50	+50	−475	−50
Sum = −175	Sum = −300	Sum = −175	Sum = −350	Sum = −825	Sum = −600
Av = $\frac{-175}{5}$ = −35	Av = $\frac{-300}{5}$ = −60	Av = $\frac{-175}{5}$ = −35	Av = $\frac{-350}{5}$ = −70	Av = $\frac{-825}{5}$ = −165	Av = $\frac{-600}{5}$ = −120

The 5-day moving average is the list of means: −35, −60, −35, −70, −165, and −120. If the list tends to increase, the price of the stock is showing an upward trend; if it decreases, the price of the stock is showing a downward trend. These trends help an analyst recommend stocks.

Example 6

The daily high temperatures (in degrees Celsius) for six days in Anchorage, Alaska, were −14°, 3°, 0°, −8°, 2°, and −1°. Find the average daily high temperature.

Strategy

To find the average daily high temperature:

- Add the six temperature readings.
- Divide the sum by 6.

Solution

$-14 + 3 + 0 + (-8) + 2 + (-1) = -18$

$-18 \div 6 = -3$

The average daily high temperature was 3°C.

You Try It 6

The daily low temperatures (in degrees Celsius) during one week were recorded as −6°, −7°, 0°, −5°, −8°, −1°, and −1°. Find the average daily low temperature.

Your strategy

Your solution

Solution on p. S1

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1.3 Exercises

Objective A

Multiply.

1. $(14)3$ **2.** $17(6)$ **3.** $-7 \cdot 4$ **4.** $-8 \cdot 7$ **5.** $(-12)(-5)$ **6.** $(-13)(-9)$

7. $-11(23)$ **8.** $-8(21)$ **9.** $(-17)14$ **10.** $(-15)12$ **11.** $6(-19)$ **12.** $17(-13)$

13. $7(5)(-3)$ **14.** $(-3)(-2)8$ **15.** $9(-7)(-4)$ **16.** $(-2)(6)(-4)$

17. $16(-3)5$ **18.** $20(-4)3$ **19.** $-4(-3)8$ **20.** $-5(-9)6$

21. $-3(-8)(-9)$ **22.** $-7(-6)(-5)$ **23.** $(-9)7(5)$ **24.** $(-8)7(10)$

25. $7(-2)(5)(-6)$ **26.** $(-3)7(-2)8$ **27.** $-9(-4)(-8)(-10)$ **28.** $-11(-3)(-5)(-2)$

29. $7(9)(-11)4$ **30.** $-12(-4)7(-2)$ **31.** $(-14)9(-11)0$ **32.** $(-13)(15)(-19)0$

Objective B

Divide.

33. $12 \div (-6)$ **34.** $18 \div (-3)$ **35.** $(-72) \div (-9)$ **36.** $(-64) \div (-8)$ **37.** $-42 \div 6$

38. $(-56) \div 8$ **39.** $(-144) \div 12$ **40.** $(-93) \div (-3)$ **41.** $48 \div (-8)$ **42.** $57 \div (-3)$

43. $\dfrac{-49}{7}$ **44.** $\dfrac{-45}{5}$ **45.** $\dfrac{-44}{-4}$ **46.** $\dfrac{-36}{-9}$ **47.** $\dfrac{98}{-7}$

48. $\dfrac{85}{-5}$ **49.** $-\dfrac{-120}{8}$ **50.** $-\dfrac{-72}{4}$ **51.** $-\dfrac{-80}{-5}$ **52.** $-\dfrac{-114}{-6}$

53. $0 \div (-9)$ **54.** $0 \div (-14)$ **55.** $\dfrac{-261}{9}$ **56.** $\dfrac{-128}{4}$ **57.** $9 \div 0$

58. $(-21) \div 0$ **59.** $\dfrac{132}{-12}$ **60.** $\dfrac{250}{-25}$ **61.** $\dfrac{0}{0}$ **62.** $\dfrac{-58}{0}$

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Objective C Application Problems

63. The high temperatures for a six-day period in Barrow, Alaska, were −23°F, −29°F, −21°F, −28°F, −28°F, and −27°F. Calculate the average daily high temperature.

64. The low temperatures for a ten-day period in a midwestern city were −4°F, −9°F, −5°F, −2°F, 4°F, −1°F, −1°F, −2°F, −2°F, and 2°F. Calculate the average daily low temperature for this city.

65. The value of a share of McDonald's stock on April 4, 1997, was $48.50. The table below shows the amount of increase or decrease, to the nearest 25 cents, from the April 4 closing price of the stock for a 10-day period. Calculate the five-day moving average for this stock.

Day 1	*Day 2*	*Day 3*	*Day 4*	*Day 5*	*Day 6*	*Day 7*	*Day 8*	*Day 9*	*Day 10*
−25	50	100	−25	−50	−125	75	50	125	−50

66. The value of a share of Disney stock on April 4, 1997, was $72.625. The table below shows the amount of increase or decrease, to the nearest 25 cents, from the April 4 closing price of the stock for a 10-day period. Calculate the five-day moving average for this stock.

Day 1	*Day 2*	*Day 3*	*Day 4*	*Day 5*	*Day 6*	*Day 7*	*Day 8*	*Day 9*	*Day 10*
100	0	25	25	25	−175	−125	275	175	25

67. To discourage random guessing on a multiple-choice exam, a professor assigns 5 points for a correct answer, −2 points for an incorrect answer, and 0 points for leaving the question blank. What is the score for a student who had 20 correct answers, 13 incorrect answers, and left 7 questions blank?

68. To discourage random guessing on a multiple-choice exam, a professor assigns 7 points for a correct answer, −3 points for an incorrect answer, and −1 point for leaving the question blank. What is the score for a student who had 17 correct answers, 8 incorrect answers, and left 2 questions blank?

APPLYING THE CONCEPTS

69. If $x \in \{-6, -2, 7\}$, for which value of x does the expression $-3x$ have the greatest value?

70. Explain why $0 \div 0$ is not defined.

71. If $-4x$ equals a positive integer, is x a positive or a negative integer? Explain your answer.

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1.4 Operations with Rational Numbers

Objective A — To write a rational number in simplest form and as a decimal



POINT OF INTEREST

As early as 630 A.D. the Hindu mathematician Brahmagupta wrote a fraction as one number over another separated by a space. The Arab mathematician al Hassar (around 1050 A.D.) was the first to show a fraction with the horizontal bar separating the numerator and denominator.

A *rational number* is the quotient of two integers. A rational number written in this way is commonly called a fraction. Here are some examples of rational numbers.

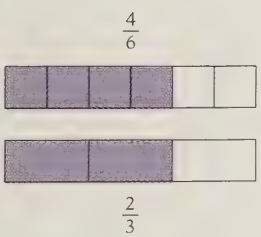
$$\frac{3}{4}, \quad \frac{-4}{9}, \quad \frac{15}{-4}, \quad \frac{8}{1}, \quad -\frac{5}{6}$$

Rational Numbers

A **rational number** is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Because an integer can be written as the quotient of the integer and 1, every integer is a rational number. For instance,

$$\frac{6}{1} = 6 \qquad \frac{8}{1} = -8$$



A fraction is in **simplest form** when there are no common factors in the numerator and the denominator. The fractions $\frac{4}{6}$ and $\frac{2}{3}$ are equivalent fractions because they represent the same part of a whole. However, the fraction $\frac{2}{3}$ is in simplest form because there are no common factors (other than 1) in the numerator and denominator.

To write a fraction in simplest form, eliminate the common factors from the numerator and denominator by using the fact that $1 \cdot \frac{a}{b} = \frac{a}{b}$.

$$\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{2} \cdot \frac{2}{3} = 1 \cdot \frac{2}{3} = \frac{2}{3}$$

The process of eliminating common factors is usually written as shown at the right.

$$\frac{4}{6} = \frac{\overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot 3} = \frac{2}{3}$$

If you have difficulty determining the common factors, write the numerator and denominator in terms of prime factors. (Recall that a prime number is a number divisible only by itself and 1. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.)

➡ Write $\frac{18}{30}$ in simplest form.

$$\frac{18}{30} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 5} = \frac{3}{5}$$

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A number written in **decimal notation** is also a rational number.

three tenths $0.3 = \frac{3}{10}$ forty-three thousandths $0.043 = \frac{43}{1000}$

A rational number written as a fraction can be written in decimal notation by dividing the numerator of the fraction by the denominator. Think of the fraction bar as meaning "divided by."

TAKE NOTE

Think of the fraction bar as "divided by." $\frac{5}{8}$ is 5 divided by 8.

➡ Write $\frac{5}{8}$ as a decimal.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

- Divide the numerator, 5, by the denominator, 8.
- Dividing the numerator by the denominator resulted in a remainder of 0. The decimal 0.625 is called a terminating decimal.

$\frac{5}{8} = 0.625$

➡ Write $\frac{4}{11}$ in decimal notation.

$$\begin{array}{r} 0.3636 \\ 11\overline{)4.0000} \\ -33 \\ \hline 70 \\ -66 \\ \hline 40 \\ -33 \\ \hline 70 \\ -66 \\ \hline 4 \end{array}$$

- Divide the numerator, 4, by the denominator, 11.
- No matter how long we continue to divide, the remainder is never zero. The decimal 0.36 is a repeating decimal. The bar over the 36 indicates that these digits repeat.

$\frac{4}{11} = 0.\overline{36}$

TAKE NOTE

Rational numbers are fractions such as $-\frac{6}{7}$ or $\frac{10}{3}$ where the numerator and denominator are integers. Rational numbers are also represented by repeating decimals, for instance 0.25767676..., or terminating decimals such as 1.73. An irrational number is neither a repeating decimal nor a terminating decimal. For instance 2.45445444544445... is an irrational number.

Every rational number can be written as a terminating or a repeating decimal. Some numbers—for example, $\sqrt{7}$ and π—have decimal representations that never terminate or repeat. These numbers are called **irrational numbers.**

$$\sqrt{7} \approx 2.6457513\ldots \quad \pi \approx 3.1415926\ldots$$

The rational numbers and the irrational numbers taken together are called the **real numbers.**

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Example 1 Write $\frac{3}{20}$ as a decimal.

Solution $\frac{3}{20} = 3 \div 20 = 0.15$

You Try It 1 Write $\frac{4}{9}$ as a decimal. Place a bar over the repeating digits of the decimal.

Your solution

Solution on p. S1

Objective B To convert between percents, fractions, and decimals

"A population growth rate of 3%," "a manufacturer's discount of 25%," and "an 8% increase in pay" are typical examples of the many ways in which percent is used in applied problems. **Percent** means "parts of 100." Thus 27% means 27 parts of 100.

In applied problems involving a percent, it may be necessary to rewrite a percent as a fraction or decimal, or to rewrite a fraction or decimal as a percent.

To write a percent as a fraction, remove the percent sign and multiply by $\frac{1}{100}$.

$$27\% = 27\left(\frac{1}{100}\right) = \frac{27}{100}$$

To write a percent as a decimal, remove the percent sign and multiply by 0.01.

$$33\% = 33(0.01) = 0.33$$

Move the decimal point two places to the left. Then remove the percent sign.

A fraction or decimal can be written as a percent by multiplying by 100%. For example, $\frac{5}{8}$ is changed to a percent as follows.

$$\frac{5}{8} = \frac{5}{8}(100\%) = \frac{500}{8}\% = 62.5\%, \quad \text{or} \quad 62\frac{1}{2}\%$$

To write a decimal as a percent, multiply by 100%.

$$0.82 = 0.82(100\%) = 82\%$$

Move the decimal point two places to the right. Then write the percent sign.

Example 2
Write 130% as a fraction and as a decimal.

Solution

$$130\% = 130\left(\frac{1}{100}\right) = \frac{130}{100} = \frac{13}{10}$$

$$130\% = 130(0.01) = 1.30$$

You Try It 2
Write 125% as a fraction and as a decimal.

Your solution

Solution on p. S1

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Example 3

Write $33\frac{1}{3}\%$ as a fraction.

Solution

$$33\frac{1}{3}\% = 33\frac{1}{3}\left(\frac{1}{100}\right) = \frac{100}{3}\left(\frac{1}{100}\right) = \frac{1}{3}$$

You Try It 3

Write $16\frac{2}{3}\%$ as a fraction.

Your solution

Example 4

Write $\frac{5}{6}$ as a percent.

Solution

$$\frac{5}{6} = \frac{5}{6}(100\%) = \frac{500}{6}\% = 83\frac{1}{3}\%$$

You Try It 4

Write $\frac{9}{16}$ as a percent.

Your solution

Example 5

Write 0.027 as a percent.

Solution

$0.027 = 0.027(100\%) = 2.7\%$

You Try It 5

Write 0.043 as a percent.

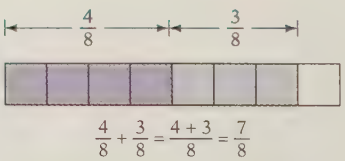
Your solution

Solutions on p. S1

Objective C To add or subtract rational numbers

3 CT

Four of the 8 squares have dark shading. This is $\frac{4}{8}$ of the entire rectangle. Three of the 8 squares have light shading. This is $\frac{3}{8}$ of the entire rectangle. So 7 of the 8 squares, or $\frac{7}{8}$, of the entire rectangle is shaded.



Addition of Fractions

To add two fractions with the same denominator, add the numerators and place the sum over the common denominator.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To add fractions with different denominators, first rewrite the fractions as equivalent fractions with a common denominator. Then add the fractions. The common denominator is the **least common multiple** (LCM) of the denominators. This is the smallest number that is a multiple of each of the denominators. It can be found by first writing each denominator as a product of prime factors. The LCM must contain the factors of each denominator.

$$6 = 2 \cdot 3$$
$$10 = 2 \cdot 5$$

$$\text{LCM} = \overbrace{2 \cdot 3}^{\text{Factors of 6}} \cdot \overbrace{5}^{\text{Factors of 10}} = 30$$

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➡ Add: $-\frac{5}{6} + \frac{3}{10}$

TAKE NOTE

The least common multiple of the denominators is frequently called the **least common denominator** (LCD).

The LCM of 6 and 10 is 30. Rewrite each fraction as an equivalent fraction with a denominator of 30. Then add the fractions.

$$-\frac{5}{6} + \frac{3}{10} = -\frac{25}{30} + \frac{9}{30} = \frac{-25 + 9}{30} = \frac{-16}{30} = -\frac{8}{15}$$

To subtract fractions with the same denominator, subtract the numerators and place the difference over the common denominator.

➡ Subtract: $-\frac{4}{9} - \left(-\frac{7}{12}\right)$

The LCM of 9 and 12 is 36. Rewrite each fraction as an equivalent fraction with a denominator of 36. Then subtract the fractions.

$$-\frac{4}{9} - \left(-\frac{7}{12}\right) = -\frac{16}{36} - \left(-\frac{21}{36}\right) = \frac{-16 - (-21)}{36} = \frac{-16 + 21}{36} = \frac{5}{36}$$

To add or subtract decimals, write the numbers so that the decimal points are in a vertical line. Then proceed as in the addition or subtraction of integers. Write the decimal point in the answer directly below the decimal points in the problem.

➡ Add: $-114.039 + 84.76$

$$\begin{array}{r} 114.039 \\ -\ 84.76 \\ \hline 29.279 \end{array}$$

- The signs are different. Find the difference between the absolute values of the numbers. $|-114.039| = 114.039$; $|84.76| = 84.76$

$$-114.039 + 84.76 = -29.279$$

- Attach the sign of the number with the larger absolute value. Because $|-114.039| > |84.76|$, use the sign of -114.039.

Example 6

Subtract: $\frac{5}{16} - \frac{7}{40}$

Solution

The LCM of 16 and 40 is 80.

$$\frac{5}{16} - \frac{7}{40} = \frac{25}{80} - \frac{14}{80} = \frac{25 - 14}{80} = \frac{11}{80}$$

You Try It 6

Subtract: $\frac{5}{9} - \frac{11}{12}$

Your solution

Example 7

Simplify: $-\frac{3}{4} + \frac{1}{6} - \frac{5}{8}$

Solution

The LCM of 4, 6, and 8 is 24.

$$-\frac{3}{4} + \frac{1}{6} - \frac{5}{8} = -\frac{18}{24} + \frac{4}{24} - \frac{15}{24}$$

$$= \frac{-18 + 4 - 15}{24} = \frac{-29}{24} = -\frac{29}{24}$$

You Try It 7

Simplify: $-\frac{7}{8} - \frac{5}{6} + \frac{3}{4}$

Your solution

Solutions on p. S2

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Example 8 Subtract: $42.987 - 98.61$

Solution
$$42.987 - 98.61 = 42.987 + (-98.61) = -55.623$$

You Try It 8 Subtract: $16.127 - 67.91$

Your solution

Solution on p. S2

Objective D To multiply or divide rational numbers

The product of two fractions is the product of the numerators divided by the product of the denominators.

Multiply: $\frac{3}{8} \times \frac{12}{17}$

$$\frac{3}{8} \times \frac{12}{17} = \frac{3 \cdot 12}{8 \cdot 17}$$

• Multiply the numerators. Multiply the denominators.

$$= \frac{3 \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3}{2 \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 17} = \frac{9}{34}$$

• Divide by the common factors.

The **reciprocal** of a fraction is the fraction with the numerator and denominator interchanged. For instance, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, and the reciprocal of $-\frac{5}{2}$ is $-\frac{2}{5}$. To divide fractions, multiply by the reciprocal of the divisor.

TAKE NOTE

The method for dividing fractions is sometimes stated, "To divide fractions, invert the divisor and then multiply." Inverting the divisor means writing its reciprocal.

Divide: $\frac{3}{10} \div \left(-\frac{18}{25}\right)$

$$\frac{3}{10} \div \left(-\frac{18}{25}\right) = -\left(\frac{3}{10} \div \frac{18}{25}\right) = -\left(\frac{3}{10} \cdot \frac{25}{18}\right) = -\left(\frac{3 \cdot 25}{10 \cdot 18}\right)$$

• The signs are different. The quotient is negative.

$$= -\left(\frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot 5}{2 \cdot \underset{1}{\cancel{5}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 3}\right) = -\frac{5}{12}$$

To multiply decimals, multiply as with integers. Write the decimal point in the product so that the number of decimal places in the product equals the sum of the number of decimal places in the factors.

Multiply: -6.89×0.00035

6.89	2 decimal places
$\times$ 0.00035	5 decimal places
3445	
2067	
0.0024115	7 decimal places

• Multiply the absolute values.

$-6.89 \times 0.00035 = -0.0024115$

• The signs are different. The product is negative.

To divide decimals, move the decimal point in the divisor to the right so that it becomes a whole number. Move the decimal point in the dividend the same number of places to the right. Place the decimal point in the quotient directly over the decimal point in the dividend. Then divide as with whole numbers.

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TAKE NOTE

The symbol ≈ is used to indicate that the quotient is an approximate value that has been rounded off.

➡ Divide: $1.32 \div 0.27$. Round to the nearest tenth.

$$\begin{array}{r} 4.88 \approx 4.9 \\ 0.27\overline{)1.32.00} \\ -108 \\ \hline 240 \\ -216 \\ \hline 240 \\ -216 \\ \hline 24 \end{array}$$

- Move the decimal point 2 places to the right in the divisor and then in the dividend. Place the decimal point in the quotient directly over the decimal point in the dividend.

Example 9

Multiply: $\frac{2}{3}\left(-\frac{9}{10}\right)$

Solution

The product is negative.

$$\frac{2}{3}\left(-\frac{9}{10}\right) = -\left(\frac{2}{3}\cdot\frac{9}{10}\right) = -\frac{2\cdot 9}{3\cdot 10}$$

$$= -\frac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{3}}\cdot 3}{\underset{1}{\cancel{3}}\cdot\underset{1}{\cancel{2}}\cdot 5} = -\frac{3}{5}$$

You Try It 9

Multiply: $-\frac{7}{12}\left(\frac{9}{14}\right)$

Your solution

Example 10

Divide: $-\frac{5}{8} \div \left(-\frac{5}{40}\right)$

Solution

The quotient is positive.

$$-\frac{5}{8} \div \left(-\frac{5}{40}\right) = \frac{5}{8} \div \frac{5}{40} = \frac{5}{8}\cdot\frac{40}{5} = \frac{5\cdot 40}{8\cdot 5}$$

$$= \frac{\overset{1}{\cancel{5}}\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot 5}{\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{5}}} = \frac{5}{1} = 5$$

You Try It 10

Divide: $-\frac{3}{8} \div \left(-\frac{5}{12}\right)$

Your solution

Example 11

Multiply: -4.29×8.2

Solution

Multiply the absolute values.
The product is negative.

$-4.29 \times 8.2 = -35.178$

You Try It 11

Multiply: -5.44×3.8

Your solution

Example 12

Divide: $-0.0792 \div (-0.42)$
Round to the nearest hundredth.

Solution

Divide the absolute values.
The quotient is positive.

$-0.0792 \div (-0.42) \approx 0.19$

You Try It 12

Divide: $-0.394 \div 1.7$
Round to the nearest hundredth.

Your solution

Solutions on p. S2

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Objective E To solve application problems

One of the applications of percent is to express a portion of a total as a percent. For instance, a recent survey of 450 mall shoppers found that 270 preferred the mall closest to their home even though it did not have the same store variety as a mall farther from home. The percent of shoppers who preferred the mall closest to home can be found by converting a fraction to a percent.

$$\frac{\text{Portion preferring mall closest to home}}{\text{Total number surveyed}} = \frac{270}{450}$$

$$= 0.60 = 60\%$$

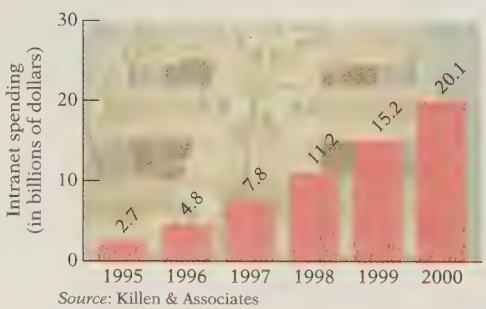
According to the Congressional Budget Office, the U.S. federal budget deficit in 1996 was $107.3 billion. The number 107.3 billion means

$$107.3 \times \underbrace{1{,}000{,}000{,}000}_{\text{1 billion}} = 107{,}300{,}000{,}000$$

Numbers such as 107.3 billion are used in many instances because they are easy to read and offer an approximation to the actual number. Such numbers are used in Example 13 and You Try It 13.

Example 13

The actual and projected spending for software, equipment, and services for intranets is shown in the bar graph below. Find the difference between expenditures in 2000 and in 1998.



Source: Killen & Associates

Strategy

To find the difference, subtract the expenditures in 1998 ($11.2 billion) from those in 2000 ($20.1 billion).

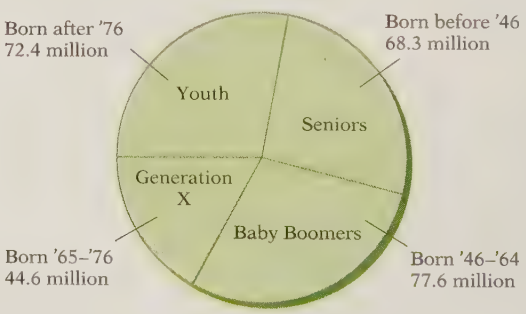
Solution

$20.1 - 11.2 = 8.9$

The difference is $8.9 billion.

You Try It 13

The circle graph below shows the breakdown of the population of the United States by generation. What percent of the population of the United States is in the baby-boomer generation? Round to the nearest percent.



Source: Data from U.S. Census Bureau

Your strategy

Your solution

Solution on p. S2

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1.4 Exercises

Objective A

Write each fraction in simplest form.

1. $\frac{7}{21}$ **2.** $\frac{10}{15}$ **3.** $\frac{8}{22}$ **4.** $\frac{8}{60}$ **5.** $\frac{50}{75}$

6. $\frac{20}{44}$ **7.** $\frac{12}{8}$ **8.** $\frac{36}{9}$ **9.** $\frac{0}{36}$ **10.** $\frac{12}{18}$

11. $\frac{60}{100}$ **12.** $\frac{14}{45}$ **13.** $\frac{44}{60}$ **14.** $\frac{19}{51}$ **15.** $\frac{23}{46}$

Write as a decimal. Place a bar over the repeating digits of a repeating decimal.

16. $\frac{4}{5}$ **17.** $\frac{1}{6}$ **18.** $\frac{5}{6}$ **19.** $\frac{1}{8}$ **20.** $\frac{7}{8}$

21. $\frac{2}{9}$ **22.** $\frac{8}{9}$ **23.** $\frac{5}{11}$ **24.** $\frac{7}{12}$ **25.** $\frac{11}{12}$

26. $\frac{9}{16}$ **27.** $\frac{15}{16}$ **28.** $\frac{7}{18}$ **29.** $\frac{17}{18}$ **30.** $\frac{1}{20}$

Objective B

Write as a fraction and a decimal.

31. 75% **32.** 40% **33.** 64% **34.** 88% **35.** 125%

36. 160% **37.** 19% **38.** 87% **39.** 5% **40.** 8%

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Write as a fraction.

41. $11\frac{1}{9}\%$ **42.** $4\frac{2}{7}\%$ **43.** $12\frac{1}{2}\%$ **44.** $37\frac{1}{2}\%$ **45.** $66\frac{2}{3}\%$

46. $\frac{1}{4}\%$ **47.** $\frac{1}{2}\%$ **48.** $6\frac{1}{4}\%$ **49.** $83\frac{1}{3}\%$ **50.** $5\frac{3}{4}\%$

Write as a decimal.

51. 7.3% **52.** 9.1% **53.** 15.8% **54.** 16.7% **55.** 0.3%

56. 0.9% **57.** 9.9% **58.** 9.15% **59.** 121.2% **60.** 18.23%

Write as a percent.

61. 0.15 **62.** 0.37 **63.** 0.05 **64.** 0.02 **65.** 0.175

66. 0.125 **67.** 1.15 **68.** 1.36 **69.** 0.008 **70.** 0.004

71. $\frac{27}{50}$ **72.** $\frac{83}{100}$ **73.** $\frac{1}{3}$ **74.** $\frac{3}{8}$ **75.** $\frac{5}{11}$

76. $\frac{4}{9}$ **77.** $\frac{7}{8}$ **78.** $\frac{9}{20}$ **79.** $1\frac{2}{3}$ **80.** $2\frac{1}{2}$

Objective C

Simplify.

81. $\frac{2}{3}+\frac{5}{12}$ **82.** $\frac{1}{2}+\frac{3}{8}$ **83.** $\frac{5}{8}-\frac{5}{6}$ **84.** $\frac{1}{9}-\frac{5}{27}$

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85. $-\frac{5}{12}-\frac{3}{8}$ **86.** $-\frac{5}{6}-\frac{5}{9}$ **87.** $-\frac{6}{13}+\frac{17}{26}$ **88.** $-\frac{7}{12}+\frac{5}{8}$

89. $\frac{5}{8}-\left(-\frac{3}{4}\right)$ **90.** $\frac{3}{5}-\frac{11}{12}$ **91.** $\frac{11}{12}-\frac{5}{6}$ **92.** $-\frac{2}{3}-\left(-\frac{11}{18}\right)$

93. $-\frac{5}{8}-\left(-\frac{11}{12}\right)$ **94.** $\frac{1}{3}+\frac{5}{6}-\frac{2}{9}$ **95.** $\frac{1}{2}-\frac{2}{3}+\frac{1}{6}$ **96.** $-\frac{3}{8}-\frac{5}{12}-\frac{3}{16}$

97. $-\frac{5}{16}+\frac{3}{4}-\frac{7}{8}$ **98.** $\frac{1}{2}-\frac{3}{8}-\left(-\frac{1}{4}\right)$ **99.** $\frac{3}{4}-\left(-\frac{7}{12}\right)-\frac{7}{8}$ **100.** $\frac{1}{3}-\frac{1}{4}-\frac{1}{5}$

101. $\frac{2}{3}-\frac{1}{2}+\frac{5}{6}$ **102.** $\frac{5}{16}+\frac{1}{8}-\frac{1}{2}$ **103.** $\frac{5}{8}-\left(-\frac{5}{12}\right)+\frac{1}{3}$ **104.** $\frac{1}{8}-\frac{11}{12}+\frac{1}{2}$

105. $7.56 + 0.462$ **106.** $1.09 + 6.2$ **107.** $-32.1 - 6.7$ **108.** $5.13 - 8.179$

109. $-13.092 + 6.9$ **110.** $2.54 - 3.6$ **111.** $5.43 + 7.925$ **112.** $-16.92 - 6.925$

113. $-3.87 + 8.546$ **114.** $6.9027 - 17.692$ **115.** $2.09 - 6.72 - 5.4$

116. $-18.39 + 4.9 - 23.7$ **117.** $19 - (-3.72) - 82.75$ **118.** $-3.07 - (-2.97) - 17.4$

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119. $16.4 - (-3.09) - 7.93$

120. $-3.09 - 4.6 - (-27.3)$

121. $2.66 - (-4.66) - 8.2$

122. $-0.34 - (-4.35) - 3.2$

123. $7.5 - 12.8 - (-0.57)$

124. $2.8 - (-3.44) + 2.3$

Objective D

Simplify.

125. $\frac{1}{2} \times \left(-\frac{3}{4}\right)$

126. $-\frac{2}{9} \times \left(-\frac{3}{14}\right)$

127. $\left(-\frac{3}{8}\right)\left(-\frac{4}{15}\right)$

128. $\left(-\frac{3}{4}\right)\left(-\frac{8}{27}\right)$

129. $-\frac{1}{2}\left(\frac{8}{9}\right)$

130. $\frac{5}{12}\left(-\frac{8}{15}\right)$

131. $\frac{5}{8}\left(-\frac{7}{12}\right)\frac{16}{25}$

132. $\left(\frac{5}{12}\right)\left(-\frac{8}{15}\right)\left(-\frac{1}{3}\right)$

133. $\frac{1}{2}\left(-\frac{3}{4}\right)\left(-\frac{5}{8}\right)$

134. $\frac{3}{8} \div \frac{1}{4}$

135. $\frac{5}{6} \div \left(-\frac{3}{4}\right)$

136. $-\frac{5}{12} \div \frac{15}{32}$

137. $-\frac{7}{8} \div \frac{4}{21}$

138. $\frac{7}{10} \div \frac{2}{5}$

139. $-\frac{15}{64} \div \left(-\frac{3}{40}\right)$

140. $\frac{1}{8} \div \left(-\frac{5}{12}\right)$

141. $-\frac{4}{9} \div \left(-\frac{2}{3}\right)$

142. $-\frac{6}{11} \div \frac{4}{9}$

143. $1.2(3.47)$

144. $(-0.8)6.2$

145. $(-1.89)(-2.3)$

146. $(6.9)(-4.2)$

147. $1.06(-3.8)$

148. $-2.7(-3.5)$

149. $1.2(-0.5)(3.7)$

150. $-2.4(6.1)(0.9)$

151. $2.3(-0.6)(0.8)$

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152. $-1.2(-0.55)(1.9)$

153. $0.44(-2.3)(-0.5)$

154. $-3.4(-22.1)(-0.5)$

155. $1.8(0.33)(-0.4)$

156. $4.5(-0.22)(-0.8)$

157. $-24.7 \div 0.09$

Simplify. Round to the nearest hundredth.

158. $-1.27 \div (-1.7)$

159. $9.07 \div (-3.5)$

160. $0.0976 \div 0.042$

161. $-6.904 \div 1.35$

162. $-7.894 \div (-2.06)$

163. $-354.2086 \div 0.1719$

Objective E Application Problems

164. When a U.S. company does business with another country, it is necessary to convert U.S. currency into the currency of the other country. These *exchange rates* are determined by various factors. The exchange rates for some currencies during one day in July 1997 are shown in the table at the right.

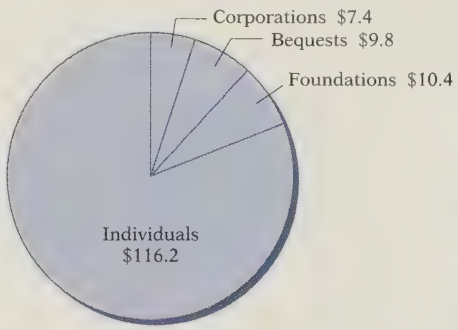
Foreign Currency per U.S. Dollar	
Canadian dollar	1.3936
French franc	5.7827
Libyan dinar	0.3555

a. If you sold goods worth 1.2 million U.S. dollars to France, how many French francs would you receive?

b. How many U.S. dollars, to the nearest cent, are equivalent to 1 Libyan dinar?

165. The circle graph at the right shows the sources of donations to social causes. What percent of the total is contributed by corporations? Round to the nearest percent.

Giving, by Source (in billions of dollars)



Source: Giving USA, 1996, AAFRC Trust for Philanthropy

166. The table below shows the first-quarter profits and losses for 1997 for three companies in the toy industry. Profits are shown as positive numbers; losses are shown as negative numbers. One-quarter of a year is three months.

a. If earnings were to continue throughout the year at the same level, what would the annual earnings or losses be for Mattel, Inc.?

b. For the quarter shown, what was the average monthly profit or loss for Acclaim Entertainment?

Toy Company	First Quarter 1997 Profits
Acclaim Entertainment	–16.842 million
Hasbro, Inc.	25.694 million
Mattel, Inc.	–204.624 million

Source: The Wall Street Journal, May 5, 1997

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167. Movies made in the United States that are not blockbusters in this country may make considerable money abroad. At the right is a table showing five films and their box-office grosses, in millions of dollars, at home and in foreign countries.

a. What percent of *Waterworld*'s total gross was from foreign countries? Round to the nearest percent.

b. What percent of *Judge Dredd*'s total gross here and abroad was its U.S. box-office gross? Round to the nearest percent.

c. Which of the films listed grossed more than half their box-office income in foreign countries?

Box-Office Grosses (in millions of dollars)

Film	U. S.	Foreign
Judge Dredd	34.7	44.1
Spy Hard	26.6	30.6
Waterworld	88.2	78.2
The Juror	22.7	17.6
Highlander III	13.8	9.1

Source: Time Magazine, July 7, 1997

168. According to the Federal Highway Administration, the average car is driven approximately 10,300 mi per year and uses approximately 495 gal of gas. Assuming that the average cost of gasoline is \$1.097 per gallon, which includes \$.443 for all taxes, and that there are 1.53 million cars on the road, determine how much total tax is paid for gasoline in one year. (You may not need all the data given in this problem.)

APPLYING THE CONCEPTS

169. A magic square is one in which the numbers in every row, column, and diagonal sum to the same number. Complete the magic square at the right.

$\frac{2}{3}$		
	$\frac{1}{6}$	$\frac{5}{6}$
		$-\frac{1}{3}$

170. If a and b are rational numbers and $a < b$, is it always possible to find a rational number c such that $a < c < b$? If not, explain why. If so, show how to find one.

171. For each part below, find a rational number r that satisfies the condition.

a. $r^2 < r$ **b.** $r^2 = r$ **c.** $r^2 > r$

172. In a survey of consumers, approximately 43% said they would be willing to pay between \$1000 and \$2000 more for a new car if the car had an EPA rating of 80 mpg. If your car now gets 28 mpg and you drive approximately 10,000 mi per year, in how many months would your savings on gasoline pay for the increased cost of such a car? Assume the average cost for gasoline is \$1.06 per gallon.

173. Find three different natural numbers a, b, and c such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is a natural number.

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1.5 Exponents and the Order of Operations Agreement

Objective A To evaluate exponential expressions

Repeated multiplication of the same factor can be written using an exponent.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$ ← exponent, ↑ base $\qquad a \cdot a \cdot a \cdot a = a^4$ ← exponent, ↑ base

POINT OF INTEREST

Rene Descartes (1596–1650) was the first mathematician to extensively use exponential notation as it is used today. However, for some unknown reason, he always used xx for x^2.

The **exponent** indicates how many times the factor, called the **base,** occurs in the multiplication. The multiplication $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is in **factored form.** The exponential expression 2^5 is in **exponential form.**

2^1 is read "2 to the first power" or just "2." Usually the exponent 1 is not written.

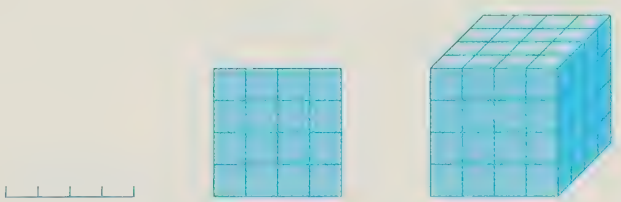
2^2 is read "2 to the second power" or "2 squared."

2^3 is read "2 to the third power" or "2 cubed."

2^4 is read "2 to the fourth power."

a^4 is read "a to the fourth power."

There is a geometric interpretation of the first three natural-number powers.



$4^1 = 4$
Length 4 ft

$4^2 = 16$
Area 16 ft^2

$4^3 = 64$
Volume 64 ft^3

To evaluate an exponential expression, write each factor as many times as indicated by the exponent. Then multiply.

➡ Evaluate $(-2)^4$.

$(-2)^4 = (-2)(-2)(-2)(-2)$ • Write -2 as a factor 4 times.

$= 16$ • Multiply.

TAKE NOTE

Note the difference between $(-2)^4$ and -2^4.

$(-2)^4 = 16$
$-2^4 = -16$

➡ Evaluate -2^4.

$-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2)$ • Write 2 as a factor 4 times.

$= -16$ • Multiply.

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Example 1 Evaluate -5^3.

Solution $-5^3 = -(5 \cdot 5 \cdot 5) = -125$

You Try It 1 Evaluate -6^3.

Your solution

Example 2 Evaluate $(-4)^4$.

Solution $(-4)^4 = (-4)(-4)(-4)(-4)$
$= 256$

You Try It 2 Evaluate $(-3)^4$.

Your solution

Example 3 Evaluate $(-3)^2 \cdot 2^3$.

Solution $(-3)^2 \cdot 2^3 = (-3)(-3) \cdot (2)(2)(2)$
$= 9 \cdot 8 = 72$

You Try It 3 Evaluate $(3^3)(-2)^3$.

Your solution

Example 4 Evaluate $\left(-\frac{2}{3}\right)^3$.

Solution $\left(-\frac{2}{3}\right)^3 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)$
$= -\frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = -\frac{8}{27}$

You Try It 4 Evaluate $\left(-\frac{2}{5}\right)^2$.

Your solution

Example 5 Evaluate $-4(0.7)^2$.

Solution $-4(0.7)^2 = -4(0.7)(0.7)$
$= -2.8(0.7) = -1.96$

You Try It 5 Evaluate $-3(0.3)^3$.

Your solution

Solutions on p. S2

Objective B To use the Order of Operations Agreement to simplify expressions

Let's evaluate $2 + 3 \cdot 5$.

There are two arithmetic operations, addition and multiplication, in this expression. The operations could be performed in different orders.

Multiply first.	$2 + \underbrace{3 \cdot 5}$	Add first.	$\underbrace{2 + 3} \cdot 5$
Then add.	$\underbrace{2 + 15}$	Then multiply.	$\underbrace{5 \cdot 5}$
	17		25

In order to prevent more than one answer for a numerical expression, an Order of Operations Agreement has been established.

The Order of Operations Agreement

Step 1 Perform operations inside grouping symbols. Grouping symbols include parentheses ( ), brackets [ ], braces { }, and the fraction bar.

Step 2 Simplify exponential expressions.

Step 3 Do multiplication and division as they occur from left to right.

Step 4 Do addition and subtraction as they occur from left to right.

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➡ Evaluate $12 - 24(8 - 5) \div 2^2$.

$12 - 24(8 - 5) \div 2^2 = 12 - 24(3) \div 2^2$ • Perform operations inside grouping symbols.

$= 12 - 24(3) \div 4$ • Simplify exponential expressions.

$= 12 - 72 \div 4$ • Do multiplication and division as they occur from left to right.

$= 12 - 18$

$= -6$ • Do addition and subtraction as they occur from left to right.

One or more of the above steps may not be needed to evaluate an expression. In that case, proceed to the next step in the Order of Operations Agreement.

➡ Evaluate $\frac{4 + 8}{2 + 1} - (3 - 1) + 2$.

$\frac{4 + 8}{2 + 1} - (3 - 1) + 2 = \frac{12}{3} - 2 + 2$ • Perform operations above and below the fraction bar and inside parentheses.

$= 4 - 2 + 2$ • Do multiplication and division as they occur from left to right.

$= 2 + 2$ • Do addition and subtraction as they occur from left to right.

$= 4$

When an expression has grouping symbols inside grouping symbols, perform the operations inside the inner grouping symbols first.

➡ Evaluate $6 \div [4 - (6 - 8)] + 2^2$.

$6 \div [4 - (6 - 8)] + 2^2 = 6 \div [4 - (-2)] + 2^2$ • Perform operations inside grouping symbols.

$= 6 \div 6 + 2^2$

$= 6 \div 6 + 4$ • Simplify exponential expressions.

$= 1 + 4$ • Do multiplication and division as they occur from left to right.

$= 5$ • Do addition and subtraction as they occur from left to right.

Example 6

Evaluate $(1.75 - 1.3)^2 \div 0.025 + 6.1$.

Solution

$(1.75 - 1.3)^2 \div 0.025 + 6.1$
$= (0.45)^2 \div 0.025 + 6.1$
$= 0.2025 \div 0.025 + 6.1$
$= 8.1 + 6.1$
$= 14.2$

You Try It 6

Evaluate $(6.97 - 4.72)^2 \cdot 4.5 \div 0.05$.

Your solution

Solution on p. S2

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Example 7

Evaluate $4 - 3[4 - 2(6 - 3)] \div 2$.

Solution

$$\begin{aligned} &4 - 3[4 - 2(6 - 3)] \div 2 \\ &\quad = 4 - 3[4 - 2 \cdot 3] \div 2 \\ &\quad = 4 - 3[4 - 6] \div 2 \\ &\quad = 4 - 3[-2] \div 2 \\ &\quad = 4 + 6 \div 2 \\ &\quad = 4 + 3 \\ &\quad = 7 \end{aligned}$$

You Try It 7

Evaluate $18 - 5[8 - 2(2 - 5)] \div 10$.

Your solution

Example 8

Evaluate $27 \div (5 - 2)^2 + (-3)^2 \cdot 4$.

Solution

$$\begin{aligned} &27 \div (5 - 2)^2 + (-3)^2 \cdot 4 \\ &\quad = 27 \div 3^2 + (-3)^2 \cdot 4 \\ &\quad = 27 \div 9 + 9 \cdot 4 \\ &\quad = 3 + 9 \cdot 4 \\ &\quad = 3 + 36 \\ &\quad = 39 \end{aligned}$$

You Try It 8

Evaluate $36 \div (8 - 5)^2 - (-3)^2 \cdot 2$.

Your solution

Example 9

Evaluate $\frac{5}{8} - \left(\frac{2}{5} - \frac{1}{2}\right) \div \left(\frac{2}{3}\right)^2$.

Solution

$$\begin{aligned} &\frac{5}{8} - \left(\frac{2}{5} - \frac{1}{2}\right) \div \left(\frac{2}{3}\right)^2 \\ &\quad = \frac{5}{8} - \left(-\frac{1}{10}\right) \div \left(\frac{2}{3}\right)^2 \\ &\quad = \frac{5}{8} + \frac{1}{10} \div \frac{4}{9} \\ &\quad = \frac{5}{8} + \frac{1}{10} \cdot \frac{9}{4} \\ &\quad = \frac{5}{8} + \frac{9}{40} \\ &\quad = \frac{25}{40} + \frac{9}{40} \\ &\quad = \frac{34}{40} = \frac{17}{20} \end{aligned}$$

You Try It 9

Evaluate $\frac{5}{8} \div \left(\frac{1}{3} - \frac{3}{4}\right) + \frac{7}{12}$.

Your solution

Solutions on pp. S2–S3

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1.5 Exercises

Objective A

Evaluate.

1. 6^2 **2.** 7^4 **3.** -7^2 **4.** -4^3 **5.** $(-3)^2$

6. $(-2)^3$ **7.** $(-3)^4$ **8.** $(-5)^3$ **9.** $\left(\frac{1}{2}\right)^2$ **10.** $\left(-\frac{3}{4}\right)^3$

11. $(0.3)^2$ **12.** $(1.5)^3$ **13.** $\left(\frac{2}{3}\right)^2 \cdot 3^3$ **14.** $\left(-\frac{1}{2}\right)^3 \cdot 8$ **15.** $(0.3)^3 \cdot 2^3$

16. $(0.5)^2 \cdot 3^3$ **17.** $(-3) \cdot 2^2$ **18.** $(-5) \cdot 3^4$ **19.** $(-2) \cdot (-2)^3$

20. $(-2) \cdot (-2)^2$ **21.** $2^3 \cdot 3^3 \cdot (-4)$ **22.** $(-3)^3 \cdot 5^2 \cdot 10$ **23.** $(-7) \cdot 4^2 \cdot 3^2$

24. $(-2) \cdot 2^3 \cdot (-3)^2$ **25.** $\left(\frac{2}{3}\right)^2 \cdot \frac{1}{4} \cdot 3^3$ **26.** $\left(\frac{3}{4}\right)^2 \cdot (-4) \cdot 2^3$ **27.** $8^2 \cdot (-3)^5 \cdot 5$

Objective B

Evaluate by using the Order of Operations Agreement.

28. $4 - 8 \div 2$ **29.** $2^2 \cdot 3 - 3$ **30.** $2(3 - 4) - (-3)^2$

31. $16 - 32 \div 2^3$ **32.** $24 - 18 \div 3 + 2$ **33.** $8 - (-3)^2 - (-2)$

34. $8 - 2(3)^2$ **35.** $16 - 16 \cdot 2 \div 4$ **36.** $12 + 16 \div 4 \cdot 2$

37. $16 - 2 \cdot 4^2$ **38.** $27 - 18 \div (-3^2)$ **39.** $4 + 12 \div 3 \cdot 2$

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40. $16 + 15 \div (-5) - 2$

41. $14 - 2^2 - (4 - 7)$

42. $3 - 2[8 - (3 - 2)]$

43. $-2^2 + 4[16 \div (3 - 5)]$

44. $6 + \dfrac{16 - 4}{2^2 + 2} - 2$

45. $24 \div \dfrac{3^2}{8 - 5} - (-5)$

46. $96 \div 2[12 + (6 - 2)] - 3^2$

47. $4[16 - (7 - 1)] \div 10$

48. $18 \div 2 - 4^2 - (-3)^2$

49. $18 \div (9 - 2^3) + (-3)$

50. $16 - 3(8 - 3)^2 \div 5$

51. $4(-8) \div [2(7 - 3)^2]$

52. $\dfrac{(-19) + (-2)}{6^2 - 30} \div (2 - 4)$

53. $16 - 4 \cdot \dfrac{3^3 - 7}{2^3 + 2} - (-2)^2$

54. $(0.2)^2 \cdot (-0.5) + 1.72$

55. $0.3(1.7 - 4.8) + (1.2)^2$

56. $(1.8)^2 - 2.52 \div 1.8$

57. $(1.65 - 1.05)^2 \div 0.4 + 0.8$

58. $\dfrac{3}{8} \div \left(\dfrac{5}{6} + \dfrac{2}{3}\right)$

59. $\left(\dfrac{5}{12} - \dfrac{9}{16}\right)\dfrac{3}{7}$

60. $\left(\dfrac{3}{4}\right)^2 - \left(\dfrac{1}{2}\right)^3 \div \dfrac{3}{5}$

APPLYING THE CONCEPTS

61. The following was offered as the simplification of $6 + 2(4 - 9)$.

$$\begin{aligned} 6 + 2(4 - 9) &= 6 + 2(-5) \\ &= 8(-5) \\ &= -40 \end{aligned}$$

If this is a correct simplification, write yes for the answer. If it is incorrect, write no and explain the incorrect step.

62. The following was offered as the simplification of $2 \cdot 3^3$.

$$2 \cdot 3^3 = 6^3 = 216$$

If this is a correct simplification, write yes for the answer. If it is incorrect, write no and explain the incorrect step.

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Focus on Problem Solving

Inductive Reasoning

Suppose you take 9 credit hours each semester. The total number of credit hours you have taken at the end of each semester can be described in a list of numbers.

$$9, 18, 27, 36, 45, 54, 63, \ldots$$

The list of numbers that indicates the total credit hours is an ordered list of numbers, called a **sequence.** Each number in a sequence is called a **term** of the sequence. The list is ordered because the position of a number in the list indicates the semester in which that number of credit hours has been taken. For example, the 7th term of the sequence is 63, and a total of 63 credit hours have been taken after the 7th semester.

Assuming the pattern is continued, find the next three numbers in the pattern

$$6, -10, -14, -18, \ldots$$

This list of numbers is a sequence. The first step in solving this problem is to observe the pattern in the list of numbers. In this case, each number in the list is 4 less than the previous number. The next three numbers are $-22, -26, -30$.

This process of discovering the pattern in a list of numbers is inductive reasoning. **Inductive reasoning** involves making generalizations from specific examples; in other words, we reach a conclusion by making observations about particular facts or cases.

Try the following exercises. Each exercise requires inductive reasoning.

Name the next two terms in the sequence.

1. 1, 3, 5, 7, 1, 3, 5, 7, 1,...

2. 1, 4, 2, 5, 3, 6, 4,...

3. 1, 2, 4, 7, 11, 16,...

4. A, B, C, G, H, I, M,...

Draw the next shape in the sequence.

5. 

6. 

Solve.

7. Convert $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}$, and $\frac{5}{11}$ to decimals. Then use the pattern you observe to convert $\frac{6}{11}, \frac{7}{11}$, and $\frac{9}{11}$ to decimals.

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8. Convert $\frac{1}{33}$, $\frac{2}{33}$, $\frac{4}{33}$, $\frac{5}{33}$, and $\frac{7}{33}$ to decimals. Then use the pattern you observe to convert $\frac{8}{33}$, $\frac{13}{33}$, and $\frac{19}{33}$ to decimals.

Projects and Group Activities

The [+/−] Key on a Calculator

Using your calculator to simplify numerical expressions sometimes requires use of the [+/−] key or, on some calculators, the negative key, which is frequently shown as [(−)]. These keys change the sign of the number currently in the display. To enter −4:

- For those calculators with [+/−], press 4 and then [+/−].
- For those calculators with [(−)], press [( )] and then 4.

Here are the keystrokes for evaluating the expression $3(-4) - (-5)$.

Calculators with [+/−] key: 3 [×] 4 [+/−] − 5 [+/−] [=]

Calculators with [(−)] key: 3 [×] [(−)] 4 [−] [(−)] 5 [=]

This example illustrates that calculators make a distinction between negative and minus. To perform the operation $3 - (-3)$, you cannot enter 3 [−] [−] 3. This would result in 0, which is not the correct answer. You must enter

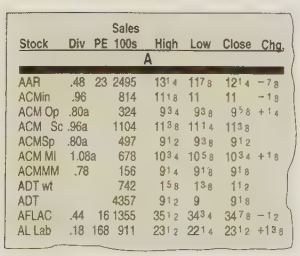
3 [−] 3 [+/−] [=] or 3 [−] [(−)] 3 [=]

Use a calculator to evaluate each of the following exercises.

1. $-16 \div 2$ **2.** $3(-8)$ **3.** $47 - (-9)$

4. $-50 - (-14)$ **5.** $4 - (-3)^2$ **6.** $-8 + (-6)^2 - 7$

Moving Averages



Stock	Div	PE	Sales 100s	High	Low	Close	Chg.
			A				
AAR	.48	23	2495	13 1/4	11 7/8	12 1/4	−7/8
ACMin	.96		814	11 1/8	11	11	−1/8
ACM Op	.80a		324	9 3/4	9 3/8	9 5/8	+1/4
ACM Sc	.96a		1104	11 3/8	11 1/4	11 3/8	
ACMSp	.80a		497	9 1/2	9 3/8	9 1/2	
ACM MI	1.08a		678	10 3/4	10 5/8	10 3/4	+1/8
ACMMM	.78		156	9 1/4	9 1/8	9 1/8	
ADT wt			742	1 5/8	1 3/8	1 1/2	
ADT			4357	9 1/2	9	9 1/8	
AFLAC	.44	16	1355	35 1/2	34 3/4	34 7/8	−1/2
AL Lab	.18	168	911	23 1/2	22 1/4	23 1/2	+13/8

Objective 1.3C on page 20 describes how to find the moving average of a stock. Use this method to calculate the five-day moving average for at least three different stocks. Discuss and compare the results for the different stocks.

For this project, you will need to use stock tables, which are printed in the business section of major newspapers. Your college library should have copies of these publications. In a stock table, the column headed "Chg." provides the change in the price of a share of the stock; that is, it gives the difference between the closing price for the day shown and the closing price for the previous day. The symbol + indicates the change was an increase in price; the symbol − indicates the change was a decrease in price.

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www.fedstats.gov

Information regarding the history of the federal budget can be found on the World Wide Web. Go to the web site www.fedstats.gov. Click on "Fast Facts" and then on "Frequently Requested Tables." In the list of tables printed to the screen, find the table entitled "Federal Budget - Summary." Click on it. When the federal budget table appears on the screen, look for the column that lists each year's surplus or deficit. You will see that a negative sign (−) is used to show a deficit. Note that it states near the top of the screen that the figures in the table are in millions of dollars.

1. During which years shown in the table was there a surplus?
2. During which year was the deficit the greatest?
3. Find the difference between the surplus or deficit this year and the surplus or deficit five years ago.
4. What is the difference between the surplus or deficit this year and the surplus or deficit a decade ago?
5. Determine what two numbers in the table are being subtracted in each row in order to arrive at the number in the surplus or deficit column.
6. Describe the trend of the federal deficit over the last ten years.

Chapter Summary

Key Words

A *set* is a collection of objects. The objects in a set are called the *elements* of the set.

The *roster method* of writing sets encloses a list of the elements in braces.

The set of *natural numbers* is $\{1, 2, 3, 4, 5, 6, 7, \ldots\}$.

The set of *integers* is $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$.

A number a *is less than* another number b, written $a < b$, if a is to the left of b on the number line.

A number a *is greater than* another number b, written $a > b$, if a is to the right of b on the number line.

The symbol $\leq$ means *is less than or equal to*. The symbol $\geq$ means *is greater than or equal to*.

Two numbers that are the same distance from zero on the number line but on opposite sides of zero are *opposite numbers*, or *opposites*. The opposite of a number is also called its *additive inverse*.

The *absolute value* of a number is its distance from zero on the number line.

A *rational number* is a number of the form $\frac{a}{b}$, where a and b are integers and b is not equal to zero. A rational number written in this form is commonly called a fraction.

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A fraction is in *simplest form* when there are no common factors in the numerator and the denominator.

The *reciprocal* of a fraction is the fraction with the numerator and denominator interchanged.

An *irrational number* is a number that has a decimal representation that never terminates or repeats.

The rational numbers and the irrational numbers taken together are called the *real numbers*.

Percent means "parts of 100." Thus 63% means 63 parts of 100 parts.

An expression of the form a^n is in *exponential form*, where a is the *base* and n is the *exponent*.

Essential Rules

To add two numbers with the same sign, add the absolute values of the numbers. Then attach the sign of the addends.

To add two numbers with different signs, find the absolute value of each number. Subtract the smaller of the two numbers from the larger. Then attach the sign of the number with the larger absolute value.

To subtract two integers, add the opposite of the second integer to the first integer.

To multiply two numbers with the same sign, multiply the absolute values of the factors. The product is positive.

To multiply two numbers with different signs, multiply the absolute values of the factors. The product is negative.

To divide two numbers with the same sign, divide the absolute values of the numbers. The quotient is positive.

To divide two numbers with different signs, divide the absolute values of the numbers. The quotient is negative.

Properties of Zero and One in Division

If $a \neq 0$, $0 \div a = 0$.
If $a \neq 0$, $a \div a = 1$.
$a \div 1 = a$.
$a \div 0$ is undefined.

To convert a percent to a decimal, remove the percent sign and multiply by 0.01.

To convert a percent to a fraction, remove the percent sign and multiply by $\frac{1}{100}$.

To convert a fraction to a percent, multiply by 100%.

To convert a decimal to a percent, multiply by 100%.

The Order of Operations Agreement

Step 1 Perform all operations inside grouping symbols. Grouping symbols include parentheses, brackets, braces, and the fraction bar.

Step 2 Simplify any numerical expressions containing exponents.

Step 3 Do multiplication and division as they occur from left to right.

Step 4 Do addition and subtraction as they occur from left to right.

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Chapter Review

1. Add: $-13 + 7$

2. Write $\frac{7}{25}$ as a decimal.

3. Evaluate -5^2.

4. Evaluate $5 - 2^2 + 9$.

5. Place the correct symbol, $<$ or $>$, between the two numbers.
$-4 \quad 2$

6. Write 6.2% as a decimal.

7. Multiply: $(-6)(7)$

8. Simplify: $\frac{1}{3} - \frac{1}{6} + \frac{5}{12}$

9. Evaluate $15 \cdot (6 - 4)^2$.

10. Given $A = \{-4, 0, 11\}$, which elements of set A are less than -1?

11. Subtract: $5.17 - 6.238$

12. Write $\frac{5}{8}$ as a percent.

13. Write $\frac{2}{15}$ as a decimal. Place a bar over the repeating digits of the decimal.

14. Subtract: $9 - 13$

15. Find the additive inverse of -4.

16. Divide: $-32 \div (-4)$

17. Divide: $-100 \div 5$

18. Write $79\frac{1}{2}\%$ as a fraction.

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19. Evaluate $-3^2 + 4[18 + (12 - 20)]$.

20. Add: $-3 + (-12) + 6 + (-4)$

21. Write $\frac{19}{35}$ as a percent. Write the remainder in fractional form.

22. Evaluate $\left(\frac{2}{3}\right)^4$.

23. Multiply: $4.32(-1.07)$

24. Evaluate $-|-5|$.

25. Subtract: $16 - (-3) - 18$

26. Divide: $-\frac{18}{35} \div \frac{27}{28}$

27. Given $C = \{-7, 0, 9\}$ find the absolute value of each element of set C.

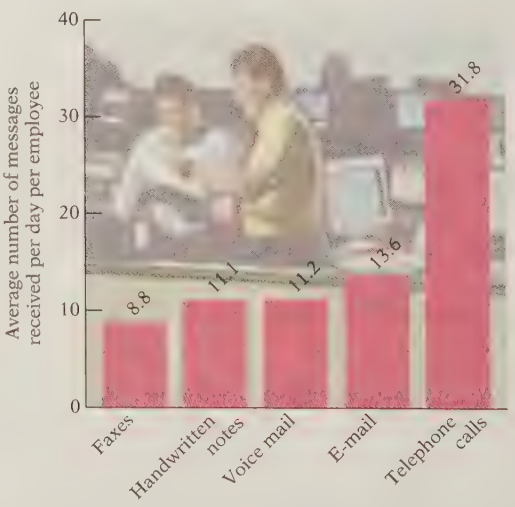
28. Multiply: $-9(-9)$

29. Place the correct symbol between the two numbers.
$-|6| \quad |-10|$

30. Evaluate $\frac{5^2 + 11}{2^2 + 5} \div (2^3 - 2^2)$.

31. To discourage random guessing on a multiple-choice exam, a professor assigns 6 points for a correct answer, -4 points for an incorrect answer, and -2 points for leaving a question blank. What is the score for a student who had 21 correct answers, 5 incorrect answers, and left 4 questions blank?

32. The graph at the right shows the average number of messages received each day per employee. What percent of all the messages is E-mail? Round to the nearest tenth of a percent.



Source: Institute for the Future/Gallup Organization

33. The temperature at which mercury boils is 357°C. The temperature at which mercury freezes is -39°C. Find the difference between the boiling point and the freezing point of mercury.

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Chapter Test

1. Divide: $-561 \div (-33)$

2. Write $\frac{5}{6}$ as a percent. Write the remainder in fractional form.

3. Evaluate $\frac{3}{4} \cdot (4)^2$.

4. Multiply: $6.02(-0.89)$

5. Subtract: $16 - 30$

6. Write $37\frac{1}{2}\%$ as a fraction.

7. Add: $-\frac{2}{5} + \frac{7}{15}$

8. Evaluate $\frac{-10 + 2}{2 + (-4)} \div 2 + 6$.

9. Multiply: $-5(-6)(3)$

10. Find the opposite of -4.

11. Evaluate $(-3^3) \cdot 2^2$.

12. Given $B = \{-5, -3, 0, 4\}$, which elements of set B are greater than 2?

13. Place the correct symbol, $<$ or $>$, between the two numbers.
$-2 \quad -40$

14. Evaluate $-|-4|$.

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15. Write 45% as a fraction and as a decimal.

16. Add: $-22 + 14 + (-8)$

17. Multiply: $-4 \cdot 12$

18. Evaluate $16 \div 2[8 - 3(4 - 2)] + 1$.

19. Subtract: $16 - (-30) - 42$

20. Divide: $\frac{5}{12} \div \left(-\frac{5}{6}\right)$

21. Write 1.025 as a percent.

22. Evaluate $3^2 - 4 + 20 \div 5$.

23. Write $\frac{7}{9}$ as a decimal. Place a bar over the repeating digit of the decimal.

24. The table below shows the first-quarter profits and losses for 1997 for five companies in the paper industry. Profits are shown as positive numbers; losses are shown as negative numbers. One-quarter of a year is three months.

a. If earnings were to continue throughout the year at the same level, what would be the annual earnings or losses for Bowater, Inc.?

b. For the quarter shown, what was the average monthly profit or loss for Boise Cascade?

Paper Company	First Quarter 1997 Profits
Boise Cascade	–15,210,000
Bowater, Inc.	–3,140,000
Champion International	–37,083,000
Consolidated Paper	28,056,000
International Paper Company	34,000,000

Source: The Wall Street Journal, May 5, 1997

25. At the end of the trading day on July 30, the price of one share of Nike stock was $\$62\frac{3}{4}$. The change in the price of a share on July 31 was $-\frac{1}{8}$. What was the price of one share of Nike stock at the end of the trading day on July 31?

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CHAPTER

2 Variable Expressions



Nutritionists are hired by schools, hospitals, and other institutions to plan appealing meals that supply the nutrition needed to maintain good health. To plan a balanced diet, they need to know the recommended daily requirements of vitamins and minerals and the vitamin and mineral content of different foods. As meal planning requires dealing with so many "variables," a knowledge of variable expressions is very helpful in this career.

Objectives

Section 2.1

To evaluate a variable expression

Section 2.2

To simplify a variable expression using the Properties of Addition

To simplify a variable expression using the Properties of Multiplication

To simplify a variable expression using the Distributive Property

To simplify general variable expressions

Section 2.3

To translate a verbal expression into a variable expression, given the variable

To translate a verbal expression into a variable expression and then simplify

To translate application problems



History of Variables

Prior to the 16th century, unknown quantities were represented by words. In Latin, the language in which most scholarly works were written, the word *res*, meaning "thing," was used. In Germany the word *zahl*, meaning "number," was used. In Italy the word *cosa*, also meaning "thing," was used.

Then in 1637, René Descartes, a French mathematician, began using the letters x, y, and z to represent variables. It is interesting to note, on examining Descartes's work, that toward the end of the book the letters y and z were no longer used and x became the choice for a variable.

One explanation of why the letters y and z appeared less frequently has to do with the nature of printing presses during Descartes's time. A printer had a large tray that contained all the letters of the alphabet. There were many copies of each letter, especially those letters that are used frequently. For example, there were more *e*'s than *q*'s. Because the letters y and z do not occur frequently in French, a printer would have few of these letters on hand. Consequently, when Descartes started using these letters as variables, it quickly depleted the printer's supply and x's had to be used instead.

Today, x is used by most nations as the standard letter for a single unknown. In fact, x-rays were so named because the scientists who discovered them did not know what they were and thus labeled them the "unknown rays" or x-rays.

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2.1 Evaluating Variable Expressions

Objective A To evaluate a variable expression

Often we discuss a quantity without knowing its exact value—for example, the price of gold next month, the cost of a new automobile next year, or the tuition cost for next semester. Recall that a letter of the alphabet is used to stand for a quantity that is unknown or that can change, or *vary*. The letter is called a variable. An expression that contains one or more variables is called a **variable expression.**

POINT OF INTEREST

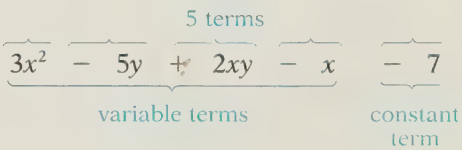
Historical manuscripts indicate that mathematics is at least 4000 years old. Yet it was only 400 years ago that mathematicians started using variables to stand for numbers. The idea that a letter can stand for some number was a critical turning point in mathematics.

A variable expression is shown at the right. The expression can be rewritten by writing subtraction as the addition of the opposite.

$$3x^2 - 5y + 2xy - x - 7$$

$$3x^2 + (-5y) + 2xy + (-x) + (-7)$$

Note that the expression has 5 addends. The **terms** of a variable expression are the addends of the expression. The expression has 5 terms.

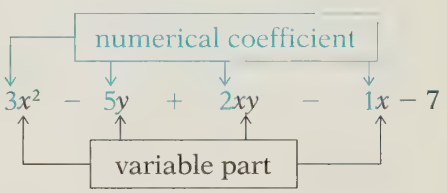


The terms $3x^2$, $-5y$, $2xy$, and $-x$ are **variable terms.**

The term -7 is a **constant term,** or simply a **constant.**

Each variable term is composed of a **numerical coefficient** and a **variable part** (the variable or variables and their exponents).

When the numerical coefficient is 1 or -1, the 1 is usually not written ($x = 1x$ and $-x = -1x$).



Variable expressions occur naturally in science. In a physics lab, a student may discover that a weight of 1 pound will stretch a spring $\frac{1}{2}$ inch. Two pounds will stretch the spring 1 inch. By experimenting, the student can discover that the distance the spring will stretch is found by multiplying the weight by $\frac{1}{2}$. By letting W represent the weight attached to the spring, the student can represent the distance the spring stretches by the variable expression $\frac{1}{2}W$.

With a weight of W pounds, the spring will stretch $\frac{1}{2} \cdot W = \frac{1}{2}W$ inches.

With a weight of 10 pounds, the spring will stretch $\frac{1}{2} \cdot 10 = 5$ inches. The number 10 is called the **value** of the variable W.

With a weight of 3 pounds, the spring will stretch $\frac{1}{2} \cdot 3 = 1\frac{1}{2}$ inches.

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Replacing each variable by its value and then simplifying the resulting numerical expression is called **evaluating the variable expression.**

CALCULATOR NOTE

See the appendix "Guidelines for Using Graphing Calculators" for instructions on using a graphing calculator to evaluate variable expressions.

➡ Evaluate $ab - b^2$ when $a = 2$ and $b = -3$.

Replace each variable in the expression by its value. Then use the Order of Operations Agreement to simplify the resulting numerical expression.

$ab - b^2$

$2(-3) - (-3)^2 = -6 - 9$

$= -15$

When $a = 2$ and $b = -3$, the value of $ab - b^2$ is -15.

Example 1 Name the variable terms of the expression $2a^2 - 5a + 7$.

Solution $2a^2$ and $-5a$

You Try It 1 Name the constant term of the expression $6n^2 + 3n - 4$.

Your solution

Example 2 Evaluate $x^2 - 3xy$ when $x = 3$ and $y = -4$.

Solution

$x^2 - 3xy$

$3^2 - 3(3)(-4) = 9 - 3(3)(-4)$

$= 9 - 9(-4)$

$= 9 - (-36)$

$= 9 + 36 = 45$

You Try It 2 Evaluate $2xy + y^2$ when $x = -4$ and $y = 2$.

Your solution

Example 3 Evaluate $\frac{a^2 - b^2}{a - b}$ when $a = 3$ and $b = -4$.

Solution

$\frac{a^2 - b^2}{a - b}$

$\frac{3^2 - (-4)^2}{3 - (-4)} = \frac{9 - 16}{3 - (-4)}$

$= \frac{-7}{7} = -1$

You Try It 3 Evaluate $\frac{a^2 + b^2}{a + b}$ when $a = 5$ and $b = -3$.

Your solution

Example 4 Evaluate $x^2 - 3(x - y) - z^2$ when $x = 2$, $y = -1$, and $z = 3$.

Solution

$x^2 - 3(x - y) - z^2$

$2^2 - 3[2 - (-1)] - 3^2$

$= 2^2 - 3(3) - 3^2$

$= 4 - 3(3) - 9$

$= 4 - 9 - 9$

$= -5 - 9$

$= -14$

You Try It 4 Evaluate $x^3 - 2(x + y) + z^2$ when $x = 2$, $y = -4$, and $z = -3$.

Your solution

Solutions on p. S3

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2.1 Exercises

Objective A

Name the terms of the variable expression. Then underline the constant term.

1. $2x^2 + 5x - 8$ **2.** $-3n^2 - 4n + 7$ **3.** $6 - a^4$

Name the variable terms of the expression. Then underline the variable part of each term.

4. $9b^2 - 4ab + a^2$ **5.** $7x^2y + 6xy^2 + 10$ **6.** $5 - 8n - 3n^2$

Name the coefficients of the variable terms.

7. $x^2 - 9x + 2$ **8.** $12a^2 - 8ab - b^2$ **9.** $n^3 - 4n^2 - n + 9$

Evaluate the variable expression when $a = 2$, $b = 3$, and $c = -4$.

10. $3a + 2b$ **11.** $a - 2c$ **12.** $-a^2$

13. $2c^2$ **14.** $-3a + 4b$ **15.** $3b - 3c$

16. $b^2 - 3$ **17.** $-3c + 4$ **18.** $16 \div (2c)$

19. $6b \div (-a)$ **20.** $bc \div (2a)$ **21.** $b^2 - 4ac$

22. $a^2 - b^2$ **23.** $b^2 - c^2$ **24.** $(a + b)^2$

25. $a^2 + b^2$ **26.** $2a - (c + a)^2$ **27.** $(b - a)^2 + 4c$

28. $b^2 - \frac{ac}{8}$ **29.** $\frac{5ab}{6} - 3cb$ **30.** $(b - 2a)^2 + bc$

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Evaluate the variable expression when $a = -2$, $b = 4$, $c = -1$, and $d = 3$.

31. $\dfrac{b + c}{d}$ **32.** $\dfrac{d - b}{c}$ **33.** $\dfrac{2d + b}{-a}$

34. $\dfrac{b + 2d}{b}$ **35.** $\dfrac{b - d}{c - a}$ **36.** $\dfrac{2c - d}{-ad}$

37. $(b + d)^2 - 4a$ **38.** $(d - a)^2 - 3c$ **39.** $(d - a)^2 \div 5$

40. $3(b - a) - bc$ **41.** $\dfrac{b - 2a}{bc^2 - d}$ **42.** $\dfrac{b^2 - a}{ad + 3c}$

43. $\dfrac{1}{3}d^2 - \dfrac{3}{8}b^2$ **44.** $\dfrac{5}{8}a^4 - c^2$ **45.** $\dfrac{-4bc}{2a - b}$

46. $-\dfrac{3}{4}b + \dfrac{1}{2}(ac + bd)$ **47.** $-\dfrac{2}{3}d - \dfrac{1}{5}(bd - ac)$ **48.** $(b - a)^2 - (d - c)^2$

49. $(b + c)^2 + (a + d)^2$ **50.** $4ac + (2a)^2$ **51.** $3dc - (4c)^2$

APPLYING THE CONCEPTS

52. Explain in your own words the meaning of "evaluate an algebraic expression."

Evaluate the following expressions for $x = 2$, $y = 3$, and $z = -2$.

53. $3^x - x^3$ **54.** z^x **55.** $x^x - y^y$ **56.** $y^{(x^2)}$

57. For each of the following, determine the first natural number x, greater than 1, for which the second expression is larger than the first.

a. x^3, 3^x **b.** x^4, 4^x **c.** x^5, 5^x **d.** x^6, 6^x

58. On the basis of your answer to Exercise 57, make a conjecture that appears to be true about the expressions x^n and n^x, where $n = 3, 4, 5, 6, 7, \ldots$ and x is a natural number greater than 1.

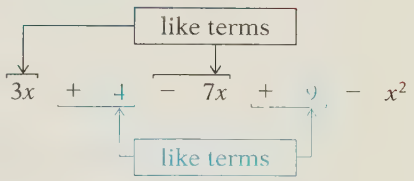
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2.2 Simplifying Variable Expressions

Objective A To simplify a variable expression using the Properties of Addition

Like terms of a variable expression are terms with the same variable part. (Because $x^2 = x \cdot x$, x^2 and x are not like terms.)

Constant terms are like terms. 4 and 9 are like terms.



To simplify a variable expression, use the Distributive Property to combine like terms by adding the numerical coefficients. The variable part remains unchanged.

Distributive Property

If a, b, and c are real numbers, then $a(b + c) = ab + ac$.

The Distributive Property can also be written $ba + ca = (b + c)a$. This form is used to simplify a variable expression.

To simplify $2x + 3x$, use the Distributive Property to add the numerical coefficients of the like variable terms. This is called **combining like terms.**

$$\begin{aligned} 2x + 3x &= (2 + 3)x \\ &= 5x \end{aligned}$$

➡ Simplify: $5y - 11y$

$$\begin{aligned} 5y - 11y &= (5 - 11)y \\ &= -6y \end{aligned}$$

• Use the Distributive Property.

TAKE NOTE

Simplifying an expression means combining like terms. A constant term (5) and a variable term ($7p$) are not like terms and therefore cannot be combined.

➡ Simplify: $5 + 7p$

The terms 5 and $7p$ are not like terms.

The expression $5 + 7p$ is in simplest form.

The Associative Property of Addition

If a, b, and c are real numbers, then $(a + b) + c = a + (b + c)$.

When three or more terms are added, the terms can be grouped (with parentheses, for example) in any order. The sum is the same. For example,

$$\begin{aligned} (3x + 5x) + 9x &= 3x + (5x + 9x) \\ 8x + 9x &= 3x + 14x \\ 17x &= 17x \end{aligned}$$

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The Commutative Property of Addition

If a and b are real numbers, then $a + b = b + a$.

When two like terms are added, the terms can be added in either order. The sum is the same. For example,

$$2x + (-4x) = -4x + 2x$$
$$-2x = -2x$$

The Addition Property of Zero

If a is a real number, then $a + 0 = 0 + a = a$.

The sum of a term and zero is the term. For example,

$$5x + 0 = 0 + 5x = 5x$$

The Inverse Property of Addition

If a is a real number, then $a + (-a) = (-a) + a = 0$.

The sum of a term and its opposite is zero. The opposite of a number is called its **additive inverse.**

$$7x + (-7x) = -7x + 7x = 0$$

➡ Simplify: $8x + 4y - 8x + y$

$8x + 4y - 8x + y$
$= (8x - 8x) + (4y + y)$ • Use the Commutative and Associative Properties of Addition to rearrange and group like terms.

$= 0 + 5y = 5y$ • Combine like terms.

➡ Simplify: $4x^2 + 5x - 6x^2 - 2x + 1$

$4x^2 + 5x - 6x^2 - 2x + 1$
$= (4x^2 - 6x^2) + (5x - 2x) + 1$ • Use the Commutative and Associative Properties of Addition to rearrange and group like terms.

$= -2x^2 + 3x + 1$ • Combine like terms.

Example 1 Simplify: $3x + 4y - 10x + 7y$

Solution $3x + 4y - 10x + 7y = -7x + 11y$

You Try It 1 Simplify: $3a - 2b - 5a + 6b$

Your solution

Example 2 Simplify: $x^2 - 7 + 4x^2 - 16$

Solution $x^2 - 7 + 4x^2 - 16 = 5x^2 - 23$

You Try It 2 Simplify: $-3y^2 + 7 + 8y^2 - 14$

Your solution

Solutions on p. S3

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Objective B To simplify a variable expression using the Properties of Multiplication

In simplifying variable expressions, the following Properties of Multiplication are used.

The Associative Property of Multiplication

If a, b, and c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

When three or more factors are multiplied, the factors can be grouped in any order. The product is the same. For example,

$$2(3x) = (2 \cdot 3)x = 6x$$

The Commutative Property of Multiplication

If a and b are real numbers, then $a \cdot b = b \cdot a$.

Two factors can be multiplied in either order. The product is the same. For example,

$$(2x) \cdot 3 = 3 \cdot (2x) = 6x$$

The Multiplication Property of One

If a is a real number, then $a \cdot 1 = 1 \cdot a = a$.

The product of a term and one is the term. For example,

$$(8x)(1) = (1)(8x) = 8x$$

The Inverse Property of Multiplication

If a is a real number, and a is not equal to zero, then

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$$

$\frac{1}{a}$ is called the **reciprocal** of a. $\frac{1}{a}$ is also called the **multiplicative inverse** of a. The product of a number and its reciprocal is one. For example,

$$7 \cdot \frac{1}{7} = \frac{1}{7} \cdot 7 = 1$$

The multiplication properties just discussed are used to simplify variable expressions.

➡ Simplify: $2(-x)$

$$\begin{aligned} 2(-x) &= 2(-1 \cdot x) \\ &= [2(-1)]x \\ &= -2x \end{aligned}$$

• Use the Associative Property of Multiplication to group factors.

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➡ Simplify: $\frac{3}{2}\left(\frac{2x}{3}\right)$

$$\frac{3}{2}\left(\frac{2x}{3}\right) = \frac{3}{2}\left(\frac{2}{3}x\right)$$

• Note that $\frac{2x}{3} = \frac{2}{3}x$.

$$= \left(\frac{3}{2} \cdot \frac{2}{3}\right)x$$

• Use the Associative Property of Multiplication to group factors.

$$= 1 \cdot x$$

$$= x$$

➡ Simplify: $(16x)2$

$$(16x)2 = 2(16x)$$

• Use the Commutative and Associative Properties of Multiplication to rearrange and group factors.

$$= (2 \cdot 16)x$$

$$= 32x$$

Example 3 Simplify: $-2(3x^2)$

Solution $-2(3x^2) = -6x^2$

You Try It 3 Simplify: $-5(4y^2)$

Your solution

Example 4 Simplify: $-5(-10x)$

Solution $-5(-10x) = 50x$

You Try It 4 Simplify: $-7(-2a)$

Your solution

Example 5 Simplify: $(6x)(-4)$

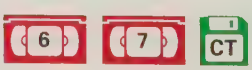
Solution $(6x)(-4) = -24x$

You Try It 5 Simplify: $(-5x)(-2)$

Your solution

Solutions on p. S3

Objective C *To simplify a variable expression using the Distributive Property*



Recall that the Distributive Property states that if a, b, and c are real numbers, then

$$a(b + c) = ab + ac$$

The Distributive Property is used to remove parentheses from a variable expression.

➡ Simplify: $3(2x + 7)$

$$3(2x + 7) = 3(2x) + 3(7)$$

• Use the Distributive Property. Multiply each term inside the parentheses by 3.

$$= 6x + 21$$

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➡ Simplify: $-5(4x + 6)$

$$\begin{aligned} -5(4x + 6) &= -5(4x) + (-5) \cdot 6 \\ &= -20x - 30 \end{aligned}$$

• Use the Distributive Property.

➡ Simplify: $-(2x - 4)$

$$\begin{aligned} -(2x - 4) &= -1(2x - 4) \\ &= 1(2x) - (-1)(4) \\ &= -2x + 4 \end{aligned}$$

• Use the Distributive Property.

Note: When a negative sign immediately precedes the parentheses, the sign of each term inside the parentheses is changed.

➡ Simplify: $-\frac{1}{2}(8x - 12y)$

$$\begin{aligned} -\frac{1}{2}(8x - 12y) &= \frac{1}{2}(8x) - \left(-\frac{1}{2}\right)(12y) \\ &= -4x + 6y \end{aligned}$$

• Use the Distributive Property.

An extension of the Distributive Property is used when an expression contains more than two terms.

➡ Simplify: $3(4x - 2y - z)$

$$\begin{aligned} 3(4x - 2y - z) &= 3(4x) - 3(2y) - 3(z) \\ &= 12x - 6y - 3z \end{aligned}$$

• Use the Distributive Property.

Example 6
Simplify: $7(4 + 2x)$

Solution
$7(4 + 2x) = 28 + 14x$

You Try It 6
Simplify: $5(3 + 7b)$

Your solution

Example 7
Simplify: $(2x - 6)2$

Solution
$(2x - 6)2 = 4x - 12$

You Try It 7
Simplify: $(3a - 1)5$

Your solution

Example 8
Simplify: $-3(-5a + 7b)$

Solution
$-3(-5a + 7b) = 15a - 21b$

You Try It 8
Simplify: $-8(-2a + 7b)$

Your solution

Solutions on p. S3

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Example 9 Simplify: $3(x^2 - x - 5)$

Solution $3(x^2 - x - 5) = 3x^2 - 3x - 15$

You Try It 9 Simplify: $3(12x^2 - x + 8)$

Your solution

Example 10 Simplify: $-2(x^2 + 5x - 4)$

Solution $-2(x^2 + 5x - 4)$
$= -2x^2 - 10x + 8$

You Try It 10 Simplify: $3(-a^2 - 6a + 7)$

Your solution

Solutions on p. S3

Objective D To simplify general variable expressions

6 CT

When simplifying variable expressions, use the Distributive Property to remove parentheses and brackets used as grouping symbols.

Simplify: $4(x - y) - 2(-3x + 6y)$

$4(x - y) - 2(-3x + 6y)$
$= 4x - 4y + 6x - 12y$ • Use the Distributive Property.
$= 10x - 16y$ • Combine like terms.

Example 11 Simplify: $2x - 3(2x - 7y)$

Solution $2x - 3(2x - 7y) = 2x - 6x + 21y$
$= -4x + 21y$

You Try It 11 Simplify: $3y - 2(y - 7x)$

Your solution

Example 12 Simplify:
$7(x - 2y) - (-x - 2y)$

Solution $7(x - 2y) - (-x - 2y)$
$= 7x - 14y + x + 2y$
$= 8x - 12y$

You Try It 12 Simplify:
$-2(x - 2y) - (-x + 3y)$

Your solution

Example 13 Simplify:
$2x - 3[2x - 3(x + 7)]$

Solution $2x - 3[2x - 3(x + 7)]$
$= 2x - 3[2x - 3x - 21]$
$= 2x - 3[-x - 21]$
$= 2x + 3x + 63$
$= 5x + 63$

You Try It 13 Simplify:
$3y - 2[x - 4(2 - 3y)]$

Your solution

Solutions on p. S3

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2.2 Exercises

Objective A

Simplify.

1. $6x + 8x$

2. $12x + 13x$

3. $9a - 4a$

4. $12a - 3a$

5. $4y + (-10y)$

6. $8y + (-6y)$

7. $-3b - 7$

8. $-12y - 3$

9. $-12a + 17a$

10. $-3a + 12a$

11. $5ab - 7ab$

12. $9ab - 3ab$

13. $-12xy + 17xy$

14. $-15xy + 3xy$

15. $-3ab + 3ab$

16. $-7ab + 7ab$

17. $-\frac{1}{2}x - \frac{1}{3}x$

18. $-\frac{2}{5}y + \frac{3}{10}y$

19. $\frac{3}{8}x^2 - \frac{5}{12}x^2$

20. $\frac{2}{3}y^2 - \frac{4}{9}y^2$

21. $3x + 5x + 3x$

22. $8x + 5x + 7x$

23. $5a - 3a + 5a$

24. $10a - 17a + 3a$

25. $-5x^2 - 12x^2 + 3x^2$

26. $-y^2 - 8y^2 + 7y^2$

27. $7x + (-8x) + 3y$

28. $8y + (-10x) + 8x$

29. $7x - 3y + 10x$

30. $8y + 8x - 8y$

31. $3a + (-7b) - 5a + b$

32. $-5b + 7a - 7b + 12a$

33. $3x + (-8y) - 10x + 4x$

34. $3y + (-12x) - 7y + 2y$

35. $x^2 - 7x + (-5x^2) + 5x$

36. $3x^2 + 5x - 10x^2 - 10x$

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Objective B

Simplify.

37. $4(3x)$ **38.** $12(5x)$ **39.** $-3(7a)$ **40.** $-2(5a)$ **41.** $-2(-3y)$

42. $-5y(-6y)$ **43.** $(4x)2$ **44.** $(6x)12$ **45.** $(3a)(-2)$ **46.** $(7a)(-4)$

47. $(-3b)(-4)$ **48.** $(-12b)(-9)$ **49.** $-5(3x^2)$ **50.** $-8(7x^2)$ **51.** $\frac{1}{3}(3x^2)$

52. $\frac{1}{6}(6x^2)$ **53.** $\frac{1}{5}(5a)$ **54.** $\frac{1}{8}(8x)$ **55.** $-\frac{1}{2}(-2x)$ **56.** $-\frac{1}{4}(-4a)$

57. $-\frac{1}{7}(-7n)$ **58.** $-\frac{1}{9}(-9b)$ **59.** $(3x)\left(\frac{1}{3}\right)$ **60.** $(12x)\left(\frac{1}{12}\right)$ **61.** $(-6y)\left(-\frac{1}{6}\right)$

62. $(-10n)\left(-\frac{1}{10}\right)$ **63.** $\frac{1}{3}(9x)$ **64.** $\frac{1}{7}(14x)$ **65.** $-\frac{1}{5}(10x)$ **66.** $-\frac{1}{8}(16x)$

67. $-\frac{2}{3}(12a^2)$ **68.** $-\frac{5}{8}(24a^2)$ **69.** $-\frac{1}{2}(-16y)$ **70.** $-\frac{3}{4}(-8y)$ **71.** $(16y)\left(\frac{1}{4}\right)$

72. $(33y)\left(\frac{1}{11}\right)$ **73.** $(-6x)\left(\frac{1}{3}\right)$ **74.** $(-10x)\left(\frac{1}{5}\right)$ **75.** $(-8a)\left(-\frac{3}{4}\right)$ **76.** $(21y)\left(-\frac{3}{7}\right)$

Objective C

Simplify.

77. $-(x + 2)$ **78.** $-(x + 7)$ **79.** $2(4x - 3)$ **80.** $5(2x - 7)$

81. $-2(a + 7)$ **82.** $-5(a + 16)$ **83.** $-3(2y - 8)$ **84.** $-5(3y - 7)$

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85. $(5 - 3b)7$ **86.** $(10 - 7b)2$ **87.** $\frac{1}{3}(6 - 15y)$ **88.** $\frac{1}{2}(-8x + 4y)$

89. $3(5x^2 + 2x)$ **90.** $6(3x^2 + 2x)$ **91.** $-2(-y + 9)$ **92.** $-5(-2x + 7)$

93. $(-3x - 6)5$ **94.** $(-2x + 7)7$ **95.** $2(-3x^2 - 14)$ **96.** $5(-6x^2 - 3)$

97. $-3(2y^2 - 7)$ **98.** $-8(3y^2 - 12)$ **99.** $3(x^2 - y^2)$ **100.** $5(x^2 + y^2)$

101. $-\frac{2}{3}(6x - 18y)$ **102.** $-\frac{1}{2}(x - 4y)$ **103.** $-(6a^2 - 7b^2)$

104. $3(x^2 + 2x - 6)$ **105.** $4(x^2 - 3x + 5)$ **106.** $-2(y^2 - 2y + 4)$

107. $\frac{1}{2}(2x - 6y + 8)$ **108.** $-\frac{1}{3}(6x - 9y + 1)$ **109.** $4(-3a^2 - 5a + 7)$

110. $-5(-2x^2 - 3x + 7)$ **111.** $-3(-4x^2 + 3x - 4)$ **112.** $3(2x^2 + xy - 3y^2)$

113. $5(2x^2 - 4xy - y^2)$ **114.** $-(3a^2 + 5a - 4)$ **115.** $-(8b^2 - 6b + 9)$

Objective D

Simplify.

116. $4x - 2(3x + 8)$ **117.** $6a - (5a + 7)$ **118.** $9 - 3(4y + 6)$

119. $10 - (11x - 3)$ **120.** $5n - (7 - 2n)$ **121.** $8 - (12 + 4y)$

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122. $3(x + 2) - 5(x - 7)$

123. $2(x - 4) - 4(x + 2)$

124. $12(y - 2) + 3(7 - 3y)$

125. $6(2y - 7) - (3 - 2y)$

126. $3(a - b) - (a + b)$

127. $2(a + 2b) - (a - 3b)$

128. $4[x - 2(x - 3)]$

129. $2[x + 2(x + 7)]$

130. $-2[3x + 2(4 - x)]$

131. $-5[2x + 3(5 - x)]$

132. $-3[2x - (x + 7)]$

133. $-2[3x - (5x - 2)]$

134. $2x - 3[x - (4 - x)]$

135. $-7x + 3[x - (3 - 2x)]$

136. $-5x - 2[2x - 4(x + 7)] - 6$

APPLYING THE CONCEPTS

137. Determine whether the statement is true or false. If the statement is false, give an example that illustrates that it is false.

a. Division is a commutative operation.
b. Division is an associative operation.
c. Subtraction is an associative operation.
d. Subtraction is a commutative operation.

138. Is the statement "any number divided by itself is 1" a true statement? If not, for what number or numbers is the statement not true?

139. Does every number have a multiplicative inverse? If not, which real number or numbers do not have a multiplicative inverse?

140. In your own words, explain the distributive property.

141. Give examples of two operations that occur in everyday experience that are not commutative (for example, putting on socks and then shoes).

142. Define an operation $\otimes$ as $a \otimes b = (a \cdot b) - (a + b)$. For example, $7 \otimes 5 = (7 \cdot 5) - (7 + 5) = 35 - 12 = 23$.

a. Is $\otimes$ a commutative operation? Support your answer.

b. Is $\otimes$ an associative operation? Support your answer.

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2.3 Translating Verbal Expressions into Variable Expressions

Objective A To translate a verbal expression into a variable expression, given the variable

One of the major skills required in applied mathematics is to translate a verbal expression into a variable expression. This requires recognizing the verbal phrases that translate into mathematical operations. A partial list of the verbal phrases used to indicate the different mathematical operations follows.

Addition	added to	6 added to y	$y + 6$
	more than	8 more than x	$x + 8$
	the sum of	the sum of x and z	$x + z$
	increased by	t increased by 9	$t + 9$
	the total of	the total of 5 and y	$5 + y$
Subtraction	minus	x minus 2	$x - 2$
	less than	7 less than t	$t - 7$
	decreased by	m decreased by 3	$m - 3$
	the difference between	the difference between y and 4	$y - 4$
Multiplication	times	10 times t	$10t$
	of	one half of x	$\frac{1}{2}x$
	the product of	the product of y and z	yz
	multiplied by	y multiplied by 11	$11y$
Division	divided by	x divided by 12	$\frac{x}{12}$
	the quotient of	the quotient of y and z	$\frac{y}{z}$
	the ratio of	the ratio of t to 9	$\frac{t}{9}$
Power	the square of	the square of x	x^2
	the cube of	the cube of a	a^3

POINT OF INTEREST

The way in which expressions are symbolized has changed over time. Here are how some of the expressions shown at the right may have appeared in the early 16th century.

R p. 9 for $x + 9$. The symbol R was used for a variable to the first power. The symbol p. was used for plus.

R m. 3 for $x - 3$. The symbol R is still used for the variable. The symbol m. is used for minus.

The square of a variable was designated by Q and the cube was designated by C. The expression $x^2 + x^3$ was written **Q p. C.**

➡ Translate "14 less than the cube of x" into a variable expression.

14 *less than* the *cube* of x

$x^3 - 14$

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Translating a phrase that contains the word *sum, difference, product,* or *quotient* can sometimes cause a problem. In the examples at the right, note where the operation symbol is placed.

the *sum* of x *and* y $\quad x + y$

the *difference* between x *and* y $\quad x - y$

the *product* of x *and* y $\quad x \cdot y$

the *quotient* of x *and* y $\quad \frac{x}{y}$

➡ Translate "the difference between the square of x and the sum of y and z" into a variable expression.

the difference between the square of x and the sum of y and z

$x^2 - (y + z)$

Example 1
Translate "the total of 3 times n and n" into a variable expression.

Solution
the total of 3 times n and n
$3n + n$

You Try It 1
Translate "the difference between twice n and one-third of n" into a variable expression.

Your solution

Example 2
Translate "m decreased by the sum of n and 12" into a variable expression.

Solution
m decreased by the sum of n and 12
$m - (n + 12)$

You Try It 2
Translate "the quotient of 7 less than b and 15" into a variable expression.

Your solution

Solutions on p. S3

Objective B To translate a verbal expression into a variable expression and then simplify

In most applications that involve translating phrases into variable expressions, the variable to be used is not given. To translate these phrases, a variable must be assigned to an unknown quantity before the variable expression can be written.

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➡ Translate "a number multiplied by the total of six and the cube of the number" into a variable expression.

the unknown number: n — Assign a variable to one of the unknown quantities.

the cube of the number: n^3
the total of six and the cube of the number: $6 + n^3$ — Use the assigned variable to write an expression for any other unknown quantity.

$n(6 + n^3)$ — Use the assigned variable to write the variable expression.

Example 3
Translate "a number added to the product of four and the square of the number" into a variable expression.

Solution
the unknown number: n
the square of the number: n^2
the product of four and the square of the number: $4n^2$
$n + 4n^2$

You Try It 3
Translate "negative four multiplied by the total of ten and the cube of a number" into a variable expression.

Your solution

Example 4
Translate "four times the sum of half of a number and fourteen" into a variable expression. Then simplify.

Solution
the unknown number: n
half of the number: $\frac{1}{2}n$
the sum of half of the number and fourteen: $\frac{1}{2}n + 14$
$4\left(\frac{1}{2}n + 14\right)$
$2n + 56$

You Try It 4
Translate "five times the difference between a number and sixty" into a variable expression. Then simplify.

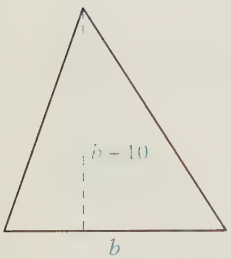
Your solution

Solutions on p. S4

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Objective C To translate application problems

Many of the applications of mathematics require that you identify the unknown quantity, assign a variable to that quantity, and then attempt to express other unknown quantities in terms of the variable.



➡ The height of a triangle is 10 ft longer than the base of the triangle. Express the height of the triangle in terms of the base of the triangle.

the base of the triangle: b • **Assign a variable to the base of the triangle.**

the height is 10 more than the base: $b + 10$ • **Express the height of the triangle in terms of b.**

Example 5
The length of a swimming pool is 4 ft less than two times the width. Express the length of the pool in terms of the width.

Solution
the width of the pool: w
the length is 4 ft less than two times the width: $2w - 4$

You Try It 5
The speed of a new jet plane is twice the speed of an older model. Express the speed of the new model in terms of the speed of the older model.

Your solution

Example 6
A banker divided $5000 between two accounts, one paying 10% annual interest and the second paying 8% annual interest. Express the amount invested in the 10% account in terms of the amount invested in the 8% account.

Solution
the amount invested at 8%: x
the amount invested at 10%: $5000 - x$

You Try It 6
A guitar string 6 ft long was cut into two pieces. Express the length of the shorter piece in terms of the length of the longer piece.

Your solution

Solutions on p. S4

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2.3 Exercises

Objective A

Translate into a variable expression.

1. the sum of 8 and y
2. a less than 16
3. t increased by 10
4. p decreased by 7
5. z added to 14
6. q multiplied by 13
7. 20 less than the square of x
8. 6 times the difference between m and 7
9. the sum of three-fourths of n and 12
10. b decreased by the product of 2 and b
11. 8 increased by the quotient of n and 4
12. the product of -8 and y
13. the product of 3 and the total of y and 7
14. 8 divided by the difference between x and 6
15. the product of t and the sum of t and 16
16. the quotient of 6 less than n and twice n
17. 15 more than one-half of the square of x
18. 19 less than the product of n and -2
19. the total of 5 times the cube of n and the square of n
20. the ratio of 9 more than m to m
21. r decreased by the quotient of r and 3
22. four-fifths of the sum of w and 10
23. the difference between the square of x and the total of x and 17
24. s increased by the quotient of 4 and s
25. the product of 9 and the total of z and 4
26. n increased by the difference between 10 times n and 9

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Objective B

Translate into a variable expression. Then simplify.

27. twelve minus a number

28. a number divided by eighteen

29. two-thirds of a number

30. twenty more than a number

31. the quotient of twice a number and nine

32. ten times the difference between a number and fifty

33. eight less than the product of eleven and a number

34. the sum of five-eighths of a number and six

35. nine less than the total of a number and two

36. the difference between a number and three more than the number

37. the quotient of seven and the total of five and a number

38. four times the sum of a number and nineteen

39. five increased by one-half of the sum of a number and three

40. the quotient of fifteen and the sum of a number and twelve

41. a number added to the difference between twice the number and four

42. the product of two-thirds and the sum of a number and seven

43. the product of five less than a number and seven

44. the difference between forty and the quotient of a number and twenty

45. the quotient of five more than twice a number and the number

46. the sum of the square of a number and twice the number

47. a number decreased by the difference between three times the number and eight

48. the sum of eight more than a number and one-third of the number

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49. a number added to the product of three and the number

50. a number increased by the total of the number and nine

51. five more than the sum of a number and six

52. a number decreased by the difference between eight and the number

53. a number minus the sum of the number and ten

54. the difference between one-third of a number and five-eighths of the number

55. the sum of one-sixth of a number and four-ninths of the number

56. two more than the total of a number and five

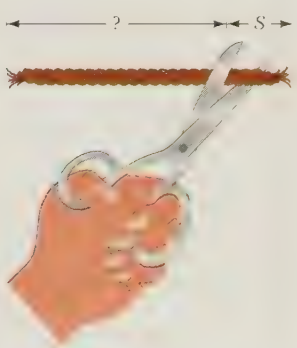
57. the sum of a number divided by three and the number

58. twice the sum of six times a number and seven

Objective C Application Problems

59. In November 1996, the average sale price of a home in Vail, Colorado, was $143,600 more than the average sale price of a home in Carmel, California. Express the average sale price of a home in Vail in terms of the average sale price of a home in Carmel.

60. According to *USA Today*, in Norway, annual spending per person for books is $40 more than it is in the United States. Express the annual spending per person for books in Norway in terms of the annual spending per person for books in the United States.



61. A rope 12 ft long was cut into two pieces of different length. Use one variable to express the lengths of the two pieces.

62. Twenty gallons of crude oil were poured into two containers of different sizes. Use one variable to express the amount of oil poured into each container.

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63. Two cars start at the same place and travel at different rates in opposite directions. Two hours later the cars are 200 mi apart. Express the distance traveled by the slower car in terms of the distance traveled by the faster car.



64. According to the *Wall Street Journal*, in a recent year, retail sales in Boston were one-half the retail sales in Los Angeles. Express the retail sales in Boston in terms of the retail sales in Los Angeles.

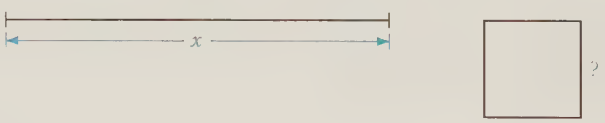
65. According to the IRS, it should take about one-fourth the time to prepare Schedule A than to prepare Form 1040. Express the time it should take to prepare Schedule A in terms of the time it should take to prepare Form 1040.

66. The diameter of a basketball is approximately 4 times the diameter of a baseball. Express the diameter of a basketball in terms of the diameter of a baseball.

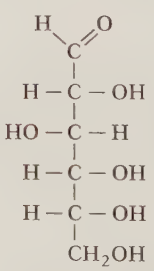
67. The world population in the year 2050 is expected to be twice the world population in 1990. Express the world population in 2050 in terms of the world population in 1990.

APPLYING THE CONCEPTS

68. A wire whose length is given as x inches is bent into a square. Express the length of a side of the square in terms of x.



69. The chemical formula for glucose (sugar) is $C_6H_{12}O_6$. This formula means that there are 12 hydrogen atoms for every 6 carbon atoms and 6 oxygen atoms in each molecule of glucose (see the figure to the right). If x represents the number of atoms of oxygen in a pound of sugar, express the number of hydrogen atoms in the pound of sugar.



70. Translate the expressions $5x + 8$ and $5(x + 8)$ into phrases.

71. In your own words, explain how variables are used.

72. Explain the similarities and the differences between the expressions "the difference between x and 5" and "5 less than x."

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Focus on Problem Solving

From Concrete to Abstract

In your study of algebra, you will find that the problems are less concrete than those you studied in arithmetic. Problems that are concrete provide information pertaining to a specific instance. Algebra is more abstract. Abstract problems are theoretical; they are stated without reference to a specific instance. Let's look at an example of an abstract problem.

How many minutes are in h hours?

A strategy that can be used to solve this problem is to solve the same problem after substituting a number for the variable.

How many minutes are in 5 hours?

You know that there are 60 minutes in 1 hour. To find the number of minutes in 5 hours, multiply 5 by 60.

$60 \cdot 5 = 300$ There are 300 minutes in 5 hours.

Use the same procedure to find the number of minutes in h hours: multiply h by 60.

$60 \cdot h = 60h$ There are $60h$ minutes in h hours.

This problem might be taken a step further:

If you walk one mile in x minutes, how far can you walk in h hours?

Consider the same problem using numbers in place of the variables.

If you walk one mile in 20 minutes, how far can you walk in 3 hours?

To solve this problem, you need to calculate the number of minutes in 3 hours (multiply 3 by 60), and divide the result by the number of minutes it takes to walk one mile (20 minutes).

$\frac{60 \cdot 3}{20} = \frac{180}{20} = 9$ If you walk one mile in 20 minutes, you can walk 9 miles in 3 hours.

Use the same procedure to solve the related abstract problem. Calculate the number of minutes in h hours (multiply h by 60), and divide the result by the number of minutes it takes to walk one mile (x minutes).

$\frac{60 \cdot h}{x} = \frac{60h}{x}$ If you walk one mile in x minutes, you can walk $\frac{60h}{x}$ miles in h hours.

At the heart of the study of algebra is the use of variables. It is the variables in the problems above that make them abstract. But it is variables that allow us to generalize situations and state rules about mathematics.

Try each of the following problems.

1. How many hours are in d days?

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2. You earn d dollars an hour. What are your wages for working h hours?
3. If p is the price of one share of stock, how many shares can you purchase with d dollars?
4. A company pays a television station d dollars to air a commercial lasting s seconds. What is the cost per second?
5. After every v video tape rentals, you are entitled to one free rental. You have rented t tapes, where $t < v$. How many more do you need to rent before you are entitled to a free rental?
6. Your car gets g miles per gallon. How many gallons of gasoline does your car consume traveling t miles?
7. If you drink j ounces of juice each day, how many days will q quarts of the juice last?
8. A TV station has m minutes of commercials each hour. How many ads lasting s seconds each can be sold for each hour of programming?
9. A factory worker can assemble p products in m minutes. How many products can the factory worker assemble in h hours?
10. If one candy bar costs n nickels, how many candy bars can be purchased with q quarters?

Projects and Group Activities

Prime and Composite Numbers

A **prime number** is a natural number greater than 1 whose only natural-number factors are itself and 1. The number 11 is a prime number because the only natural-number factors of 11 are 11 and 1.

Eratosthenes, a Greek philosopher and astronomer who lived from 270 to 190 B.C., devised a method of identifying prime numbers. It is called the **Sieve of Eratosthenes.** The procedure is illustrated below.

~~1~~	(2)	(3)	~~4~~	(5)	~~6~~	(7)	~~8~~	~~9~~	~~10~~
(11)	~~12~~	(13)	~~14~~	~~15~~	~~16~~	(17)	~~18~~	(19)	~~20~~
~~21~~	~~22~~	(23)	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	(29)	~~30~~
(31)	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	(37)	~~38~~	~~39~~	~~40~~
(41)	~~42~~	(43)	~~44~~	~~45~~	~~46~~	(47)	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	(53)	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	(59)	~~60~~
(61)	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	(67)	~~68~~	~~69~~	~~70~~
(71)	~~72~~	(73)	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	(79)	~~80~~
~~81~~	~~82~~	(83)	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	(89)	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	(97)	~~98~~	~~99~~	~~100~~

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List all the natural numbers from 1 to 100. Cross out the number 1, because it is not a prime number. The number 2 is prime; circle it. Cross out all the other multiples of 2 (4, 6, 8,...), because they are not prime. The number 3 is prime; circle it. Cross out all the other multiples of 3 (6, 9, 12,...) that are not already crossed out. The number 4, the next consecutive number in the list, has already been crossed out. The number 5 is prime; circle it. Cross out all the other multiples of 5 that are not already crossed out. Continue in this manner until all the prime numbers less than 100 are circled.

A **composite number** is a natural number greater than 1 that has a natural-number factor other than itself and 1. The number 21 is a composite number because it has factors of 3 and 7. All the numbers crossed out in the preceding table, except the number 1, are composite numbers.

1. Use the Sieve of Eratosthenes to find the prime numbers between 100 and 200.
2. How many prime numbers are even numbers?
3. Find the "twin primes" between 1 and 200. Twin primes are two prime numbers whose difference is 2. For instance, 3 and 5 are twin primes; 5 and 7 are also twin primes.
4. **a.** List two prime numbers that are consecutive natural numbers.
   b. Can there be any other pairs of prime numbers that are consecutive natural numbers?
5. Some primes are the sum of a square and 1. For example, $5 = 2^2 + 1$. Find another prime p such that $p = n^2 + 1$, where n is a natural number.
6. Find a prime number p such that $p = n^2 - 1$, where n is a natural number.
7. **a.** 4! (which is read "4 factorial") is equal to $4 \cdot 3 \cdot 2 \cdot 1$. Show that $4! + 2$, $4! + 3$, and $4! + 4$ are all composite numbers.
   b. 5! (which is read "5 factorial") is equal to $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. Will $5! + 2$, $5! + 3$, $5! + 4$, and $5! + 5$ generate four consecutive composite numbers?
   c. Use the notation 6! to represent a list of five consecutive composite numbers.

Investigation into Operations with Even and Odd Integers

Complete each statement with the word *even* or *odd*.

1. If k is an odd integer, then $k + 1$ is an _____ integer.
2. If k is an odd integer, then $k - 2$ is an _____ integer.
3. If n is an integer, then $2n$ is an _____ integer.
4. If m and n are even integers, then $m - n$ is an _____ integer.
5. If m and n are even integers, then mn is an _____ integer.
6. If m and n are odd integers, then $m + n$ is an _____ integer.
7. If m and n are odd integers, then $m - n$ is an _____ integer.
8. If m and n are odd integers, then mn is an _____ integer.
9. If m is an even integer and n is an odd integer, then $m - n$ is an _____ integer.
10. If m is an even integer and n is an odd integer, then $m + n$ is an _____ integer.

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Chapter Summary

Key Words

A *variable* is a letter that is used to stand for a quantity that is unknown.

TAKE NOTE

"To the Student" on page xix provides suggestions on how best to make use of the "Chapter Summary," the "Chapter Revew," and the "Chapter Test."

A *variable expression* is an expression that contains one or more variables.

The *terms* of a variable expression are the addends of the expression. A *variable term* is composed of a *numerical coefficient* and a *variable part*. A *constant term* has no variable part.

Replacing the variables in a variable expression with numbers and then simplifying the numerical expression is called *evaluating the variable expression.*

Like terms of a variable expression are terms that have the same variable part. Adding or subtracting the like terms of a variable expression is called *combining like terms.*

The *additive inverse* of a number is the opposite of the number.

The *multiplicative inverse* of a number is the reciprocal of the number.

Essential Rules

The Associative Property of Addition

If a, b, and c are real numbers, then $(a + b) + c = a + (b + c)$.

The Commutative Property of Addition

If a and b are real numbers, then $a + b = b + a$.

The Addition Property of Zero

If a is a real number, then $a + 0 = 0 + a = a$.

The Inverse Property of Addition

If a is a real number, then $a + (-a) = (-a) + a = 0$.

The Associative Property of Multiplication

If a, b, and c are real numbers, then $(ab)c = a(bc)$.

The Commutative Property of Multiplication

If a and b are real numbers, then $ab = ba$.

The Multiplication Property of One

If a is a real number, then $1 \cdot a = a \cdot 1 = a$.

The Inverse Property of Multiplication

If a is a nonzero real number, then $a\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)a = 1$.

The Distributive Property

If a, b, and c are real numbers, then $a(b + c) = ab + ac$.

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Chapter Review

1. Simplify: $3(x^2 - 8x - 7)$

2. Simplify: $7x + 4x$

3. Simplify: $6a - 4b + 2a$

4. Simplify: $(-50n)\left(\frac{1}{10}\right)$

5. Evaluate $(5c - 4a)^2 - b$ when $a = -1$, $b = 2$, and $c = 1$.

6. Simplify: $5(2x - 7)$

7. Simplify: $2(6y^2 + 4y - 5)$

8. Simplify: $\frac{1}{4}(-24a)$

9. Simplify: $-6(7x^2)$

10. Simplify: $-9(7 + 4x)$

11. Simplify: $12y - 17y$

12. Evaluate $2bc \div (a + 7)$ when $a = 3$, $b = -5$, and $c = 4$.

13. Simplify: $6(8y - 3) - 8(3y - 6)$

14. Simplify: $5c + (-2d) - 3d - (-4c)$

15. Simplify: $5(4x)$

16. Simplify: $-4(2x - 9) + 5(3x + 2)$

17. Evaluate $(b - a)^2 + c$ when $a = -2$, $b = 3$, and $c = 4$.

18. Simplify: $-9r + 2s - 6s + 12s$

19. Simplify: $4x - 3x^2 + 2x - x^2$

20. Simplify: $5[2 - 3(6x - 1)]$

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21. Simplify: $(7a^2 - 2a + 3)4$

22. Simplify: $18 - (4x - 2)$

23. Evaluate $a^2 - b^2$ when $a = 3$ and $b = 4$.

24. Simplify: $-3(-12y)$

25. Translate "two-thirds of the total of x and 10" into a variable expression.

26. A candy bar contains eight more calories than twice the number of calories in an apple. Express the number of calories in a candy bar in terms of the number of calories in an apple.

27. Translate "three times a number plus the product of five and one less than the number" into a variable expression. Then simplify.

28. Translate "the product of 4 and x" into a variable expression.

29. A club treasurer has some five-dollar bills and some ten-dollar bills. The treasurer has a total of 35 bills. Express the number of five-dollar bills in terms of the number of ten-dollar bills.

30. Translate "the difference between twice a number and one-half of the number" into a variable expression. Then simplify.

31. A baseball card collection contains five times as many National League players' cards as American League players' cards. Express the number of National League players' cards in the collection in terms of the number of American League players' cards.

32. Translate "6 less than x" into a variable expression.

33. Translate "a number plus twice the number" into a variable expression. Then simplify.

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Chapter Test

1. Simplify: $3x - 5x + 7x$

2. Simplify: $-3(2x^2 - 7y^2)$

3. Simplify: $2x - 3(x - 2)$

4. Simplify: $2x + 3[4 - (3x - 7)]$

5. Simplify: $3x - 7y - 12x$

6. Evaluate $b^2 - 3ab$ when $a = 3$ and $b = -2$.

7. Simplify: $\frac{1}{5}(10x)$

8. Simplify: $5(2x + 4) - 3(x - 6)$

9. Simplify: $-5(2x^2 - 3x + 6)$

10. Simplify: $3x + (-12y) - 5x - (-7y)$

11. Evaluate $\frac{-2ab}{2b - a}$ when $a = -4$ and $b = 6$.

12. Simplify: $(12x)\left(\frac{1}{4}\right)$

13. Simplify: $-7y^2 + 6y^2 - (-2y^2)$

14. Simplify: $-2(2x - 4)$

15. Simplify: $\frac{2}{3}(-15a)$

16. Simplify: $-2[x - 2(x - y)] + 5y$

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17. Simplify: $(-3)(-12y)$

18. Simplify: $5(3 - 7b)$

19. Translate "the difference between the squares of a and b" into a variable expression.

20. Translate "ten times the difference between a number and 3" into a variable expression. Then simplify.

21. Translate "the sum of a number and twice the square of the number" into a variable expression.

22. The speed of a pitcher's fastball is twice the speed of the catcher's return throw. Express the speed of the fastball in terms of the speed of the return throw.

23. Translate "three less than the quotient of six and a number" into a variable expression.

24. Translate "b decreased by the product of b and seven" into a variable expression.

25. A wire is cut into two lengths. The length of the longer piece is 3 in. less than four times the length of the shorter piece. Express the length of the longer piece in terms of the length of the shorter piece.

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Cumulative Review

1. Add: $-4 + 7 + (-10)$

2. Subtract: $-16 - (-25) - 4$

3. Multiply: $(-2)(3)(-4)$

4. Divide: $(-60) \div 12$

5. Write $1\frac{1}{4}$ as a decimal.

6. Simplify: $\frac{7}{12} - \frac{11}{16} - \left(\frac{1}{3}\right)$

7. Simplify: $\frac{5}{12} \div \left(2\frac{1}{2}\right)$

8. Simplify: $\left(-\frac{9}{16}\right) \cdot \left(\frac{8}{27}\right) \cdot \left(-\frac{3}{2}\right)$

9. Write $\frac{3}{4}$ as a percent.

10. Simplify: $-2^5 \div (3 - 5)^2 - (-3)$

11. Simplify: $\left(-\frac{3}{4}\right)^2 - \left(\frac{3}{8} - \frac{11}{12}\right)$

12. Evaluate $a^2 - 3b$ when $a = 2$ and $b = -4$.

13. Simplify: $-2x^2 - (-3x^2) + 4x^2$

14. Simplify: $5a - 10b - 12a$

15. Simplify: $\frac{1}{2}(12a)$

16. Simplify: $\left(-\frac{5}{6}\right)(-36b)$

17. Simplify: $3(8 - 2x)$

18. Simplify: $-2(-3y + 9)$

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19. Write $37\frac{1}{2}\%$ as a fraction.

20. Write 1.05% as a decimal.

21. Simplify: $-4(2x^2 - 3y^2)$

22. Simplify: $-3(3y^2 - 3y - 7)$

23. Simplify: $-3x - 2(2x - 7)$

24. Simplify: $4(3x - 2) - 7(x + 5)$

25. Simplify: $2x + 3[x - 2(4 - 2x)]$

26. Simplify: $3[2x - 3(x - 2y)] + 3y$

27. Translate "the sum of one-half of b and b" into a variable expression.

28. Translate "10 divided by the difference between y and 2" into a variable expression.

29. Translate "the difference between eight and the quotient of a number and twelve" into a variable expression.

30. Translate "the sum of a number and two more than the number" into a variable expression. Then simplify.

31. Translate and simplify "twelve more than the product of three plus a number and five."

32. A "triple-speed" CD-ROM drive spins three times faster than a normal CD-ROM drive. Express the speed of the "triple-speed" CD-ROM drive in terms of that of the normal CD-ROM drive.

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CHAPTER

3 Solving Equations



It is the job of construction electricians to install heating, lighting, air conditioning, controls, communications, and other types of electrical equipment. Electricians follow specifications such as blueprints to connect equipment to power sources and circuit breakers. In their work, they must apply various algebraic and scientific formulas. One equation in Ohm's Law is $IR = V$, where I is current, R is resistance, and V is voltage. For example, if $I = 10$ amps and $R = 12$ ohms, then $V = 120$ volts.

Objectives

Section 3.1

To determine whether a given number is a solution of an equation

To solve an equation of the form $x + a = b$

To solve an equation of the form $ax = b$

To solve application problems using the basic percent equation

Section 3.2

To solve an equation of the form $ax + b = c$

To solve application problems using formulas

Section 3.3

To solve an equation of the form $ax + b = cx + d$

To solve an equation containing parentheses

To solve application problems using formulas

Section 3.4

To solve integer problems

To translate a sentence into an equation and solve

Section 3.5

To solve perimeter problems

To solve problems involving angles formed by intersecting lines

To solve problems involving the angles of a triangle

Section 3.6

To solve value mixture problems

To solve percent mixture problems

To solve uniform motion problems

Mersenne Primes

A prime number that can be written in the form $2^n - 1$, where n is also prime, is called a **Mersenne prime.** The table at the right shows some Mersenne primes.

$$3 = 2^2 - 1$$
$$7 = 2^3 - 1$$
$$31 = 2^5 - 1$$
$$127 = 2^7 - 1$$



Not every prime number is a Mersenne prime. For example, 5 is a prime number but not a Mersenne prime. Also, not all numbers in the form $2^n - 1$, where n is prime, yield a prime number. For example, $2^{11} - 1 = 2047$, which is not a prime number.

The search for Mersenne primes has been quite extensive, especially since the advent of the computer. One reason for the extensive research into large prime numbers (not only Mersenne primes) involves cryptology.

Cryptology is the study of making or breaking secret codes. One method of making a code that is difficult to break is called public-key cryptology. For this method to work, it is necessary to use very large prime numbers. To keep anyone from breaking the code, each prime should have at least 200 digits.

Today, the largest known Mersenne prime is $2^{6972593} - 1$. This number has 2,098,960 digits in its representation.

Another Mersenne prime got special recognition in a postage-meter stamp. It is the number $2^{11213} - 1$. This number has 3276 digits in its representation.

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3.1 Introduction to Equations

Objective A To determine whether a given number is a solution of an equation

POINT OF INTEREST

One of the most famous equations ever stated is $E = mc^2$. This equation, stated by Albert Einstein, shows that there is a relationship between mass m and energy E. As a side note, the chemical element einsteinium was named in honor of Einstein.

An **equation** expresses the equality of two mathematical expressions. The expressions can be either numerical or variable expressions.

$9 + 3 = 12$
$3x - 2 = 10$
$y^2 + 4 = 2y - 1$
$z = 2$
} Equations

The equation at the right is true if the variable is replaced by 5.

$x + 8 = 13$
$5 + 8 = 13$ A true equation

The equation is false if the variable is replaced by 7.

$7 + 8 = 13$ A false equation

A **solution** of an equation is a number that, when substituted for the variable, results in a true equation. 5 is a solution of the equation $x + 8 = 13$. 7 is not a solution of the equation $x + 8 = 13$.

➡ Is -2 a solution of $2x + 5 = x^2 - 3$?

$2x + 5 = x^2 - 3$	
$2(-2) + 5$	$(-2)^2 - 3$
$-4 + 5$	$4 - 3$
$1 = 1$	

Yes, -2 is a solution of the equation.

- Replace x by -2.
- Evaluate the numerical expressions.
- If the results are equal, -2 is a solution of the equation. If the results are not equal, -2 is not a solution of the equation.

TAKE NOTE

The Order of Operations Agreement applies to evaluating $2(-2) + 5$ and $(-2)^2 - 3$.

Example 1 Is -4 a solution of $5x - 2 = 6x + 2$?

Solution

$5x - 2 = 6x + 2$	
$5(-4) - 2$	$6(-4) + 2$
$-20 - 2$	$-24 + 2$
$-22 = -22$	

Yes, -4 is a solution.

You Try It 1 Is $\frac{1}{4}$ a solution of $5 - 4x = 8x + 2$?

Your solution

Example 2 Is -4 a solution of $4 + 5x = x^2 - 2x$?

Solution

$4 + 5x = x^2 - 2x$	
$4 + 5(-4)$	$(-4)^2 - 2(-4)$
$4 + (-20)$	$16 - (-8)$
$-16 \neq 24$	

($\neq$ means "is not equal to")

No, -4 is not a solution.

You Try It 2 Is 5 a solution of $10x - x^2 = 3x - 10$?

Your solution

Solutions on p. S4

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Objective B To solve an equation of the form $x + a = b$

To **solve an equation** means to find a solution of the equation. The simplest equation to solve is an equation of the form *variable* = *constant*, because the constant is the solution.

The solution of the equation $x = 5$ is 5 because $5 = 5$ is a true equation.

The solution of the equation at the right is 7 because $7 + 2 = 9$ is a true equation.

$$x + 2 = 9 \qquad 7 + 2 = 9$$

Note that if 4 is added to each side of the equation $x + 2 = 9$, the solution is still 7.

$$\begin{aligned} x + 2 &= 9 \\ x + 2 + 4 &= 9 + 4 \\ x + 6 &= 13 \end{aligned} \qquad 7 + 6 = 13$$

If -5 is added to each side of the equation $x + 2 = 9$, the solution is still 7.

$$\begin{aligned} x + 2 &= 9 \\ x + 2 + (-5) &= 9 + (-5) \\ x - 3 &= 4 \end{aligned} \qquad 7 - 3 = 4$$

Equations that have the same solution are **equivalent equations.** The equations $x + 2 = 9$, $x + 6 = 13$, and $x - 3 = 4$ are equivalent equations; each equation has 7 as its solution. These examples suggest that adding the same number to each side of an equation produces equivalent equations. This is called the *Addition Property of Equations.*

Addition Property of Equations

The same number can be added to each side of an equation without changing its solution. In symbols, the equation $a = b$ has the same solution as the equation $a + c = b + c$.

In solving an equation, the goal is to rewrite the given equation in the form *variable* = *constant*. The Addition Property of Equations is used to remove a *term* from one side of the equation by adding the opposite of that term to each side of the equation.

Solve: $x - 4 = 2$

$$\begin{aligned} x - 4 &= 2 && \text{• The goal is to rewrite the equation as } \textit{variable} = \textit{constant}. \\ x - 4 + 4 &= 2 + 4 && \text{• Add 4 to each side of the equation.} \\ x + 0 &= 6 && \text{• Simplify.} \\ x &= 6 && \text{• The equation is in the form } \textit{variable} = \textit{constant}. \end{aligned}$$

Check:

$$\begin{array}{r|l} x - 4 = 2 & \\ \hline 6 - 4 & 2 \\ 2 = 2 & \text{A true equation} \end{array}$$

The solution is 6.

Because subtraction is defined in terms of addition, the Addition Property of Equations also makes it possible to subtract the same number from each side of an equation without changing the solution of the equation.

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Solve: $y + \frac{3}{4} = \frac{1}{2}$

$$y + \frac{3}{4} = \frac{1}{2}$$

- The goal is to rewrite the equation in the form *variable = constant*.

$$y + \frac{3}{4} - \frac{3}{4} = \frac{1}{2} - \frac{3}{4}$$

- Subtract $\frac{3}{4}$ from each side of the equation.

$$y + 0 = \frac{2}{4} - \frac{3}{4}$$

- Simplify.

$$y = -\frac{1}{4}$$

- The equation is in the form *variable = constant*.

The solution is $-\frac{1}{4}$. You should check this solution.

Example 3 Solve: $x + \frac{3}{4} = \frac{1}{3}$

Solution

$$x + \frac{3}{4} = \frac{1}{3}$$

$$x + \frac{3}{4} - \frac{3}{4} = \frac{1}{3} - \frac{3}{4}$$

$$x + 0 = \frac{4}{12} - \frac{9}{12}$$

$$x = -\frac{5}{12}$$

The solution is $-\frac{5}{12}$.

You Try It 3 Solve: $\frac{5}{6} = y - \frac{3}{8}$

Your solution

Solution on p. S4

Objective C To solve an equation of the form ax = b



The solution of the equation at the right is 3 because $2 \cdot 3 = 6$ is a true equation.

Note that if each side of $2x = 6$ is multiplied by 5, the solution is still 3.

If each side of $2x = 6$ is multiplied by -4, the solution is still 3.

$2x = 6$	$2 \cdot 3 = 6$
$2x = 6$	
$5(2x) = 5 \cdot 6$	
$10x = 30$	$10 \cdot 3 = 30$
$2x = 6$	
$(-4)(2x) = (-4) \cdot 6$	
$-8x = -24$	$-8 \cdot 3 = -24$

The equations $2x = 6$, $10x = 30$, and $-8x = -24$ are equivalent equations; each equation has 3 as its solution. These examples suggest that multiplying each side of an equation by the same number produces equivalent equations.

Multiplication Property of Equations

Each side of an equation can be multiplied by the same *nonzero* number without changing the solution of the equation. In symbols, if $c \neq 0$, then the equation $a = b$ has the same solutions as the equation $ac = bc$.

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The Multiplication Property of Equations is used to remove a coefficient by multiplying each side of the equation by the reciprocal of the coefficient.

➡ Solve: $\frac{3}{4}z = 9$

$$\frac{3}{4}z = 9$$ • The goal is to rewrite the equation in the form *variable* = *constant*.

$$\frac{4}{3} \cdot \frac{3}{4}z = \frac{4}{3} \cdot 9$$ • Multiply each side of the equation by $\frac{4}{3}$.

$$1 \cdot z = 12$$ • Simplify.

$$z = 12$$ • The equation is in the form *variable* = *constant*.

The solution is 12. You should check this solution.

Because division is defined in terms of multiplication, each side of an equation can be divided by the same nonzero number without changing the solution of the equation.

➡ Solve: $6x = 14$

$$6x = 14$$ • The goal is to rewrite the equation in the form *variable* = *constant*.

$$\frac{6x}{6} = \frac{14}{6}$$ • Divide each side of the equation by 6.

$$x = \frac{7}{3}$$ • Simplify. The equation is in the form *variable* = *constant*.

The solution is $\frac{7}{3}$.

TAKE NOTE

Remember to check the solution.

Check:

$$\begin{array}{r|l} 6x = 14 & \\ \hline 6\left(\frac{7}{3}\right) & 14 \\ 14 = 14 & \end{array}$$

When using the Multiplication Property of Equations, multiply each side of the equation by the reciprocal of the coefficient when the coefficient is a fraction. Divide each side of the equation by the coefficient when the coefficient is an integer or decimal.

Example 4 Solve: $\frac{3x}{4} = -9$

Solution

$$\frac{3x}{4} = -9$$

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3}(-9) \quad \left| \frac{3x}{4} = \frac{3}{4}x \right.$$

$$x = -12$$

The solution is -12.

You Try It 4 Solve: $-\frac{2x}{5} = 6$

Your solution

Example 5 Solve: $5x - 9x = 12$

Solution

$$5x - 9x = 12$$

$$-4x = 12$$ • Combine like terms.

$$\frac{-4x}{-4} = \frac{12}{-4}$$

$$x = -3$$

The solution is -3.

You Try It 5 Solve: $4x - 8x = 16$

Your solution

Solutions on p. S4

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Objective D To solve application problems using the basic percent equation

10 CT

An equation that is used frequently in mathematics applications is the basic percent equation.

Basic Percent Equation

Percent · Base = Amount

$P \cdot B = A$

In many application problems involving percent, the base follows the word "of."

➡ 20% of what number is 30?

$$P \cdot B = A$$
- Use the basic percent equation.

$$0.20B = 30$$
- $P = 20\% = 0.20$, $A = 30$, and B is unknown.

$$\frac{0.20B}{0.20} = \frac{30}{0.20}$$
- Solve for B.

$$B = 150$$

The number is 150.

TAKE NOTE

We have written $P(80) = 70$ because that is the form of the basic percent equation. We could have written $80P = 70$. The important point is that each side of the equation is divided by 80, the coefficient of P.

➡ 70 is what percent of 80?

$$P \cdot B = A$$
- Use the basic percent equation.

$$P(80) = 70$$
- $B = 80$, $A = 70$, and P is unknown.

$$\frac{P(80)}{80} = \frac{70}{80}$$
- Solve for P.

$$P = 0.875$$
- The question asked for a percent. Convert the decimal to a percent.

$$P = 87.5\%$$

70 is 87.5% of 80.

POINT OF INTEREST

As noted in the problem at the right, $735 of a typical traveler's vacation bill is paid for in cash. Of the remainder spent, $433 (or 30%) is paid by credit card, $144 (or 10%) is paid for in traveler's checks, and $130 (or 9%) is paid for in personal checks.

➡ According to the Travel Industry Association of America, a typical traveler's total vacation bill is $1442. Of that amount, $735 is paid in cash. To the nearest percent, what percent of a typical traveler's vacation bill is paid in cash?

Strategy To determine the percent, use the basic percent equation. $B = 1442$, $A = 735$, and P is unknown.

Solution

$$P \cdot B = A$$

$$P(1442) = 735$$

$$\frac{P(1442)}{1442} = \frac{735}{1442}$$

$$P \approx 0.51 = 51\%$$

A typical traveler pays 51% of the vacation bill in cash.

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In most cases, you should write the percent as a decimal before solving the basic percent equation. However, some percents are more easily written as a fraction. For example,

$$33\frac{1}{3}\% = \frac{1}{3} \qquad 66\frac{2}{3}\% = \frac{2}{3} \qquad 16\frac{2}{3}\% = \frac{1}{6} \qquad 83\frac{1}{3}\% = \frac{5}{6}$$

Example 6

12 is $33\frac{1}{3}\%$ of what number?

Solution

$$P \cdot B = A$$

$$\frac{1}{3}B = 12 \qquad 33\frac{1}{3}\% = \frac{1}{3}$$

$$3 \cdot \frac{1}{3}B = 3 \cdot 12$$

$$B = 36$$

12 is $33\frac{1}{3}\%$ of 36.

You Try It 6

18 is $16\frac{2}{3}\%$ of what number?

Your solution

Example 7

In a recent year, 238 U.S. airports collected \$1.1 billion in passenger taxes. Of this amount, \$88 million was spent on noise reduction. What percent of the passenger taxes collected was spent on noise reduction?

Strategy

To find the percent, solve the basic percent equation using $B = 1.1$ billion $= 1100$ million and $A = 88$ million. The percent is unknown.

Solution

$$P \cdot B = A$$

$$P(1100) = 88$$

$$\frac{P(1100)}{1100} = \frac{88}{1100}$$

$$P = 0.08$$

8% of the passenger taxes collected was spent on noise reduction.

You Try It 7

The federal government ran a deficit of \$23.1 billion in March of 1997. The deficit in March of 1996 was \$47.1 billion. What percent of the March 1996 deficit was the March 1997 deficit? Round to the nearest tenth of a percent.

Your strategy

Your solution

Solutions on p. S4

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3.1 Exercises

Objective A

1. Is 4 a solution of $2x = 8$?

2. Is 3 a solution of $y + 4 = 7$?

3. Is -1 a solution of $2b - 1 = 3$?

4. Is -2 a solution of $3a - 4 = 10$?

5. Is 1 a solution of $4 - 2m = 3$?

6. Is 2 a solution of $7 - 3n = 2$?

7. Is 5 a solution of $2x + 5 = 3x$?

8. Is 4 a solution of $3y - 4 = 2y$?

9. Is 0 a solution of $4a + 5 = 3a + 5$?

10. Is 3 a solution of $z^2 + 1 = 4 + 3z$?

11. Is 2 a solution of $2x^2 - 1 = 4x - 1$?

12. Is -1 a solution of $y^2 - 1 = 4y + 3$?

13. Is -2 a solution of $m^2 - 4 = m + 3$?

14. Is 5 a solution of $x^2 + 2x + 1 = (x + 1)^2$?

15. Is -6 a solution of $(n - 2)^2 = n^2 - 4n + 4$?

16. Is 4 a solution of $x(x + 1) = x^2 + 5$?

17. Is 3 a solution of $2a(a - 1) = 3a + 3$?

18. Is $-\frac{1}{4}$ a solution of $8t + 1 = -1$?

19. Is $\frac{1}{2}$ a solution of $4y + 1 = 3$?

20. Is $\frac{2}{5}$ a solution of $5m + 1 = 10m - 3$?

21. Is $\frac{3}{4}$ a solution of $8x - 1 = 12x + 3$?

Objective B

Solve and check.

22. $x + 5 = 7$

23. $y + 3 = 9$

24. $b - 4 = 11$

25. $z - 6 = 10$

26. $2 + a = 8$

27. $5 + x = 12$

28. $m + 9 = 3$

29. $t + 12 = 10$

30. $n - 5 = -2$

31. $x - 6 = -5$

32. $b + 7 = 7$

33. $y - 5 = -5$

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34. $a - 3 = -5$ **35.** $x - 6 = -3$ **36.** $z + 9 = 2$ **37.** $n + 11 = 1$

38. $10 + m = 3$ **39.** $8 + x = 5$ **40.** $9 + x = -3$ **41.** $10 + y = -4$

42. $b - 5 = -3$ **43.** $t - 6 = -4$ **44.** $4 + x = 10$ **45.** $9 + a = 20$

46. $2 = x + 7$ **47.** $-8 = n + 1$ **48.** $4 = m - 11$ **49.** $-6 = y - 5$

50. $12 = 3 + w$ **51.** $-9 = 5 + x$ **52.** $4 = -10 + b$ **53.** $-7 = -2 + x$

54. $13 = -6 + a$ **55.** $m + \frac{2}{3} = -\frac{1}{3}$ **56.** $c + \frac{3}{4} = -\frac{1}{4}$ **57.** $x - \frac{1}{2} = \frac{1}{2}$

58. $x - \frac{2}{5} = \frac{3}{5}$ **59.** $\frac{5}{8} + y = \frac{1}{8}$ **60.** $\frac{4}{9} + a = -\frac{2}{9}$

61. $m + \frac{1}{2} = -\frac{1}{4}$ **62.** $b + \frac{1}{6} = -\frac{1}{3}$ **63.** $x + \frac{2}{3} = \frac{3}{4}$

64. $n + \frac{2}{5} = \frac{2}{3}$ **65.** $-\frac{5}{6} = x - \frac{1}{4}$ **66.** $-\frac{1}{4} = c - \frac{2}{3}$

67. $d + 1.3619 = 2.0148$ **68.** $w + 2.932 = 4.801$ **69.** $-0.813 + x = -1.096$

70. $-1.926 + t = -1.042$ **71.** $6.149 = -3.108 + z$ **72.** $5.237 = -2.014 + x$

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Objective C

Solve and check.

73. $5x = -15$

74. $4y = -28$

75. $3b = 0$

76. $2a = 0$

77. $-3x = 6$

78. $-5m = 20$

79. $-3x = -27$

80. $-\frac{1}{6}n = -30$

81. $20 = \frac{1}{4}c$

82. $18 = 2t$

83. $-32 = 8w$

84. $-56 = 7x$

85. $0 = -5x$

86. $0 = -8a$

87. $-32 = -4y$

88. $-54 = 6c$

89. $49 = -7t$

90. $\frac{x}{3} = 2$

91. $\frac{x}{4} = 3$

92. $-\frac{y}{2} = 5$

93. $-\frac{b}{3} = 6$

94. $\frac{3}{4}y = 9$

95. $\frac{2}{5}x = 6$

96. $-\frac{2}{3}d = 8$

97. $-\frac{3}{5}m = 12$

98. $\frac{2n}{3} = 0$

99. $\frac{5x}{6} = 0$

100. $\frac{-3z}{8} = 9$

101. $\frac{-4x}{5} = -12$

102. $-6 = -\frac{2}{3}y$

103. $-15 = -\frac{1}{5}x$

104. $\frac{2}{5}a = 3$

105. $\frac{3x}{4} = 2$

106. $\frac{3}{4}c = \frac{3}{5}$

107. $\frac{2}{9} = \frac{2}{3}y$

108. $-\frac{6}{7} = -\frac{3}{4}b$

109. $\frac{1}{5}x = -\frac{1}{10}$

110. $-\frac{2}{3}y = -\frac{8}{9}$

111. $-1 = \frac{2n}{3}$

112. $-\frac{3}{4} = \frac{a}{8}$

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113. $-\frac{2}{5}m = -\frac{6}{7}$ **114.** $5x + 2x = 14$ **115.** $3n + 2n = 20$ **116.** $7d - 4d = 9$

117. $10y - 3y = 21$ **118.** $2x - 5x = 9$ **119.** $\frac{x}{1.46} = 3.25$ **120.** $\frac{z}{2.95} = -7.88$

121. $3.47a = 7.1482$ **122.** $2.31m = 2.4255$ **123.** $-3.7x = 7.881$ **124.** $\frac{n}{2.65} = 9.08$

Objective D

125. What is 35% of 80? **126.** What percent of 8 is 0.5? **127.** Find 1.2% of 60.

128. 8 is what percent of 5? **129.** 125% of what is 80? **130.** What percent of 20 is 30?

131. 12 is what percent of 50? **132.** What percent of 125 is 50? **133.** Find 18% of 40.

134. What is 25% of 60? **135.** 12% of what is 48? **136.** 45% of what is 9?

137. What is $33\frac{1}{3}\%$ of 27? **138.** Find $16\frac{2}{3}\%$ of 30. **139.** What percent of 12 is 3?

140. 10 is what percent of 15? **141.** 60% of what is 3? **142.** 75% of what is 6?

143. 12 is what percent of 6? **144.** 20 is what percent of 16? **145.** $5\frac{1}{4}\%$ of what is 21?

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146. $37\frac{1}{2}\%$ of what is 15?

147. Find 15.4% of 50.

148. What is 18.5% of 46?

149. 1 is 0.5% of what?

150. 3 is 1.5% of what?

151. $\frac{3}{4}\%$ of what is 3?

152. $\frac{1}{2}\%$ of what is 3?

153. Find 125% of 16.

154. What is 250% of 12?

155. 16.43 is what percent of 20.45? Round to the nearest hundredth of a percent.

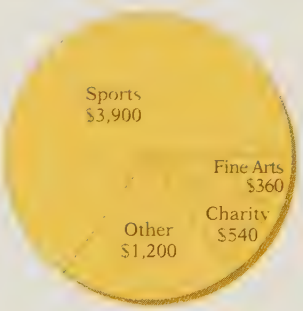
156. Find 18.37% of 625.43. Round to the nearest hundredth.

157. A university consists of three colleges: business, engineering, and fine arts. There are 2900 students in the business college, 1500 students in the engineering college, and 1000 students in the fine arts college. What percent of the total number of students in the university are in the fine arts college? Round to the nearest percent.

158. Approximately 21% of air is oxygen. Using this estimate, determine how many liters of oxygen there are in a room containing 21,600 L of air.

159. The circle graph at the right represents corporate spending in 1997 of \$6 billion to sponsor events. What percent of the amount spent by companies was spent on sports?

Corporate Sponsorships
(in millions of dollars)



Source: IEG Sponsorship Report

160. In 1950, 12% of mothers with children under age 6 worked outside the home. By 1995, that percent had increased to 64%. How many more mothers with children under age 6 worked outside the home in 1995 than in 1950?

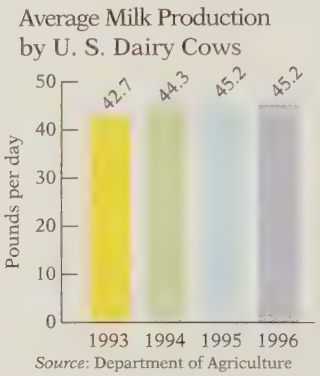
161. The Energy Information Administration reports that if every U.S. household switched 4 h of lighting per day from incandescent bulbs to compact fluorescent bulbs, we would save 31.7 billion kilowatt-hours (kWh) a year, or 33% of the total electricity used for home lighting. What is the total electricity used for home lighting in this country? Round to the nearest tenth of a billion.

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162. To override a presidential veto, at least $66\frac{2}{3}\%$ of the Senate must vote to override the veto. There are 100 senators in the Senate. What is the minimum number of votes needed to override a veto?

163. According to *Time* (March 1997), in 1996 Coca-Cola Company declared its intention to repurchase 206 million shares of the company, or 8.3% of the company's outstanding common stock. How many shares of Coca-Cola Company common stock were outstanding at the time of this announcement? Round to the nearest million.

164. A Holstein cow in Wisconsin recently set a milk production record. She averaged 174 lb of milk per day for 365 days. What percent of the average daily milk production in 1996 was the production by this Wisconsin Holstein? See the graph at the right. Round to the nearest percent.



165. According to *Working Woman* (July–August 1997), in 1996, total consumer spending in the United States was \$4.69 trillion. Of that amount, 1.58% was spent on new cars. In 1986, total consumer spending in the United States was \$3.72 trillion, of which 3.09% was spent on new cars.

a. During which year was more money spent on new cars?

b. How much more? Round to the nearest billion.

APPLYING THE CONCEPTS

166. Solve the equation $ax = b$ for x. Is the solution you have written valid for all real numbers a and b?

167. **a.** Make up an equation of the form $x + a = b$ that has 2 as a solution.

b. Make up an equation of the form $ax = b$ that has -1 as a solution.

168. Solve. **a.** $\dfrac{3}{\frac{1}{x}} = 5$ **b.** $\dfrac{2}{\frac{1}{y}} = -2$ **c.** $\dfrac{3x + 2x}{3} = 2$

169. Write out the steps for solving the equation $\frac{1}{2}x = -3$. Identify each Property of Real Numbers or Property of Equations as you use it.

170. In your own words, state the Addition Property and the Multiplication Property of Equations.

171. If a quantity increases by 100%, how many times its original value is the new value?

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3.2 General Equations—Part I

Objective A To solve an equation of the form $ax + b = c$

In solving an equation of the form $ax + b = c$, the goal is to rewrite the equation in the form *variable* = *constant*. This requires the application of both the Addition and the Multiplication Properties of Equations.

➡ Solve: $\frac{3}{4}x - 2 = -11$

The goal is to write the equation in the form *variable* = *constant*.

$$\frac{3}{4}x - 2 = -11$$

$$\frac{3}{4}x - 2 + 2 = -11 + 2$$ • Add 2 to each side of the equation.

$$\frac{3}{4}x = -9$$ • Simplify.

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3}(-9)$$ • Multiply each side of the equation by $\frac{4}{3}$.

$$x = -12$$ • The equation is of the form *variable* = *constant*.

The solution is -12.

TAKE NOTE

Check: $\frac{3}{4}x - 2 = -11$

$\frac{3}{4}(-12) - 2$	-11
$-9 - 2$	-11
$-11 = -11$	

A true equation

Example 1 Solve: $3x - 7 = -5$

Solution

$$3x - 7 = -5$$
$$3x - 7 + 7 = -5 + 7$$
$$3x = 2$$
$$\frac{3x}{3} = \frac{2}{3}$$
$$x = \frac{2}{3}$$

The solution is $\frac{2}{3}$.

You Try It 1 Solve: $5x + 7 = 10$

Your solution

Example 2 Solve: $5 = 9 - 2x$

Solution

$$5 = 9 - 2x$$
$$5 - 9 = 9 - 9 - 2x$$
$$-4 = -2x$$
$$\frac{-4}{-2} = \frac{-2x}{-2}$$
$$2 = x$$

The solution is 2.

You Try It 2 Solve: $2 = 11 + 3x$

Your solution

Solutions on pp. S4–S5

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Example 3

Solve: $2x + 4 - 5x = 10$

Solution

$$\begin{aligned} 2x + 4 - 5x &= 10 \\ -3x + 4 &= 10 \quad \text{• Combine like terms.} \\ -3x + 4 - 4 &= 10 - 4 \\ -3x &= 6 \\ \frac{-3x}{-3} &= \frac{6}{-3} \\ x &= -2 \end{aligned}$$

The solution is -2.

You Try It 3

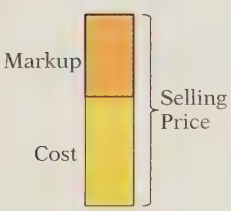
Solve: $x - 5 + 4x = 25$

Your solution

Solution on p. S5

Objective B To solve application problems using formulas

In this objective we will be solving application problems using formulas. Two of the formulas we will use are related to markup and discount.



Cost is the price a business pays for a product. **Selling price** is the price for which a business sells a product to a customer. The difference between selling price and cost is called **markup.** Markup is added to the cost to cover the expenses of operating a business. The diagram at the left illustrates these terms. The total length is the selling price. One part of the diagram is the cost, and the other part is the markup.

When the markup is expressed as a percent of the retailer's cost, it is called the **markup rate.**

The basic markup equations used by a business are

$$\begin{array}{lll} \text{Selling price} = \text{cost} + \text{markup} & & \text{Markup} = \text{markup rate} \cdot \text{cost} \\ S = C + M & & M = r \cdot C \end{array}$$

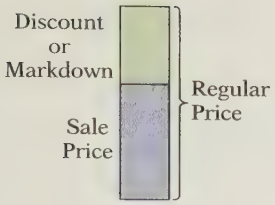
Substituting $r \cdot C$ for M in the first equation results in $S = C + (r \cdot C)$, or $S = C + rC$.

➡ The manager of a clothing store buys a jacket for \$80 and sells the jacket for \$116. Find the markup rate.

$$\begin{aligned} S &= C + rC && \text{• Use the equation } S = C + rC. \\ 116 &= 80 + 80r && \text{• Given: } C = \$80 \text{ and } S = \$116 \\ 36 &= 80r && \text{• Subtract 80 from each side of the equation.} \\ \frac{36}{80} &= \frac{80r}{80} && \text{• Divide both sides of the equation by 80.} \\ 0.45 &= r \end{aligned}$$

The markup rate is 45%.

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A retailer may reduce the regular price of a product because the goods are damaged, odd sizes, or discontinued items. The **discount,** or **markdown,** is the amount by which a retailer reduces the regular price of a product. The percent discount is called the **discount rate** and is usually expressed as a percent of the original selling price (the regular price).

The basic discount equations used by a business are

$$\text{Sale price} = \text{regular price} - \text{discount} \qquad \text{Discount} = \text{discount rate} \cdot \text{regular price}$$

$$S = R - D \qquad D = r \cdot R$$

Substituting $r \cdot R$ for D in the first equation results in $S = R - (r \cdot R)$, or $S = R - rR$.

➡ A portable computer that regularly sells for \$1850 is on sale for \$1480. Find the discount rate.

$$S = R - rR$$ • Use the equation $S = R - rR$.

$$1480 = 1850 - 1850r$$ • Given: $S = \$1480$ and $R = 1850$

$$-370 = -1850r$$ • Subtract 1850 from each side of the equation.

$$\frac{-370}{-1850} = \frac{-1850r}{-1850}$$ • Divide each side of the equation by -1850.

$$0.2 = r$$

The discount rate on the portable computer is 20%.

Example 4

A markup rate of 40% was used on a refrigerator that has a selling price of \$749. Find the cost of the refrigerator. Use the formula $S = C + rC$.

Strategy

Given: $S = \$749$
$r = 40\% = 0.40$
Unknown: C

Solution

$$S = C + rC$$
$$749 = C + 0.40C$$ • $C + 0.40C = 1C + 0.40C$
$$749 = 1.40C$$ • Combine like terms.
$$\frac{749}{1.40} = \frac{1.40C}{1.40}$$
$$535 = C$$

The cost of the refrigerator is \$535.

You Try It 4

A markup rate of 45% was used on an outboard motor that has a selling price of \$986. Find the cost of the outboard motor. Use the formula $S = C + rC$.

Your strategy

Your solution

Solution on p. S5

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Example 5

A necklace that is marked down 35% has a sale price of \$292.50. Find the regular price of the necklace. Use the formula $S = R - rR$.

Strategy

Given: $S = 292.50$
$r = 35\% = 0.35$
Unknown: R

Solution

$$S = R - rR$$
$$292.50 = R - 0.35R \quad \text{• } R - 0.35R = 1R - 0.35R$$
$$292.50 = 0.65R \quad \text{• Combine like terms.}$$
$$\frac{292.50}{0.65} = \frac{0.65R}{0.65}$$
$$450 = R$$

The regular price of the necklace is \$450.

You Try It 5

A garage door opener, marked down 25%, is on sale for \$159. Find the regular price of the garage door opener. Use the formula $S = R - rR$.

Your strategy

Your solution

Example 6

To determine the total cost of production, an economist uses the equation $T = U \cdot N + F$, where T is the total cost, U is the unit cost, N is the number of units made, and F is the fixed cost. Use this equation to find the number of units made during a month when the total cost was \$9000, the unit cost was \$25, and the fixed cost was \$3000.

Strategy

Given: $T = \$9000$
$U = \$25$
$F = \$3000$
Unknown: N

Solution

$$T = U \cdot N + F$$
$$9000 = 25N + 3000$$
$$6000 = 25N$$
$$\frac{6000}{25} = \frac{25N}{25}$$
$$240 = N$$

There were 240 units made.

You Try It 6

The pressure at a certain depth in the ocean can be approximated by the equation $P = 15 + \frac{1}{2}D$, where P is the pressure in pounds per square inch and D is the depth in feet. Use this equation to find the depth when the pressure is 45 pounds per square inch.

Your strategy

Your solution

Solutions on p. S5

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3.2 Exercises

Objective A

Solve and check.

1. $3x + 1 = 10$ **2.** $4y + 3 = 11$ **3.** $2a - 5 = 7$ **4.** $5m - 6 = 9$

5. $5 = 4x + 9$ **6.** $2 = 5b + 12$ **7.** $2x - 5 = -11$ **8.** $3n - 7 = -19$

9. $4 - 3w = -2$ **10.** $5 - 6x = -13$ **11.** $8 - 3t = 2$ **12.** $12 - 5x = 7$

13. $4a - 20 = 0$ **14.** $3y - 9 = 0$ **15.** $6 + 2b = 0$ **16.** $10 + 5m = 0$

17. $-2x + 5 = -7$ **18.** $-5d + 3 = -12$ **19.** $-12x + 30 = -6$ **20.** $-13 = -11y + 9$

21. $2 = 7 - 5a$ **22.** $3 = 11 - 4n$ **23.** $-35 = -6b + 1$ **24.** $-8x + 3 = -29$

25. $-3m - 21 = 0$ **26.** $-5x - 30 = 0$ **27.** $-4y + 15 = 15$ **28.** $-3x + 19 = 19$

29. $9 - 4x = 6$ **30.** $3t - 2 = 0$ **31.** $9x - 4 = 0$ **32.** $7 - 8z = 0$

33. $1 - 3x = 0$ **34.** $9d + 10 = 7$ **35.** $12w + 11 = 5$ **36.** $6y - 5 = -7$

37. $8b - 3 = -9$ **38.** $5 - 6m = 2$ **39.** $7 - 9a = 4$ **40.** $9 = -12c + 5$

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41. $10 = -18x + 7$

42. $2y + \frac{1}{3} = \frac{7}{3}$

43. $4a + \frac{3}{4} = \frac{19}{4}$

44. $2n - \frac{3}{4} = \frac{13}{4}$

45. $3x - \frac{5}{6} = \frac{13}{6}$

46. $5y + \frac{3}{7} = \frac{3}{7}$

47. $9x + \frac{4}{5} = \frac{4}{5}$

48. $8 = 7d - 1$

49. $8 = 10x - 5$

50. $4 = 7 \cdot 2w$

51. $7 = 9 - 5a$

52. $8t + 13 = 3$

53. $12x + 19 = 3$

54. $-6y + 5 = 13$

55. $-4x + 3 = 9$

56. $\frac{1}{2}a - 3 = 1$

57. $\frac{1}{3}m - 1 = 5$

58. $\frac{2}{5}y + 4 = 6$

59. $\frac{3}{4}n + 7 = 13$

60. $-\frac{2}{3}x + 1 = 7$

61. $-\frac{3}{8}b + 4 = 10$

62. $\frac{x}{4} - 6 = 1$

63. $\frac{y}{5} - 2 = 3$

64. $\frac{2x}{3} - 1 = 5$

65. $\frac{3c}{7} - 1 = 8$

66. $4 - \frac{3}{4}z = -2$

67. $3 - \frac{4}{5}w = -9$

68. $5 + \frac{2}{3}y = 3$

69. $17 + \frac{5}{8}x = 7$

70. $17 = 7 - \frac{5}{6}t$

71. $9 = 3 - \frac{2x}{7}$

72. $3 = \frac{3a}{4} + 1$

73. $7 = \frac{2x}{5} + 4$

74. $5 - \frac{4c}{7} = 8$

75. $7 - \frac{5}{9}y = 9$

76. $6a + 3 + 2a = 11$

77. $5y + 9 + 2y = 23$

78. $7x - 4 - 2x = 6$

79. $11z - 3 - 7z = 9$

80. $2x - 6x + 1 = 9$

81. $b - 8b + 1 = -6$

82. $3 = 7x + 9 - 4x$

83. $-1 = 5m + 7 - m$

84. $8 = 4n - 6 + 3n$

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85. If $2x - 3 = 7$, evaluate $3x + 4$.

86. If $3x + 5 = -4$, evaluate $2x - 5$.

87. If $4 - 5x = -1$, evaluate $x^2 - 3x + 1$.

88. If $2 - 3x = 11$, evaluate $x^2 + 2x - 3$.

89. If $5x + 3 - 2x = 12$, evaluate $4 - 5x$.

90. If $2x - 4 - 7x = 16$, evaluate $x^2 + 1$.

Objective B Application Problems

Solve. Use the markup equation $S = C + rC$, where S is selling price, C is cost, and r is the markup rate.

91. A watch costing \$98 is sold for \$156.80. Find the markup rate on the watch.

92. A set of golf clubs costing \$360 is sold for \$630. Find the markup rate on the set of golf clubs.

93. A markup rate of 40% was used on a basketball with a selling price of \$82.60. Find the cost of the basketball.

94. A portable tape player with a selling price of \$57 has a markup rate of 50%. Find the cost of the tape player.

95. A freezer costing \$360 is sold for \$520. Find the markup rate. Round to the nearest tenth of a percent.

96. A sofa costing \$320 is sold for \$479. Find the markup rate. Round to the nearest tenth of a percent.

97. A digitally recorded compact disk has a selling price of \$11.90. The markup rate is 40%. Find the cost of the CD.

98. A markup rate of 25% is used on a computer that has a selling price of \$2187.50. Find the cost of the computer.

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Solve. Use the discount equation $S = R - rR$, where S is the sale price, R is the regular price, and r is the discount rate.

99. An oak bedroom set with a regular price of \$1295 is on sale for \$995. Find the discount rate. Round to the nearest tenth of a percent.

100. A stereo system with a regular price of \$495 is on sale for \$395. Find the markdown rate. Round to the nearest tenth of a percent.

101. A mechanic's tool set is on sale for \$180 after a markdown of 40% off the regular price. Find the regular price.

102. A battery with a discount price of \$65 is on sale for 22% off the regular price. Find the regular price. Round to the nearest cent.

103. A compact disk player with a regular price of \$325 is on sale for \$201.50. Find the markdown rate.

104. A luggage set with a regular price of \$178 is on sale for \$103.24. Find the discount rate.

105. A telescope is on sale for \$165 after a markdown of 40% off the regular price. Find the regular price.

106. An exercise bike is on sale for \$390, having been marked down 25% of the regular price. Find the regular price.

The distance s, in feet, that an object will fall in t seconds is given by $s = 16t^2 + vt$, where v is the initial velocity of the object in feet per second.

107. Find the initial velocity of an object that falls 80 ft in 2 s.

108. Find the initial velocity of an object that falls 144 ft in 3 s.

A company uses the equation $V = C - 6000t$ to determine the depreciated value V, after t years, of a milling machine that originally cost C dollars. Equations like this are used in accounting for straight-line depreciation.

109. A milling machine originally cost \$50,000. In how many years will the depreciated value of the machine be \$38,000?

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110. A milling machine originally cost \$78,000. In how many years will the depreciated value of the machine be \$48,000?

Anthropologists approximate the height of a primate by the size of its humerus (the bone from the elbow to the shoulder) using the equation $H = 1.2L + 27.8$, where L is the length of the humerus and H is the height, in inches, of the primate.

111. An anthropologist estimates the height of a primate to be 66 in. What is the approximate length of the humerus of this primate? Round to the nearest tenth of an inch.

112. An anthropologist estimates the height of a primate to be 62 in. What is the approximate length of the humerus of this primate?

Black ice is an ice covering on roads that is especially difficult to see and therefore extremely dangerous for motorists. The distance a car traveling 30 mph will slide after its brakes are applied is related to the outside temperature by the formula $C = \frac{1}{4}D - 45$, where C is the Celsius temperature and D is the distance in feet that the car will slide.



113. Determine the distance a car will slide on black ice when the outside temperature is $-3°C$.

114. Determine the distance a car will slide on black ice when the outside temperature is $-11°C$.

A telephone company estimates that the number N of phone calls per day between two cities of population P_1 and P_2 that are d miles apart is given by the equation $N = \frac{2.51P_1P_2}{d^2}$.

115. Estimate the population (P_1) of a city given that the population of a second city (P_2) is 48,000, the number of phone calls per day between the two cities is 1,100,000, and the distance between the cities is 75 mi. Round to the nearest thousand.

116. Estimate the population (P_1) of a city given that the population of a second city (P_2) is 125,000, the number of phone calls per day between the two cities is 2,500,000, and the distance between the cities is 50 mi. Round to the nearest thousand.

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The world record time for a 1-mile race can be approximated by the equation $t = 17.08 - 0.0067y$, where t is the time in minutes and y is the year of the race.

117. Approximate the year in which the first "4-minute mile" was run. (The actual year was 1954.) Round to the nearest whole number.

118. In 1985, the world record for a 1-mile race was 3.77 min. For what year does the equation predict this record time? Round to the nearest whole number.

APPLYING THE CONCEPTS

119. A pair of shoes that now sells for \$63 has been marked up 40%. Find the markup on the pair of shoes.

120. The sale price of a typewriter is 25% off the regular price. The discount is \$70. Find the sale price.

121. The sale price of a television was \$180. Find the regular price if the sale price was computed by taking one-third off the regular price followed by an additional 15% discount on the reduced price.

122. A customer buys four tires, three at the regular price and one for 20% off the regular price. The four tires cost \$323. What was the regular price of a tire?

123. Solve: $x \div 15 = 25$ remainder 10

124. Does the sentence, "Solve $3x - 4(x - 1)$" make sense? Why or why not?

125. Explain the steps you would take to solve the equation $3x - 4 = 14$. State the Property of Real Numbers or the Property of Equations that is used at each step.

126. The following problem does not contain enough information for us to find only one solution. Supply some additional information so that the problem has exactly one solution. Then write and solve an equation.

The sum of two numbers is 15. Find the numbers.

127. The following problem does not contain enough information. What additional information is needed to answer the question?

How many hours does it take to fly from Los Angeles to New York?

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3.3 General Equations—Part II

Objective A *To solve an equation of the form $ax + b = cx + d$* ..... 9 CT

In solving an equation of the form $ax + b = cx + d$, the goal is to rewrite the equation in the form *variable* = *constant*. Begin by rewriting the equation so that there is only one variable term in the equation. Then rewrite the equation so that there is only one constant term.

Solve: $2x + 3 = 5x - 9$

$$2x + 3 = 5x - 9$$

$2x - 5x + 3 = 5x - 5x - 9$ — Subtract $5x$ from each side of the equation.

$-3x + 3 = -9$ — Simplify. There is only one variable term.

$-3x + 3 - 3 = -9 - 3$ — Subtract 3 from each side of the equation.

$-3x = -12$ — Simplify. There is only one constant term.

$\frac{-3x}{-3} = \frac{-12}{-3}$ — Divide each side of the equation by -3.

$x = 4$ — The equation is in the form *variable* = *constant*.

The solution is 4. You should verify this by checking this solution.

Example 1 Solve: $4x - 5 = 8x - 7$

Solution

$$4x - 5 = 8x - 7$$
$$4x - 8x - 5 = 8x - 8x - 7$$
$$-4x - 5 = -7$$
$$-4x - 5 + 5 = -7 + 5$$
$$-4x = -2$$
$$\frac{-4x}{-4} = \frac{-2}{-4}$$
$$x = \frac{1}{2}$$

The solution is $\frac{1}{2}$.

You Try It 1 Solve: $5x + 4 = 6 + 10x$

Your solution

Solution on p. S5

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Example 2 Solve: $3x + 4 - 5x = 2 - 4x$

Solution

$$3x + 4 - 5x = 2 - 4x$$
$$-2x + 4 = 2 - 4x$$
$$-2x + 4x + 4 = 2 - 4x + 4x$$
$$2x + 4 = 2$$
$$2x + 4 - 4 = 2 - 4$$
$$2x = -2$$
$$\frac{2x}{2} = \frac{-2}{2}$$
$$x = -1$$

The solution is -1.

You Try It 2 Solve: $5x - 10 - 3x = 6 - 4x$

Your solution

Solution on p. S5

Objective B To solve an equation containing parentheses

When an equation contains parentheses, one of the steps in solving the equation requires the use of the Distributive Property. The Distributive Property is used to remove parentheses from a variable expression.

Solve: $4 + 5(2x - 3) = 3(4x - 1)$

$$4 + 5(2x - 3) = 3(4x - 1)$$
$$4 + 10x - 15 = 12x - 3 \quad \text{Use the Distributive Property. Then simplify.}$$
$$10x - 11 = 12x - 3$$
$$10x - 12x - 11 = 12x - 12x - 3 \quad \text{Subtract } 12x \text{ from each side of the equation.}$$
$$-2x - 11 = -3 \quad \text{Simplify.}$$
$$-2x - 11 + 11 = -3 + 11 \quad \text{Add 11 to each side of the equation.}$$
$$-2x = 8 \quad \text{Simplify.}$$
$$\frac{-2x}{-2} = \frac{8}{-2} \quad \text{Divide each side of the equation by } -2.$$
$$x = -4 \quad \text{The equation is in the form } \textit{variable} = \textit{constant}.$$

The solution is -4. You should verify this by checking this solution.

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Example 3

Solve: $3x - 4(2 - x) = 3(x - 2) - 4$

Solution

$$\begin{aligned}
3x - 4(2 - x) &= 3(x - 2) - 4 \\
3x - 8 + 4x &= 3x - 6 - 4 \\
7x - 8 &= 3x - 10 \\
7x - 3x - 8 &= 3x - 3x - 10 \\
4x - 8 &= -10 \\
4x - 8 + 8 &= -10 + 8 \\
4x &= -2 \\
\frac{4x}{4} &= \frac{-2}{4} \\
x &= -\frac{1}{2}
\end{aligned}$$

The solution is $-\frac{1}{2}$.

You Try It 3

Solve: $5x - 4(3 - 2x) = 2(3x - 2) + 6$

Your solution

Example 4

Solve: $3[2 - 4(2x - 1)] = 4x - 10$

Solution

$$\begin{aligned}
3[2 - 4(2x - 1)] &= 4x - 10 \\
3[2 - 8x + 4] &= 4x - 10 \\
3[6 - 8x] &= 4x - 10 \\
18 - 24x &= 4x - 10 \\
18 - 24x - 4x &= 4x - 4x - 10 \\
18 - 28x &= -10 \\
18 - 18 - 28x &= -10 - 18 \\
-28x &= -28 \\
\frac{-28x}{-28} &= \frac{-28}{-28} \\
x &= 1
\end{aligned}$$

The solution is 1.

You Try It 4

Solve: $-2[3x - 5(2x - 3)] = 3x - 8$

Your solution

Example 5

If $7x = 3x + 12$, evaluate $3x^2 - 7$.

Solution

Solve $7x = 3x + 12$ for x.

$$\begin{aligned}
7x - 3x &= 12 \\
4x &= 12 \\
x &= 3
\end{aligned}$$

Evaluate $3x^2 - 7$ for $x = 3$.

$$\begin{aligned}
&3x^2 - 7 \\
&3(3)^2 - 7 = 3(9) - 7 \\
&\qquad = 27 - 7 \\
&\qquad = 20
\end{aligned}$$

You Try It 5

If $2x = 5x + 6$, evaluate $-2x + 7$.

Your solution

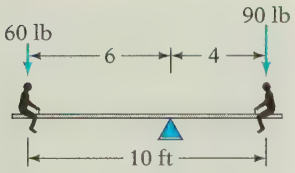
Solutions on pp. S5–S6

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Objective C To solve application problems using formulas

9 CT

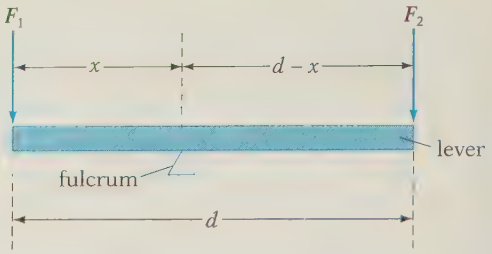
TAKE NOTE



This system balances because

$F_1x = F_2(d - x)$
$60(6) = 90(10 - 6)$
$60(6) = 90(4)$
$360 = 360$

A lever system is shown at the right. It consists of a lever, or bar; a fulcrum; and two forces, F_1 and F_2. The distance d represents the length of the lever, x represents the distance from F_1 to the fulcrum, and $d - x$ represents the distance from F_2 to the fulcrum.



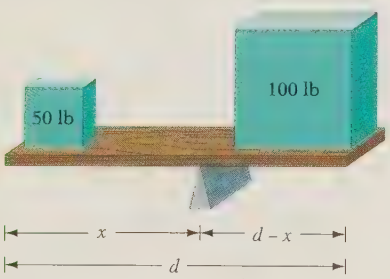
A principle of physics states that when the lever system balances, $F_1 \cdot x = F_2 \cdot (d - x)$.

Example 6

A lever is 15 ft long. A force of 50 lb is applied to one end of the lever, and a force of 100 lb is applied to the other end. Where is the fulcrum located when the system balances?

Strategy

Make a drawing.



Given: $F_1 = 50$
$F_2 = 100$
$d = 15$
Unknown: x

Solution

$$F_1 \cdot x = F_2 \cdot (d - x)$$
$$50x = 100(15 - x)$$
$$50x = 1500 - 100x$$
$$50x + 100x = 1500 - 100x + 100x$$
$$150x = 1500$$
$$\frac{150x}{150} = \frac{1500}{150}$$
$$x = 10$$

The fulcrum is 10 ft from the 50-pound force.

You Try It 6

A lever is 25 ft long. A force of 45 lb is applied to one end of the lever, and a force of 80 lb is applied to the other end. Where is the location of the fulcrum when the system balances?

Your strategy

Your solution

Solution on p. S6

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3.3 Exercises

Objective A

Solve and check.

1. $8x + 5 = 4x + 13$

2. $6y + 2 = y + 17$

3. $5x - 4 = 2x + 5$

4. $13b - 1 = 4b - 19$

5. $15x - 2 = 4x - 13$

6. $7a - 5 = 2a - 20$

7. $3x + 1 = 11 - 2x$

8. $n - 2 = 6 - 3n$

9. $2x - 3 = -11 - 2x$

10. $4y - 2 = -16 - 3y$

11. $2b + 3 = 5b + 12$

12. $m + 4 = 3m + 8$

13. $4y - 8 = y - 8$

14. $5a + 7 = 2a + 7$

15. $6 - 5x = 8 - 3x$

16. $10 - 4n = 16 - n$

17. $5 + 7x = 11 + 9x$

18. $3 - 2y = 15 + 4y$

19. $2x - 4 = 6x$

20. $2b - 10 = 7b$

21. $8m = 3m + 20$

22. $9y = 5y + 16$

23. $8b + 5 = 5b + 7$

24. $6y - 1 = 2y + 2$

25. $7x - 8 = x - 3$

26. $2y - 7 = -1 - 2y$

27. $2m - 1 = -6m + 5$

28. If $5x = 3x - 8$, evaluate $4x + 2$.

29. If $7x + 3 = 5x - 7$, evaluate $3x - 2$.

30. If $2 - 6a = 5 - 3a$, evaluate $4a^2 - 2a + 1$.

31. If $1 - 5c = 4 - 4c$, evaluate $3c^2 - 4c + 2$.

32. If $2y + 3 = 5 - 4y$, evaluate $6y - 7$.

33. If $3z + 1 = 1 - 5z$, evaluate $3z^2 - 7z + 8$.

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Objective B

Solve and check.

34. $5x + 2(x + 1) = 23$

35. $6y + 2(2y + 3) = 16$

36. $9n - 3(2n - 1) = 15$

37. $12x - 2(4x - 6) = 28$

38. $7a - (3a - 4) = 12$

39. $9m - 4(2m - 3) = 11$

40. $5(3 - 2y) + 4y = 3$

41. $4(1 - 3x) + 7x = 9$

42. $5y - 3 = 7 + 4(y - 2)$

43. $5 + 2(3b + 1) = 3b + 5$

44. $6 - 4(3a - 2) = 2(a + 5)$

45. $7 - 3(2a - 5) = 3a + 10$

46. $2a - 5 = 4(3a + 1) - 2$

47. $5 - (9 - 6x) = 2x - 2$

48. $7 - (5 - 8x) = 4x + 3$

49. $3[2 - 4(y - 1)] = 3(2y + 8)$

50. $5[2 - (2x - 4)] = 2(5 - 3x)$

51. $3a + 2[2 + 3(a - 1)] = 2(3a + 4)$

52. $5 + 3[1 + 2(2x - 3)] = 6(x + 5)$

53. $-2[4 - (3b + 2)] = 5 - 2(3b + 6)$

54. $-4[x - 2(2x - 3)] + 1 = 2x - 3$

55. If $4 - 3a = 7 - 2(2a + 5)$, evaluate $a^2 + 7a$.

56. If $9 - 5x = 12 - (6x + 7)$, evaluate $x^2 - 3x - 2$.

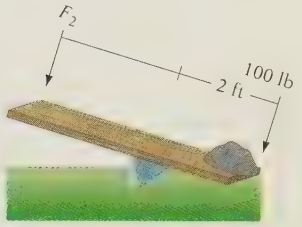
57. If $2z - 5 = 3(4z + 5)$, evaluate $\dfrac{z^2}{(z - 2)}$.

58. If $3n - 7 = 5(2n + 7)$, evaluate $\dfrac{n^2}{(2n - 6)}$.

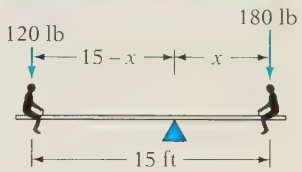
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Objective C Application Problems

Solve. Use the lever system equation $F_1x = F_2(d - x)$.



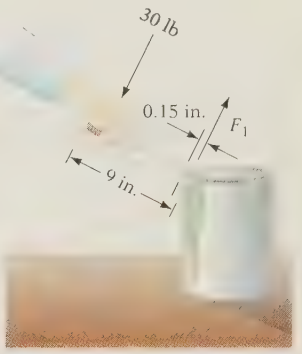
59. A lever 10 ft long is used to move a 100-pound rock. The fulcrum is placed 2 ft from the rock. What force must be applied to the other end of the lever to move the rock?



60. An adult and a child are on a seesaw 14 ft long. The adult weighs 175 lb and the child weighs 70 lb. How many feet from the child must the fulcrum be placed so that the seesaw balances?

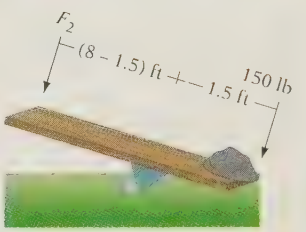
61. Two people are sitting 15 ft apart on a seesaw. One person weighs 180 lb. The second person weighs 120 lb. How far from the 180-pound person should the fulcrum be placed so that the seesaw balances?

62. Two children are sitting on a seesaw that is 12 ft long. One child weighs 60 lb. The other child weighs 90 lb. How far from the 90-pound child should the fulcrum be placed so that the seesaw balances?



63. In preparation for a stunt, two acrobats are standing on a plank 18 ft long. One acrobat weighs 128 lb and the second acrobat weighs 160 lb. How far from the 128-pound acrobat must the fulcrum be placed so that the acrobats are balanced on the plank?

64. A screwdriver 9 in. long is used as a lever to open a can of paint. The tip of the screwdriver is placed under the lip of the can with the fulcrum 0.15 in. from the lip. A force of 30 lb is applied to the other end of the screwdriver. Find the force on the lip of the can.



65. A metal bar 8 ft long is used to move a 150-pound rock. The fulcrum is placed 1.5 ft from the rock. What minimum force must be applied to the other end of the bar to move the rock? Round to the nearest tenth.

To determine the breakeven point, or the number of units that must be sold so that no profit or loss occurs, an economist uses the formula $Px = Cx + F$, where P is the selling price per unit, x is the number of units that must be sold to break even, C is the cost to make each unit, and F is the fixed cost.

66. A business analyst has determined that the selling price per unit for a laser printer is \$1600. The cost to make the laser printer is \$950, and the fixed cost is \$211,250. Find the breakeven point.

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67. A business analyst has determined that the selling price per unit for a gas barbecue is \$325. The cost to make one gas barbecue is \$175, and the fixed cost is \$39,000. Find the breakeven point.

68. A manufacturer of thermostats determines that the cost per unit for a programmable thermostat is \$38 and that the fixed cost is \$24,400. The selling price for the thermostat is \$99. Find the breakeven point.

69. A manufacturing engineer determines that the cost per unit for a desk lamp is \$12 and that the fixed cost is \$19,240. The selling price for the desk lamp is \$49. Find the breakeven point.

70. A manufacturing engineer determines the cost to make one compact disk to be \$3.35 and the fixed cost to be \$6180. The selling price for each compact disk is \$8.50. Find the number of compact disks that must be sold to break even.

71. To manufacture a softball bat requires two steps. The first step is to cut a rough shape. The second step is to sand the bat to its final form. The cost to rough-shape a bat is \$.45, and the cost to sand a bat to final form is \$1.05. The total fixed cost for the two steps is \$16,500. How many softball bats must be sold at a price of \$7.00 to break even?

APPLYING THE CONCEPTS

72. Write an equation of the form $ax + b = cx + d$ that has 4 as the solution.

Solve. If the equation has no solution, write "no solution."

73. $3(2x - 1) - (6x - 4) = -9$

74. $7(3x + 6) - 4(3 + 5x) = 13 + x$

75. $\frac{1}{5}(25 - 10a) + 4 = \frac{1}{3}(12a - 15) + 14$

76. $5[m + 2(3 - m)] = 3[2(4 - m) - 5]$

77. The equation $x = x + 1$ has no solution, whereas the solution of the equation $2x + 3 = 3$ is zero. Is there a difference between no solution and a solution of zero? Explain your answer.

78. Archimedes supposedly said, "Give me a long enough lever and I can move the world." Explain what Archimedes meant by that statement.

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3.4 Translating Sentences into Equations

Objective A To solve integer problems

An equation states that two mathematical expressions are equal. Therefore, to **translate** a sentence into an equation requires recognition of the words or phrases that mean "equals." Some of these phrases are listed below.

equals
is
is equal to
amounts to
represents
} translate to =

Once the sentence is translated into an equation, the equation can be solved by rewriting the equation in the form *variable = constant*.

Translate "five less than a number is thirteen" into an equation and solve.

The unknown number: n — Assign a variable to the unknown number.

Five less than a number	is	thirteen

Find two verbal expressions for the same value.

$n - 5 = 13$ — Write a mathematical expression for each verbal expression. Write the equals sign.

$n - 5 + 5 = 13 + 5$ — Solve the equation.

$n = 18$

The number is 18.

TAKE NOTE

You can check the solution to a translation problem.

Check:

5 less than 18 is 13

$18 - 5 \mid 13$

$13 = 13$

Recall that the integers are the numbers $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$. An **even integer** is an integer that is divisible by 2. Examples of even integers are -8, 0, and 22. An **odd integer** is an integer that is not divisible by 2. Examples of odd integers are -17, 1, and 39.

Consecutive integers are integers that follow one another in order. Examples of consecutive integers are shown at the right. (Assume that the variable n represents an integer.)

11, 12, 13
$-8, -7, -6$
$n, n + 1, n + 2$

Examples of **consecutive even integers** are shown at the right. (Assume that the variable n represents an even integer.)

24, 26, 28
$-10, -8, -6$
$n, n + 2, n + 4$

Examples of **consecutive odd integers** are shown at the right. (Assume that the variable n represents an odd integer.)

19, 21, 23
$-1, 1, 3$
$n, n + 2, n + 4$

TAKE NOTE

Both consecutive even and consecutive odd integers are represented using n, $n + 2$, $n + 4, \ldots$.

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➡ The sum of three consecutive odd integers is forty-five. Find the integers.

Strategy

- First odd integer: n
  Second odd integer: $n + 2$
  Third odd integer: $n + 4$
- The sum of the three odd integers is 45.

• Represent three consecutive odd integers.

Solution

$n + (n + 2) + (n + 4) = 45$ • Write an equation.

$3n + 6 = 45$ • Solve the equation.

$3n = 39$

$n = 13$ • The first odd integer is 13.

$n + 2 = 13 + 2 = 15$ • Find the second odd integer.

$n + 4 = 13 + 4 = 17$ • Find the third odd integer.

The three consecutive odd integers are 13, 15, and 17.

Example 1

The sum of two numbers is sixteen. The difference between four times the smaller number and two is two more than twice the larger number. Find the two numbers.

Solution

The smaller number: n
The larger number: $16 - n$

The difference between four times the smaller and two	is	two more than twice the larger

$$\begin{aligned}
4n - 2 &= 2(16 - n) + 2 \\
4n - 2 &= 32 - 2n + 2 \\
4n - 2 &= 34 - 2n \\
4n + 2n - 2 &= 34 - 2n + 2n \\
6n - 2 &= 34 \\
6n - 2 + 2 &= 34 + 2 \\
6n &= 36 \\
\frac{6n}{6} &= \frac{36}{6} \\
n &= 6
\end{aligned}$$

$16 - n = 16 - 6 = 10$

The smaller number is 6.
The larger number is 10.

You Try It 1

The sum of two numbers is twelve. The total of three times the smaller number and six amounts to seven less than the product of four and the larger number. Find the two numbers.

Your solution

Solution on p. S6

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Example 2

Find three consecutive even integers such that three times the second equals four more than the sum of the first and third.

Strategy

First even integer: n
Second even integer: $n + 2$
Third even integer: $n + 4$
Three times the second equals four more than the sum of the first and third.

Solution

$$\begin{aligned} 3(n + 2) &= n + (n + 4) + 4 \\ 3n + 6 &= 2n + 8 \\ 3n - 2n + 6 &= 2n - 2n + 8 \\ n + 6 &= 8 \\ n &= 2 \end{aligned}$$

$n + 2 = 2 + 2 = 4$
$n + 4 = 2 + 4 = 6$

The three integers are 2, 4, and 6.

You Try It 2

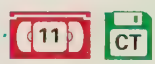
Find three consecutive integers whose sum is negative six.

Your strategy

Your solution

Solution on p. S6

Objective B To translate a sentence into an equation and solve



Example 3

A wallpaper hanger charges a fee of \$25 plus \$12 for each roll of wallpaper used in a room. If the total charge for hanging wallpaper is \$97, how many rolls of wallpaper were used?

Strategy

To find the number of rolls of wallpaper used, write and solve an equation using n to represent the number of rolls of wallpaper used.

Solution

\$25 plus \$12 for each roll of wallpaper	is	\$97

$$\begin{aligned} 25 + 12n &= 97 \\ 12n &= 72 \\ \frac{12n}{12} &= \frac{72}{12} \\ n &= 6 \end{aligned}$$

6 rolls of wallpaper were used.

You Try It 3

The fee charged by a ticketing agency for a concert is \$3.50 plus \$17.50 for each ticket purchased. If your total charge for tickets is \$161, how many tickets are you purchasing?

Your strategy

Your solution

Solution on p. S6

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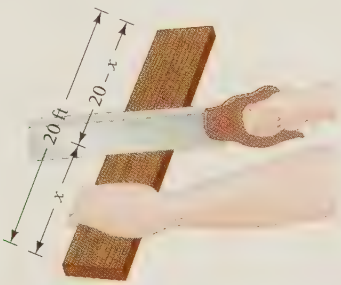
Example 4

A board 20 ft long is cut into two pieces. Five times the length of the shorter piece is 2 ft more than twice the length of the longer piece. Find the length of each piece.

Strategy

Let x represent the length of the shorter piece. Then $20 - x$ represents the length of the longer piece.

Make a drawing.



To find the lengths, write and solve an equation using x to represent the length of the shorter piece and $20 - x$ to represent the length of the longer piece.

Solution

Five times the shorter piece	is	2 ft more than twice the longer

$$\begin{aligned} 5x &= 2(20 - x) + 2 \\ 5x &= 40 - 2x + 2 \\ 5x &= 42 - 2x \\ 5x + 2x &= 42 - 2x + 2x \\ 7x &= 42 \\ \frac{7x}{7} &= \frac{42}{7} \\ x &= 6 \end{aligned}$$

$20 - x = 20 - 6 = 14$

The shorter piece is 6 ft.
The longer piece is 14 ft.

You Try It 4

A wire 22 in. long is cut into two pieces. The longer piece is 4 in. more than twice the shorter piece. Find the length of each piece.

Your strategy

Your solution

Solution on p. S6

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3.4 Exercises

Objective A

In Exercises 1 to 19, translate into an equation and solve.

1. The difference between a number and fifteen is seven. Find the number.

2. The sum of five and a number is three. Find the number.

3. The product of seven and a number is negative twenty-one. Find the number.

4. The quotient of a number and four is two. Find the number.

5. The difference between nine and a number is seven. Find the number.

6. Three-fifths of a number is negative thirty. Find the number.

7. The difference between five and twice a number is one. Find the number.

8. Four more than three times a number is thirteen. Find the number.

9. The sum of twice a number and five is fifteen. Find the number.

10. The difference between nine times a number and six is twelve. Find the number.

11. Six less than four times a number is twenty-two. Find the number.

12. Four times the sum of twice a number and three is twelve. Find the number.

13. Three times the difference between four times a number and seven is fifteen. Find the number.

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14. Twelve is six times the difference between a number and three. Find the number.

15. Twice the difference between a number and twenty-five is three times the number. Find the number.

16. The sum of two numbers is twenty. Three times the smaller is equal to two times the larger. Find the two numbers.

17. The sum of two numbers is fifteen. One less than three times the smaller is equal to the larger. Find the two numbers.

18. The sum of two numbers is fourteen. The difference between two times the smaller and the larger is one. Find the two numbers.

19. The sum of two numbers is eighteen. The total of three times the smaller and twice the larger is forty-four. Find the two numbers.

20. The sum of three consecutive odd integers is fifty-one. Find the integers.

21. Find three consecutive even integers whose sum is negative eighteen.

22. Find three consecutive odd integers such that three times the middle integer is one more than the sum of the first and third.

23. Twice the smallest of three consecutive odd integers is seven more than the largest. Find the integers.

24. Find two consecutive even integers such that three times the first equals twice the second.

25. Find two consecutive even integers such that four times the first is three times the second.

26. Seven times the first of two consecutive odd integers is five times the second. Find the integers.

27. Find three consecutive even integers such that three times the middle integer is four more than the sum of the first and third.

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Objective B Application Problems

Write an equation and solve.

28. The processor speed of a personal computer is 80 megahertz (mHz). This is one-third of the processor speed of a newer model personal computer. Find the processor speed of the newer personal computer.

29. The storage capacity of a hard-disk drive is 0.5 gigabyte. This is one-fourth of the storage capacity of a second hard-disk drive. Find the storage capacity of the second hard-disk drive.

30. A union charges monthly dues of \$4.00 plus \$.15 for each hour worked during the month. A union member's dues for March were \$29.20. How many hours did the union member work during the month of March?

31. A technical information hotline charges a customer \$9.00 plus \$.50 per minute to answer questions about software. How many minutes did a customer who received a bill for \$14.50 use this service?

32. The total cost to paint the inside of a house was \$1346. This cost included \$125 for materials and \$33 per hour for labor. How many hours of labor were required to paint the inside of the house?

33. The cellular phone service for a business executive is \$35 per month plus \$.40 per minute of phone use. In a month when the executive's cellular phone bill was \$99.80, how many minutes did the executive use the phone?

34. A computer screen consists of tiny dots of light called pixels. In a certain graphics mode, there are 640 horizontal pixels. This is 40 more than 3 times the number of vertical pixels. Find the number of vertical pixels.

35. The cost of electricity in a certain city is \$.08 for each of the first 300 kWh (kilowatt-hours) and \$.13 for each kilowatt-hour over 300 kWh. Find the number of kilowatt-hours used by a family with a \$51.95 electric bill.

36. A 12-foot board is cut into two pieces. Twice the length of the shorter piece is 3 ft less than the longer piece. Find the length of each piece.

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37. A 14-yard fishing line is cut into two pieces. Three times the length of the longer piece is four times the length of the shorter piece. Find the length of each piece.

38. Seven thousand dollars is divided into two scholarships. Twice the amount of the smaller scholarship is $1000 less than the larger scholarship. What is the amount of the larger scholarship?

39. An investment of $10,000 is divided into two accounts, one for stocks and one for mutual funds. The value of the stock account is $2000 less than twice the value of the mutual fund account. Find the amount in each account.

APPLYING THE CONCEPTS

40. Make up a word problem that has either the solution of the equation $6x = 124$ or the solution of the equation $8x + 120 = 300$ as the answer to a problem.

41. A formula is an equation that relates variables in a known way. Find two examples of formulas that are used in your college major. Explain what each of the variables represents.

An equation that is never true is called a **contradiction.** For example, $x = x + 1$ is a contradiction; there is no value of x that will make the equation true. An equation that is true for all real numbers is called an **identity.** The equation $x + x = 2x$ is an identity; this equation is true for any real number. A **conditional equation** is one that is true for some real numbers and false for some real numbers. The equation $2x = 4$ is a conditional equation; this equation is true when x is 2 and false for any other real number. Determine whether each equation below is a contradiction, an identity, or a conditional equation. If it is a conditional equation, find the solution.

42. $6x + 2 = 5 + 3(2x - 1)$

43. $3 - 2(4x + 1) = 5 + 8(1 - x)$

44. $3t - 5(t + 1) = 2(2 - t) - 9$

45. $6 + 4(2y + 1) = 5 - 8y$

46. $3v - 2 = 5v - 2(2 + v)$

47. $9z = 15z$

48. It is always important to check the answer to an application problem to be sure the answer makes sense. Consider the following problem. A 4-quart mixture of fruit juices is made from apple juice and cranberry juice. There are 6 more quarts of apple juice than of cranberry juice. Write and solve an equation for the number of quarts of each juice used. Does the answer to this question make sense? Explain.

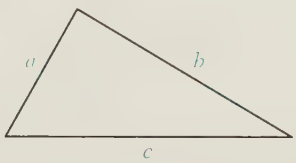
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3.5 Geometry Problems

Objective A To solve perimeter problems

The **perimeter** of a plane geometric figure is a measure of the distance around the figure. Perimeter is used in buying fencing for a lawn or determining how much baseboard is needed for a room.

The perimeter of a triangle is the sum of the lengths of the three sides. Therefore, if a, b, and c represent the lengths of the sides of a triangle, the perimeter, P, of the triangle is given by $P = a + b + c$.



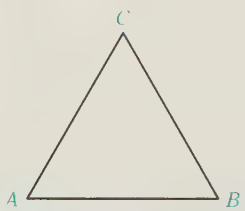
$P = a + b + c$

Two special types of triangles are shown below. An **isosceles triangle** has two sides of equal length. The three sides of an **equilateral triangle** are of equal length.



Isosceles triangle

$AC = BC$



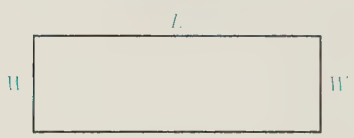
Equilateral triangle

$AB = AC = BC$

POINT OF INTEREST

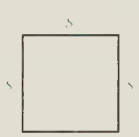
Leonardo da Vinci painted the Mona Lisa on a rectangular canvas whose height was approximately 1.6 times its width. Rectangles with these proportions, called golden rectangles, were used extensively in Renaissance art.

The perimeter of a rectangle is the sum of the lengths of the four sides. Let L represent the length and W represent the width of a rectangle. Then the perimeter, P, of the rectangle is given by $P = L + W + L + W$. After combining like terms, the formula is $P = 2L + 2W$.



$P = 2L + 2W$

A square is a rectangle in which each side has the same length. Let s represent the length of each side of a square. Then the perimeter, P, of the square is given by $P = s + s + s + s$. After combining like terms, the formula is $P = 4s$.



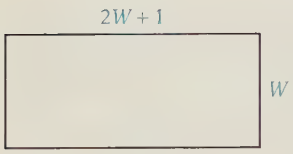
$P = 4s$

Formulas for Perimeters of Geometric Figures

Perimeter of a triangle	$P = a + b + c$
Perimeter of a rectangle	$P = 2L + 2W$
Perimeter of a square	$P = 4s$

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➡ The perimeter of a rectangle is 26 ft. The length of the rectangle is 1 ft more than twice the width. Find the width and length of the rectangle.



Strategy

- Width: W
  Length: $2W + 1$
- Use the formula for the perimeter of a rectangle.

- Let a variable represent the width.
- Represent the length in terms of that variable.

Solution

$P = 2L + 2W$

$26 = 2(2W + 1) + 2W$ • $P = 26$. Substitute $2W + 1$ for L.

$26 = 4W + 2 + 2W$ • Use the Distributive Property.

$26 = 6W + 2$ • Combine like terms.

$24 = 6W$ • Subtract 2 from each side of the equation.

$4 = W$ • Divide each side of the equation by 6.

$L = 2W + 1$

$= 2(4) + 1 = 8 + 1 = 9$ • Find the length of the rectangle by substituting 4 for W in $2W + 1$.

The width is 4 ft. The length is 9 ft.

Example 1

The perimeter of an isosceles triangle is 25 ft. The third side of the triangle is 2 ft less than the length of one of the equal sides. Find the measures of each of the three sides of the triangle.

Strategy

Each equal side: x
The third side: $x - 2$
Use the equation for the perimeter of a triangle.

Solution

$P = a + b + c$
$25 = x + x + (x - 2)$
$25 = 3x - 2$
$27 = 3x$
$9 = x$

$x - 2 = 9 - 2 = 7$

Each of the equal sides measures 9 ft.
The third side measures 7 ft.

You Try It 1

A carpenter is designing a square patio with a perimeter of 52 ft. What is the length of each side?

Your strategy

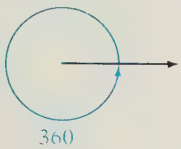
Your solution

Solution on p. S7

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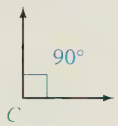
Objective B To solve problems involving angles formed by intersecting lines

A unit used to measure angles is the **degree.** The symbol for degree is °. $\angle$ is the symbol for angle.



One complete revolution is 360°. It is probable that the Babylonians chose 360° for a circle because they knew that there were 365 days in one year, and 360 is the closest number to 365 that is divisible by many numbers.

A 90° angle is called a **right angle.** The symbol $\llcorner$ represents a right angle. Angle C ($\angle C$) is a right angle.



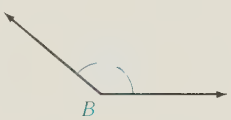
A 180° angle is called a **straight angle.** The angle at the right is a straight angle.



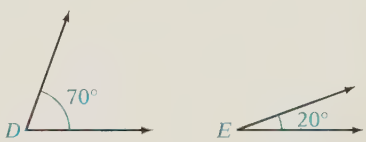
An **acute angle** is an angle whose measure is between 0° and 90°. $\angle A$ at the right is an acute angle.



An **obtuse angle** is an angle whose measure is between 90° and 180°. $\angle B$ at the right is an obtuse angle.



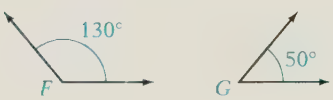
Complementary angles are two angles whose measures have the sum 90°.



$\angle D + \angle E = 70° + 20° = 90°$

$\angle D$ and $\angle E$ are complementary angles.

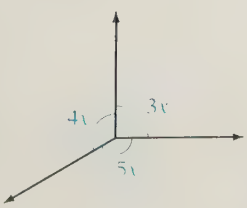
Supplementary angles are two angles whose measures have the sum 180°.



$\angle F + \angle G = 130° + 50° = 180°$

$\angle F$ and $\angle G$ are supplementary angles.

➨ Given the diagram at the left, find x.



Strategy

The sum of the measures of the three angles is 360°. To find x, write an equation and solve for x.

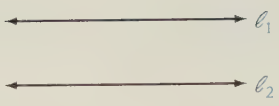
Solution

$$3x + 4x + 5x = 360°$$
$$12x = 360°$$
$$x = 30°$$

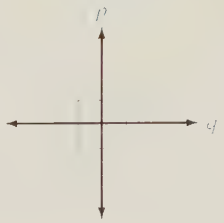
The measure of x is 30°.

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Parallel lines never meet. The distance between them is always the same. The symbol $\parallel$ means "is parallel to." In the figure at the right, $\ell_1 \parallel \ell_2$.

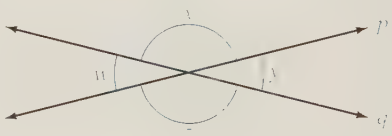


Perpendicular lines are intersecting lines that form right angles. The symbol $\perp$ means "is perpendicular to." In the figure at the right, $p \perp q$.



Four angles are formed by the intersection of two lines. If the two lines are perpendicular, each of the four angles is a right angle. If the two lines are not perpendicular, then two of the angles formed are acute angles and two of the angles are obtuse angles. The two acute angles are always opposite each other, and the two obtuse angles are always opposite each other.

In the figure at the right, $\angle w$ and $\angle y$ are acute angles, and $\angle x$ and $\angle z$ are obtuse angles.



Two angles that are on opposite sides of the intersection of two lines are called **vertical angles.** Vertical angles have the same measure. $\angle w$ and $\angle y$ are vertical angles. $\angle x$ and $\angle z$ are vertical angles.

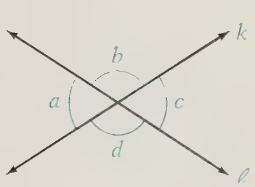
Vertical angles have the same measure.

$$\angle w = \angle y$$
$$\angle x = \angle z$$

Two angles that share a common side are called **adjacent angles.** For the figure shown above, $\angle x$ and $\angle y$ are adjacent angles, as are $\angle y$ and $\angle z$, $\angle z$ and $\angle w$, and $\angle w$ and $\angle x$. Adjacent angles of intersecting lines are supplementary angles.

Adjacent angles of intersecting lines are supplementary angles.

$$\angle x + \angle y = 180°$$
$$\angle y + \angle z = 180°$$
$$\angle z + \angle w = 180°$$
$$\angle w + \angle x = 180°$$



➡ In the diagram at the left, $\angle b = 115°$. Find the measures of angles a, c, and d.

$\angle a + \angle b = 180°$
$\angle a + 115° = 180°$
$\angle a = 65°$

• $\angle a$ is supplementary to $\angle b$ because $\angle a$ and $\angle b$ are adjacent angles of intersecting lines.

$\angle c = 65°$

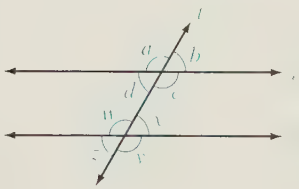
• $\angle c = \angle a$ because $\angle c$ and $\angle a$ are vertical angles.

$\angle d = 115°$

• $\angle d = \angle b$ because $\angle d$ and $\angle b$ are vertical angles.

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A line that intersects two other lines at different points is called a **transversal.** If the lines cut by a transversal t are parallel lines and the transversal is not perpendicular to the parallel lines, all four acute angles have the same measure and all four obtuse angles have the same measure. In the figure at the right,



$$\angle b = \angle d = \angle x = \angle z$$
$$\angle a = \angle c = \angle w = \angle y$$

Alternate interior angles are two angles that are on opposite sides of the transversal and between the lines. In the figure above, $\angle c$ and $\angle w$ are alternate interior angles, and $\angle d$ and $\angle x$ are alternate interior angles. Alternate interior angles have the same measure.

Alternate interior angles have the same measure.

$$\angle c = \angle w$$
$$\angle d = \angle x$$

Alternate exterior angles are two angles that are on opposite sides of the transversal and outside the parallel lines. In the figure above, $\angle a$ and $\angle y$ are alternate exterior angles, and $\angle b$ and $\angle z$ are alternate exterior angles. Alternate exterior angles have the same measure.

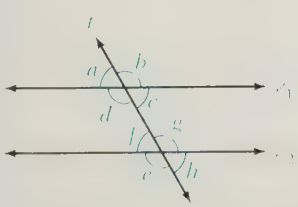
Alternate exterior angles have the same measure.

$$\angle a = \angle y$$
$$\angle b = \angle z$$

Corresponding angles are two angles that are on the same side of the transversal and are both acute angles or are both obtuse angles. In the figure above, the following pairs of angles are corresponding angles: $\angle a$ and $\angle w$, $\angle d$ and $\angle z$, $\angle b$ and $\angle x$, and $\angle c$ and $\angle y$. Corresponding angles have the same measure.

Corresponding angles have the same measure.

$$\angle a = \angle w$$
$$\angle d = \angle z$$
$$\angle b = \angle x$$
$$\angle c = \angle y$$



➡ In the diagram at the left, $\ell_1 \parallel \ell_2$ and $\angle f = 58°$. Find the measures of $\angle a$, $\angle c$, and $\angle d$.

$\angle a = \angle f = 58°$ • $\angle a$ and $\angle f$ are corresponding angles.

$\angle c = \angle f = 58°$ • $\angle c$ and $\angle f$ are alternate interior angles.

$\angle d + \angle a = 180°$ • $\angle d$ is supplementary to $\angle a$.

$\angle d + 58° = 180°$

$\angle d = 122°$

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Example 2
Find the complement of a 39° angle.

Strategy
To find the complement, let x represent the complement of a 39° angle. Use the fact that complementary angles are two angles whose sum is 90° to write an equation. Solve for x.

Solution
$x + 39° = 90°$
$x = 51°$

The complement of a 39° angle is a 51° angle.

You Try It 2
Find the supplement of a 107° angle.

Your strategy

Your solution

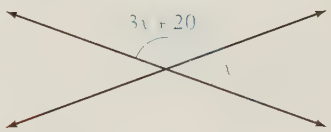
Example 3
Find x.



Strategy
The angles labeled are adjacent angles of intersecting lines and are, therefore, supplementary angles. To find x, write an equation and solve for x.

Solution
$x + (x + 70°) = 180°$
$2x + 70° = 180°$
$2x = 110°$
$x = 55°$

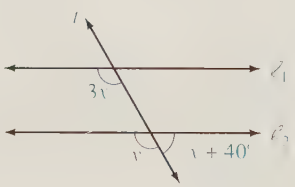
You Try It 3
Find x.



Your strategy

Your solution

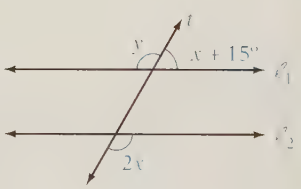
Example 4
Given $\ell_1 \parallel \ell_2$, find x.



Strategy
$3x = y$ because corresponding angles have the same measure. $y + (x + 40°) = 180°$ because adjacent angles of intersecting lines are supplementary angles. Substitute $3x$ for y and solve for x.

Solution
$3x + (x + 40°) = 180°$
$4x + 40° = 180°$
$4x = 140°$
$x = 35°$

You Try It 4
Given $\ell_1 \parallel \ell_2$, find x.



Your strategy

Your solution

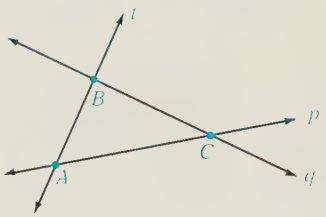
Solutions on p. S7

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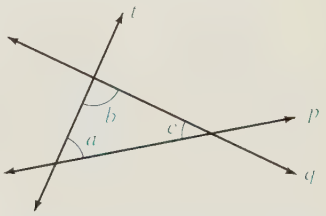
Objective C To solve problems involving the angles of a triangle

12 CT

If the lines cut by a transversal are not parallel lines, the three lines will intersect at three points. In the figure at the right, the transversal t intersects lines p and q. The three lines intersect at points A, B, and C. The geometric figure formed by AB, BC, and AC is a **triangle.**



The angles within the region enclosed by the triangle are called **interior angles.** In the figure at the right, angles a, b, and c are interior angles. The sum of the measures of the interior angles is 180°.

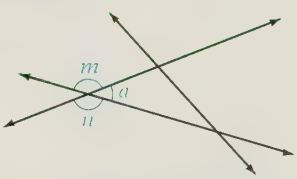


$\angle a + \angle b + \angle c = 180°$

The Sum of the Measures of the Interior Angles of a Triangle

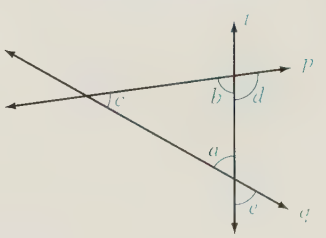
The sum of the measures of the interior angles of a triangle is 180°.

An angle adjacent to an interior angle is an **exterior angle.** In the figure at the right, angles m and n are exterior angles for angle a. The sum of the measures of an interior and an exterior angle is 180°.



$\angle a + \angle m = 180°$
$\angle a + \angle n = 180°$

➡ Given that $\angle c = 40°$ and $\angle e = 60°$, find the measure of $\angle d$.



$\angle a = \angle e = 60°$ • $\angle a$ and $\angle e$ are vertical angles.

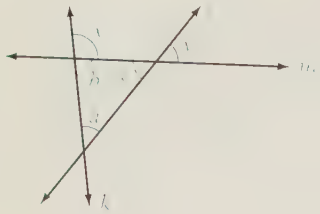
$\angle c + \angle a + \angle b = 180°$
$40° + 60° + \angle b = 180°$
$100° + \angle b = 180°$
$\angle b = 80°$ • The sum of the interior angles is 180°.

$\angle b + \angle d = 180°$
$80° + \angle d = 180°$
$\angle d = 100°$ • $\angle b$ and $\angle d$ are supplementary angles.

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Example 5

Given that $\angle a = 45°$ and $\angle x = 100°$, find the measures of angles b, c, and y.



Strategy

To find the measure of $\angle b$, use the fact that $\angle b$ and $\angle x$ are supplementary angles.
To find the measure of $\angle c$, use the fact that the sum of the measures of the interior angles of a triangle is 180°.
To find the measure of $\angle y$, use the fact that $\angle c$ and $\angle y$ are vertical angles.

Solution

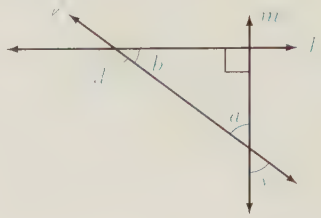
$$\angle b + \angle x = 180°$$
$$\angle b + 100° = 180°$$
$$\angle b = 80°$$

$$\angle a + \angle b + \angle c = 180°$$
$$45° + 80° + \angle c = 180°$$
$$125° + \angle c = 180°$$
$$\angle c = 55°$$

$$\angle y = \angle c = 55°$$

You Try It 5

Given that $\angle y = 55°$, find the measures of angles a, b, and d.



Your strategy

Your solution

Example 6

Two angles of a triangle measure 43° and 86°. Find the measure of the third angle.

Strategy

To find the measure of the third angle, use the fact that the sum of the measures of the interior angles of a triangle is 180°. Write an equation using x to represent the measure of the third angle. Solve the equation for x.

Solution

$$x + 43° + 86° = 180°$$
$$x + 129° = 180°$$
$$x = 51°$$

The measure of the third angle is 51°.

You Try It 6

One angle in a triangle is a right angle, and one angle measures 27°. Find the measure of the third angle.

Your strategy

Your solution

Solutions on p. S7

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3.5 Exercises

Objective A

1. In an isosceles triangle, the third side is 50% of the length of one of the equal sides. Find the length of each side when the perimeter is 125 ft.

2. In an isosceles triangle, the length of one of the equal sides is 3 times the length of the third side. The perimeter is 21 m. Find the length of each side.

3. The perimeter of a rectangle is 42 m. The length of the rectangle is 3 m less than twice the width. Find the length and width of the rectangle.

4. The width of a rectangle is 25% of the length. The perimeter is 250 cm. Find the length and width of the rectangle.

5. The perimeter of a rectangle is 120 ft. The length of the rectangle is twice the width. Find the length and width of the rectangle.

6. The perimeter of a rectangle is 50 m. The width of the rectangle is 5 m less than the length. Find the length and width of the rectangle.

7. The perimeter of a triangle is 110 cm. One side is twice the second side. The third side is 30 cm more than the second side. Find the length of each side.

8. The perimeter of a triangle is 33 ft. One side of the triangle is 1 ft longer than the second side. The third side is 2 ft longer than the second side. Find the length of each side.

9. The width of the rectangular foundation of a building is 30% of the length. The perimeter of the foundation is 338 ft. Find the length and width of the foundation.

10. The perimeter of a rectangular playground is 440 ft. If the width is 100 ft, what is the length of the playground?

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11. A rectangular vegetable garden has a perimeter of 64 ft. The length of the garden is 20 ft. What is the width of the garden?

12. Each of two sides of a triangular banner measures 18 in. If the perimeter of the banner is 46 in., what is the length of the third side of the banner?

13. The perimeter of a square picture frame is 48 in. Find the length of each side of the frame.

14. A square rug has a perimeter of 32 ft. Find the length of each side of the rug.

Objective B

15. Find the complement of a 28° angle.

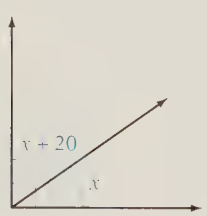
16. Find the complement of a 46° angle.

17. Find the supplement of a 73° angle.

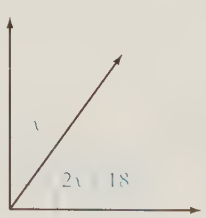
18. Find the supplement of a 119° angle.

Find the measure of $\angle x$.

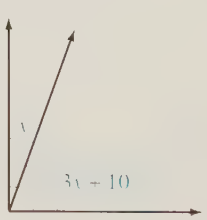
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20.

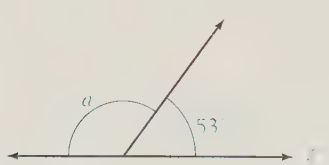


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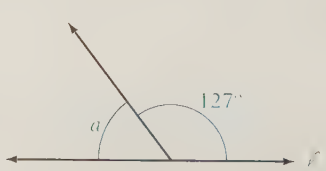


Find the measure of $\angle a$.

22.



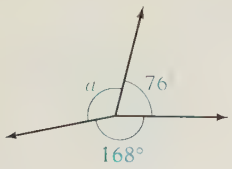
23.



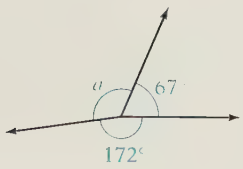
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Find the measure of $\angle a$.

24.

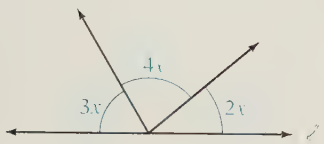


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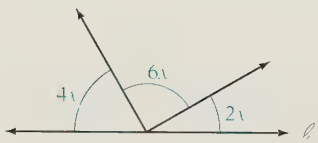


Find x.

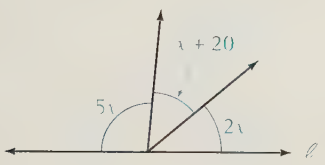
26.



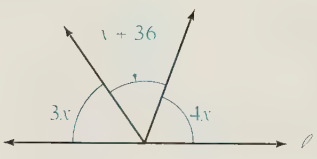
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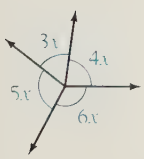
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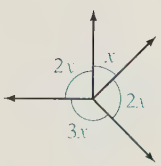
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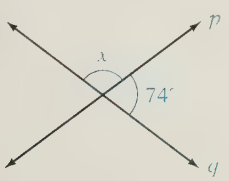


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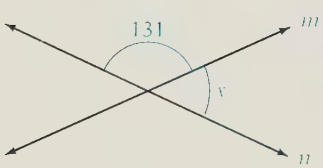


Find the measure of $\angle x$.

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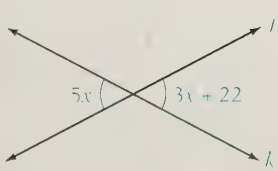


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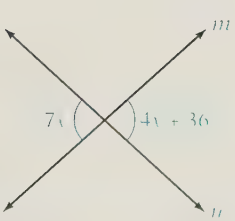


Find x.

34.



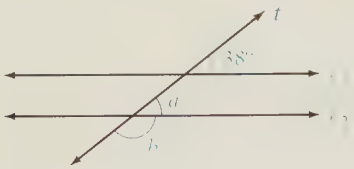
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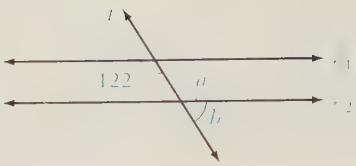
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Given that $\ell_1 \parallel \ell_2$, find the measures of angles a and b.

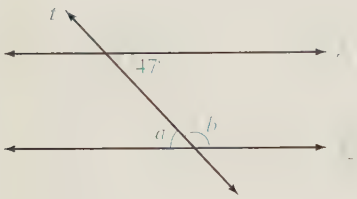
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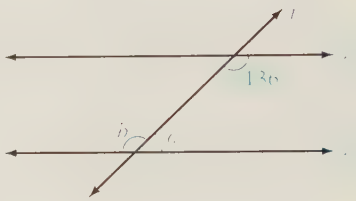
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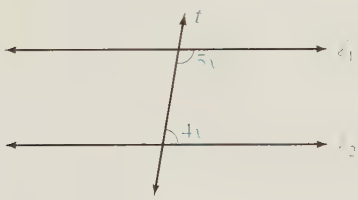


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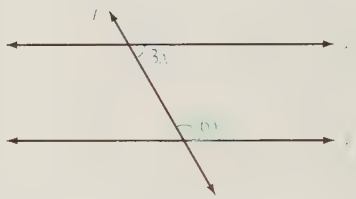


Given that $\ell_1 \parallel \ell_2$, find x.

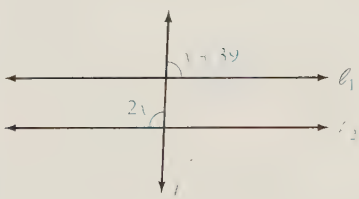
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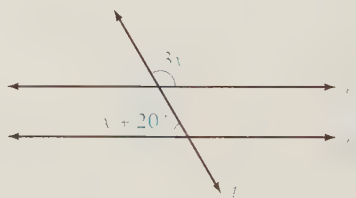
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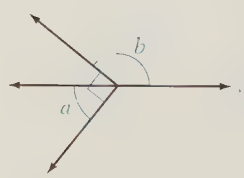
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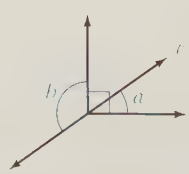
43.



44. Given that $\angle a = 51°$, find the measure of $\angle b$.



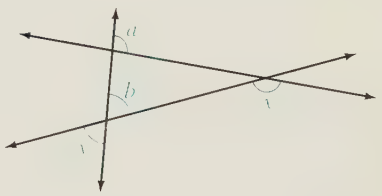
45. Given that $\angle a = 38°$, find the measure of $\angle b$.



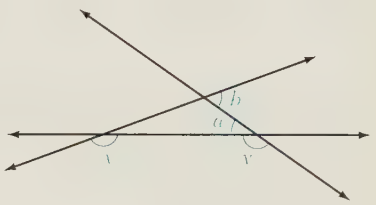
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Objective C

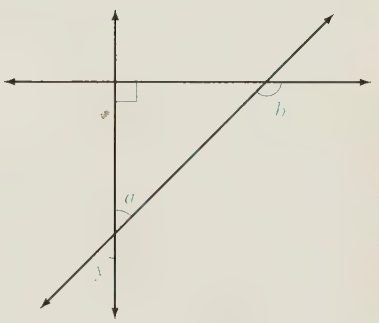
46. Given that $\angle a = 95°$ and $\angle b = 70°$, find the measures of angles x and y.



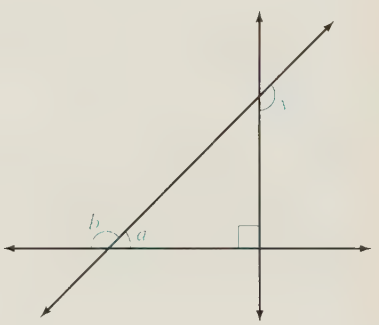
47. Given that $\angle a = 35°$ and $\angle b = 55°$, find the measures of angles x and y.



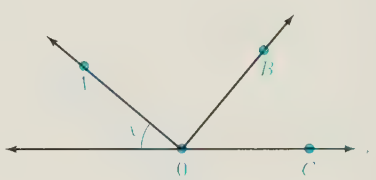
48. Given that $\angle y = 45°$, find the measures of angles a and b.



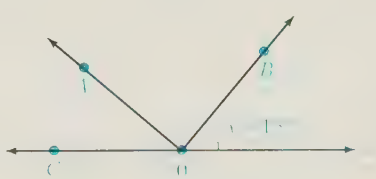
49. Given that $\angle y = 130°$, find the measures of angles a and b.



50. Given that $AO \perp OB$, express in terms of x the number of degrees in $\angle BOC$.



51. Given that $AO \perp OB$, express in terms of x the number of degrees in $\angle AOC$.



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52. One angle in a triangle is a right angle, and one angle measures 30°. What is the measure of the third angle?

53. A triangle has a 45° angle and a right angle. Find the measure of the third angle.

54. Two angles of a triangle measure 42° and 103°. Find the measure of the third angle.

55. Two angles of a triangle measure 62° and 45°. Find the measure of the third angle.

56. A triangle has a 13° angle and a 65° angle. What is the measure of the third angle?

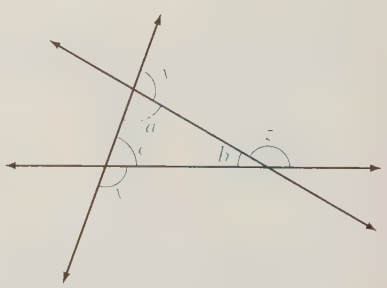
57. A triangle has a 105° angle and a 32° angle. What is the measure of the third angle?

APPLYING THE CONCEPTS

58. A rectangle and an equilateral triangle have the same perimeter. The length of the rectangle is three times the width. Each side of the triangle is 8 cm. Find the length and width of the rectangle.

59. The length of a rectangle is 1 cm more than twice the width. If the length of the rectangle is decreased by 2 cm and the width is decreased by 1 cm, the perimeter is 20 cm. Find the length and width of the original rectangle.

60. For the figure at the right, find the sum of the measures of angles x, y, and z.



61. For the figure at the right, explain why $\angle a + \angle b = \angle x$. Write a rule that describes the relationship between an exterior angle of a triangle and the opposite interior angles. Use the rule to write an equation involving angles a, c, and z.

62. The length of a rectangle is $14x$. The perimeter is $50x$. Find the width of the rectangle in terms of the variable x.

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3.6 Mixture and Uniform Motion Problems

Objective A To solve value mixture problems

A value mixture problem involves combining two ingredients that have different prices into a single blend. For example, a coffee merchant may blend two types of coffee into a single blend, or a candy manufacturer may combine two types of candy to sell as a variety pack.

TAKE NOTE

The equation $AC = V$ is used to find the value of an ingredient. For example, the value of 4 lb of cashews costing \$6 per pound is

$$AC = V$$
$$4 \cdot 6 = V$$
$$\$24 = V$$

The solution of a value mixture problem is based on the equation $AC = V$, where A is the amount of an ingredient, C is the cost per unit of the ingredient, and V is the value of the ingredient.

➡ A coffee merchant wants to make 6 lb of a blend of coffee costing \$5 per pound. The blend is made using a \$6-per-pound grade and a \$3-per-pound grade of coffee. How many pounds of each of these grades should be used?

Strategy for Solving a Value Mixture Problem

1. For each ingredient in the mixture, write a numerical or variable expression for the amount of the ingredient used, the unit cost of the ingredient, and the value of the amount used. For the blend, write a numerical or variable expression for the amount, the unit cost of the blend, and the value of the amount. The results can be recorded in a table.

The sum of the amounts is 6 lb.

Amount of \$6 coffee: x
Amount of \$3 coffee: $6 - x$

TAKE NOTE

Use the information given in the problem to fill in the amount and unit cost columns of the table. Fill in the value column by multiplying the two expressions you wrote in each row. Use the expressions in the last column to write the equation.

	Amount, A	·	*Unit Cost, C*	=	*Value, V*
\$6 grade	x	·	\$6	=	$6x$
\$3 grade	$6 - x$	·	\$3	=	$3(6 - x)$
\$5 blend	6	·	\$5	=	$5(6)$

2. Determine how the values of each ingredient are related. Use the fact that the sum of the values of all the ingredients is equal to the value of the blend.

The sum of the values of the \$6 grade and the \$3 grade is equal to the value of the \$5 blend.

$$6x + 3(6 - x) = 5(6)$$
$$6x + 18 - 3x = 30$$
$$3x + 18 = 30$$
$$3x = 12$$
$$x = 4$$

$$6 - x = 6 - 4 = 2$$

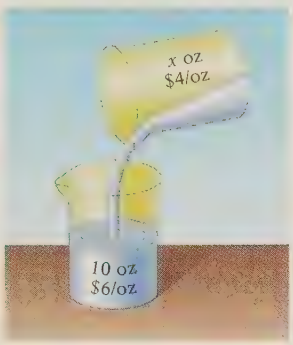
The merchant must use 4 lb of the \$6 coffee and 2 lb of the \$3 coffee.

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Example 1

How many ounces of a silver alloy that costs \$4 an ounce must be mixed with 10 oz of an alloy that costs \$6 an ounce to make a mixture that costs \$4.32 an ounce?

Strategy



Ounces of \$4 alloy: x

	Amount	*Cost*	*Value*
\$4 alloy	x	\$4	$4x$
\$6 alloy	10	\$6	$6(10)$
\$4.32 mixture	$10 + x$	\$4.32	$4.32(10 + x)$

The sum of the values before mixing equals the value after mixing.

Solution

$$4x + 6(10) = 4.32(10 + x)$$
$$4x + 60 = 43.2 + 4.32x$$
$$-0.32x + 60 = 43.2$$
$$-0.32x = -16.8$$
$$x = 52.5$$

52.5 oz of the \$4 silver alloy must be used.

You Try It 1

A gardener has 20 lb of a lawn fertilizer that costs \$.80 per pound. How many pounds of a fertilizer that costs \$.55 per pound should be mixed with this 20 lb of lawn fertilizer to produce a mixture that costs \$.75 per pound?

Your strategy

Your solution

Solution on p. S7

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Objective B To solve percent mixture problems

The amount of a substance in a solution can be given as a percent of the total solution. For example, a 5% salt water solution means that 5% of the total solution is salt. The remaining 95% is water.

Solving a percent mixture problem can be done using the equation $Ar = Q$, where A is the amount of a solution, r is the percent concentration of a substance in the solution, and Q is the quantity of the substance in the solution.

For example, a 500-milliliter bottle is filled with a 4% solution of hydrogen peroxide.

$$Ar = Q$$
$$500(0.04) = Q$$
$$20 = Q$$

The bottle contains 20 ml of hydrogen peroxide.

Example How many gallons of a 20% salt solution must be mixed with 6 gal of a 30% salt solution to make a 22% salt solution?

Strategy for Solving a Percent Mixture Problem

1. For each solution, write a numerical or variable expression for the amount of solution, the percent concentration, and the quantity of the substance in the solution. The results can be recorded in a table.

The unknown quantity of 20% solution: x

TAKE NOTE

Use the information given in the problem to fill in the amount and percent columns of the table. Fill in the quantity column by multiplying the two expressions you wrote in each row. Use the expressions in the last column to write the equation.

	Amount of Solution, A	·	*Percent Concentration, r*	=	*Quantity of Substance, Q*
20% solution	x	·	0.20	=	$0.20x$
30% solution	6	·	0.30	=	$0.30(6)$
22% solution	$x + 6$	·	0.22	=	$0.22(x + 6)$

2. Determine how the quantities of the substances in each solution are related. Use the fact that the sum of the quantities of the substances being mixed is equal to the quantity of the substance after mixing.

The sum of the quantities of the substances in the 20% solution and the 30% solution is equal to the quantity of the substance in the 22% solution.

$$0.20x + 0.30(6) = 0.22(x + 6)$$
$$0.20x + 1.80 = 0.22x + 1.32$$
$$-0.02x + 1.80 = 1.32$$
$$-0.02x = -0.48$$
$$x = 24$$

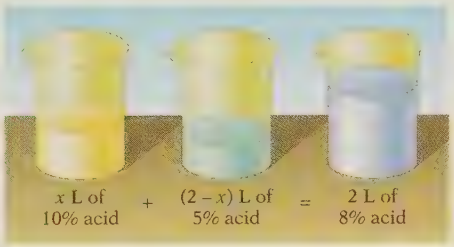
24 gal of the 20% solution are required.

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Example 2

A chemist wishes to make 2 L of an 8% acid solution by mixing a 10% acid solution and a 5% acid solution. How many liters of each solution should the chemist use?

Strategy



Liters of 10% solution: x
Liters of 5% solution: $2 - x$

	Amount	*Percent*	*Quantity*
10% solution	x	0.10	$0.10x$
5% solution	$2 - x$	0.05	$0.05(2 - x)$
8% solution	2	0.08	$0.08(2)$

The sum of the quantities before mixing is equal to the quantity after mixing.

Solution

$$0.10x + 0.05(2 - x) = 0.08(2)$$

$$0.10x + 0.10 - 0.05x = 0.16$$

$$0.05x + 0.10 = 0.16$$

$$0.05x = 0.06$$

$$x = 1.2$$

$$2 - x = 2 - 1.2 = 0.8$$

The chemist needs 1.2 L of the 10% solution and 0.8 L of the 5% solution.

You Try It 2

A pharmacist dilutes 5 L of a 12% solution with a 6% solution. How many liters of the 6% solution are added to make an 8% solution?

Your strategy

Your solution

Solution on p. S8

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Objective C To solve uniform motion problems

A train that travels constantly in a straight line at 50 mph is in *uniform motion.* **Uniform motion** means that the speed or direction of an object does not change.

The solution of a uniform motion problem is based on the equation $rt = d$, where r is the rate of travel, t is the time spent traveling, and d is the distance traveled.

For example, a car travels 50 mph for 3 h.

$$\begin{aligned} rt &= d \\ 50 \cdot 3 &= d \\ 150 &= d \end{aligned}$$

The car travels a distance of 150 mph.

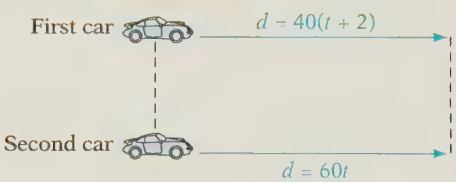
➡ A car leaves a town traveling at 40 mph. Two hours later, a second car leaves the same town, on the same road, traveling at 60 mph. In how many hours will the second car pass the first car?

Strategy for Solving a Uniform Motion Problem

1. For each object, write a numerical or variable expression for the distance, rate, and time. The results can be recorded in a table.

The first car traveled 2 h longer than the second car.

Unknown time for the second car: t
Time for the first car: $t + 2$



TAKE NOTE

Use the information given in the problem to fill in the rate and time columns of the table. Find the distance column by multiplying the two expressions you wrote in each row.

	Rate, r	·	*Time, t*	=	*Distance, d*
First car	40	·	$t + 2$	=	$40(t + 2)$
Second car	60	·	t	=	$60t$

2. Determine how the distances traveled by each object are related. For example, the total distance traveled by both objects may be known, or it may be known that the two objects traveled the same distance.

The two cars travel the same distance.

$$\begin{aligned} 40(t + 2) &= 60t \\ 40t + 80 &= 60t \\ 80 &= 20t \\ 4 &= t \end{aligned}$$

The second car will pass the first car in 4 h.

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Example 3

Two cars, one traveling 10 mph faster than the other car, start at the same time from the same point and travel in opposite directions. In 3 h they are 300 mi apart. Find the rate of each car.

Strategy

Rate of 1st car: r

Rate of 2nd car: $r + 10$

	Rate	*Time*	*Distance*
1st car	r	3	$3r$
2nd car	$r + 10$	3	$3(r + 10)$

The total distance traveled by the two cars is 300 mi.

Solution

$$3r + 3(r + 10) = 300$$
$$3r + 3r + 30 = 300$$
$$6r + 30 = 300$$
$$6r = 270$$
$$r = 45$$

$r + 10 = 45 + 10 = 55$

The first car is traveling 45 mph.
The second car is traveling 55 mph.

You Try It 3

Two trains, one traveling at twice the speed of the other, start at the same time on parallel tracks from stations that are 288 mi apart and travel toward each other. In 3 h, the trains pass each other. Find the rate of each train.

Your strategy

Your solution

Example 4

How far can the members of a bicycling club ride out into the country at a speed of 12 mph and return over the same road at 8 mph if they travel a total of 10 h?

Strategy

Time spent riding out: t

Time spent riding back: $10 - t$

	Rate	*Time*	*Distance*
Out	12	t	$12t$
Back	8	$10 - t$	$8(10 - t)$

The distance out equals the distance back.

Solution

$$12t = 8(10 - t)$$
$$12t = 80 - 8t$$
$$20t = 80$$
$$t = 4 \quad \text{(The time is 4 h.)}$$

The distance out $= 12t = 12(4) = 48$ mi.
The club can ride 48 mi into the country.

You Try It 4

A pilot flew out to a parcel of land and back in 5 h. The rate out was 150 mph and the rate returning was 100 mph. How far away was the parcel of land?

Your strategy

Your solution

Solutions on p. S8

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3.6 Exercises

Objective A Application Problems

1. An herbalist has 30 oz of herbs costing $2 per ounce. How many ounces of herbs costing $1 per ounce should be mixed with the 30 oz to produce a mixture costing $1.60 per ounce?

2. The manager of a farmer's market has 500 lb of grain that costs $1.20 per pound. How many pounds of meal costing $.80 per pound should be mixed with the 500 lb of grain to produce a mixture that costs $1.05 per pound?

3. Find the cost per pound of a meatloaf mixture made from 3 lb of ground beef costing $1.99 per pound and 1 lb of ground turkey costing $1.39 per pound.

4. Find the cost per ounce of a sunscreen made from 100 oz of a lotion that costs $2.50 per ounce and 50 oz of a lotion that costs $4.00 per ounce.

5. A snack food is made by mixing 5 lb of popcorn that costs $.80 per pound with caramel that costs $2.40 per pound. How much caramel is needed to make a mixture that costs $1.40 per pound?

6. A wild birdseed mix is made by combining 100 lb of millet seed costing $.60 per pound with sunflower seeds costing $1.10 per pound. How many pounds of sunflower seeds are needed to make a mixture that costs $.70 per pound?

7. Ten cups of a restaurant's house Italian dressing is made by blending olive oil costing $1.50 per cup with vinegar that costs $.25 per cup. How many cups of each are used if the cost of the blend is $.50 per cup?

8. A high-protein diet supplement that costs $6.75 per pound is mixed with a vitamin supplement that costs $3.25 per pound. How many pounds of each should be used to make 5 lb of a mixture that costs $4.65 per pound?

9. Find the cost per ounce of a mixture of 200 oz of a cologne that costs $5.50 per ounce and 500 oz of a cologne that costs $2.00 per ounce.



10. Find the cost per pound of a trail mix made from 40 lb of raisins that cost $4.40 per pound and 100 lb of granola that costs $2.30 per pound.

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11. A 20 oz alloy of platinum that costs $220 per ounce is mixed with an alloy that costs $400 per ounce. How many ounces of the $400 alloy should be used to make an alloy that costs $300 per ounce?

12. How many liters of a blue dye that costs $1.60 per liter must be mixed with 18 L of anil that costs $2.50 per liter to make a mixture that costs $1.90 per liter?

13. The manager of a specialty food store combined almonds that cost $4.50 per pound with walnuts that cost $2.50 per pound. How many pounds of each were used to make a 100-pound mixture that costs $3.24 per pound?

14. A goldsmith combined an alloy that cost $4.30 per ounce with an alloy that cost $1.80 per ounce. How many ounces of each were used to make a mixture of 200 oz costing $2.50 per ounce?

15. Adult tickets for a play cost $6.00 and children's tickets cost $2.50. For one performance, 370 tickets were sold. Receipts for the performance were $1723. Find the number of adult tickets sold.

16. Tickets for a piano concert sold for $4.50 for each adult. Student tickets sold for $2.00 each. The total receipts for 1720 tickets were $5980. Find the number of adult tickets sold.

17. Find the cost per pound of sugar-coated breakfast cereal made from 40 lb of sugar that costs $1.00 per pound and 120 lb of corn flakes that cost $.60 per pound.

18. Find the cost per pound of a coffee mixture made from 8 lb of coffee that costs $9.20 per pound and 12 lb of coffee that costs $5.50 per pound.



Objective B Application Problems

19. Forty ounces of a 30% gold alloy are mixed with 60 oz of a 20% gold alloy. Find the percent concentration of the resulting gold alloy.

20. One hundred ounces of juice that is 50% tomato juice are added to 200 oz of a vegetable juice that is 25% tomato juice. What is the percent concentration of tomato juice in the resulting mixture?

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21. How many gallons of a 15% acid solution must be mixed with 5 gal of a 20% acid solution to make a 16% acid solution?

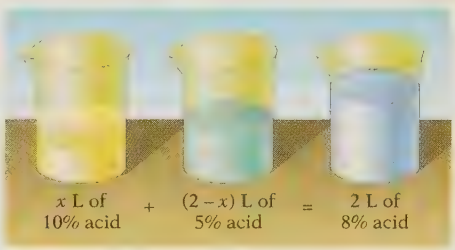
22. How many pounds of a chicken feed that is 50% corn must be mixed with 400 lb of a feed that is 80% corn to make a chicken feed that is 75% corn?

23. A rug is made by weaving 20 lb of yarn that is 50% wool with a yarn that is 25% wool. How many pounds of the yarn that is 25% wool is used if the finished rug is 35% wool?

24. Five gallons of a light green latex paint that is 20% yellow paint is combined with a darker green latex paint that is 40% yellow paint. How many gallons of the darker green paint must be used to create a green paint that is 25% yellow paint?

25. How many gallons of a plant food that is 9% nitrogen must be combined with another plant food that is 25% nitrogen to make 10 gal of a solution that is 15% nitrogen?

26. A chemist wants to make 50 ml of a 16% acid solution by mixing a 13% acid solution and an 18% acid solution. How many milliliters of each solution should the chemist use?



27. Five grams of sugar are added to a 45-gram serving of a breakfast cereal that is 10% sugar. What is the percent concentration of sugar in the resulting mixture?

28. A goldsmith mixes 8 oz of a 30% gold alloy with 12 oz of a 25% gold alloy. What is the percent concentration of the resulting alloy?

29. How many pounds of coffee that is 40% java beans must be mixed with 80 lb of coffee that is 30% java beans to make a coffee blend that is 32% java beans?

30. The manager of a garden shop mixes grass seed that is 60% rye grass with 70 lb of grass seed that is 80% rye grass to make a mixture that is 74% rye grass. How much of the 60% rye grass is used?

31. A hair dye is made by blending a 7% hydrogen peroxide solution and a 4% hydrogen peroxide solution. How many milliliters of each are used to make a 300-milliliter solution that is 5% hydrogen peroxide?

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32. A tea that is 20% jasmine is blended with a tea that is 15% jasmine. How many pounds of each tea are used to make 5 lb of tea that is 18% jasmine?

33. How many ounces of pure chocolate must be added to 150 oz of chocolate topping that is 50% chocolate to make a topping that is 75% chocolate?

34. How many ounces of pure bran flakes must be added to 50 oz of cereal that is 40% bran flakes to produce a mixture that is 50% bran flakes?

35. Thirty ounces of pure silver are added to 50 oz of a silver alloy that is 20% silver. What is the percent concentration of the resulting alloy?

36. A clothing manufacturer has some pure silk thread and some thread that is 85% silk. How many kilograms of each must be woven together to make 75 kg of cloth that is 96% silk?

Objective C Application Problems

37. Two small planes start from the same point and fly in opposite directions. The first plane is flying 25 mph slower than the second plane. In 2 h, the planes are 470 mi apart. Find the rate of each plane.



38. Two cyclists start from the same point and ride in opposite directions. One cyclist rides twice as fast as the other. In 3 h, they are 81 mi apart. Find the rate of each cyclist.

39. Two planes leave an airport at 8 A.M., one flying north at 480 km/h and the other flying south at 520 km/h. At what time will they be 3000 km apart?

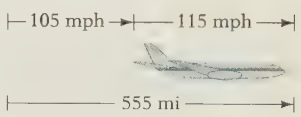
40. A long-distance runner started on a course running at an average speed of 6 mph. One-half hour later, a second runner began the same course at an average speed of 7 mph. How long after the second runner started will the second runner overtake the first runner?



41. A motorboat leaves a harbor and travels at an average speed of 9 mph toward a small island. Two hours later a cabin cruiser leaves the same harbor and travels at an average speed of 18 mph toward the same island. In how many hours after the cabin cruiser leaves will the cabin cruiser be alongside the motorboat?

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42. A 555-mile, 5-hour plane trip was flown at two speeds. For the first part of the trip, the average speed was 105 mph. For the remainder of the trip, the average speed was 115 mph. For how long did the plane fly at each speed?



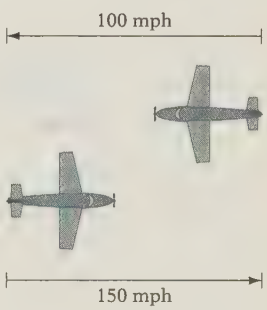
43. An executive drove from home at an average speed of 30 mph to an airport where a helicopter was waiting. The executive boarded the helicopter and flew to the corporate offices at an average speed of 60 mph. The entire distance was 150 mi. The entire trip took 3 h. Find the distance from the airport to the corporate offices.

44. After a sailboat had been on the water for 3 h, a change in the wind direction reduced the average speed of the boat by 5 mph. The entire distance sailed was 57 mi. The total time spent sailing was 6 h. How far did the sailboat travel in the first 3 h?

45. A car and a bus set out at 3 P.M. from the same point headed in the same direction. The average speed of the car is twice the average speed of the bus. In 2 h the car is 68 mi ahead of the bus. Find the rate of the car.

46. A passenger train leaves a train depot 2 h after a freight train leaves the same depot. The freight train is traveling 20 mph slower than the passenger train. Find the rate of each train if the passenger train overtakes the freight train in 3 h.

47. As part of flight training, a student pilot was required to fly to an airport and then return. The average speed on the way to the airport was 100 mph, and the average speed returning was 150 mph. Find the distance between the two airports if the total flying time was 5 h.



48. A ship traveling east at 25 mph is 10 mi from a harbor when another ship leaves the harbor traveling east at 35 mph. How long does it take the second ship to catch up to the first ship?

49. At 10 A.M. a plane leaves Boston, Massachusetts, for Seattle, Washington, a distance of 3000 mi. One hour later a plane leaves Seattle for Boston. Both planes are traveling at a speed of 500 mph. How many hours after the plane leaves Seattle will the planes pass each other?

50. At noon a train leaves Washington, D.C., headed for Charleston, South Carolina, a distance of 500 mi. The train travels at a speed of 60 mph. At 1 P.M. a second train leaves Charleston headed for Washington, D.C., traveling at 50 mph. How long after the train leaves Charleston will the two trains pass each other?

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51. Two cyclists start at the same time from opposite ends of a course that is 51 mi long. One cyclist is riding at a rate of 16 mph, and the second cyclist is riding at a rate of 18 mph. How long after they begin will they meet?

52. A bus traveled on a level road for 2 h at an average speed that was 20 mph faster than its average speed on a winding road. The time spent on the winding road was 3 h. Find the average speed on the winding road if the total trip was 210 mi.

53. A bus traveling at a rate of 60 mph overtakes a car traveling at a rate of 45 mph. If the car had a 1-hour head start, how far from the starting point does the bus overtake the car?

54. A car traveling at 48 mph overtakes a cyclist who, riding at 12 mph, had a 3-hour head start. How far from the starting point does the car overtake the cyclist?

APPLYING THE CONCEPTS

55. How many grams of pure water must be added to 50 g of pure acid to make a solution that is 40% acid?

56. How many ounces of water must be evaporated from 50 oz of a 12% salt solution to produce a 15% salt solution?

57. A radiator contains 15 gal of a 20% antifreeze solution. How many gallons must be drained from the radiator and replaced by pure antifreeze so that the radiator will contain 15 gal of a 40% antifreeze solution?

58. At 10 A.M., two campers left their campsite by canoe and paddled downstream at an average speed of 12 mph. They then turned around and paddled back upstream at an average rate of 4 mph. The total trip took 1 h. At what time did the campers turn around downstream?

59. A bicyclist rides for 2 h at a speed of 10 mph and then returns at a speed of 20 mph. Find the cyclist's average speed for the trip.

60. A car travels a 1-mile track at an average speed of 30 mph. At what average speed must the car travel the next mile so that the average speed for the 2 mi is 60 mph?

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Focus on Problem Solving

Trial and Error Approach to Problem Solving

The questions below require an answer of always true, sometimes true, or never true. These problems are best solved by the trial and error method. The trial and error method of arriving at a solution to a problem involves repeated tests or experiments.

For example, consider the statement

> Both sides of an equation can be divided by the same number without changing the solution of the equation.

The solution of the equation $6x = 18$ is 3. If we divide both sides of the equation by 2, the result is $3x = 9$ and the solution is still 3. So the answer "never true" has been eliminated. We still need to determine whether there is a case for which it is not true. Is there a number that we could divide both sides of the equation by and the result would be an equation for which the solution is not 3? If we divide both sides of the equation by 0, the result is $\frac{6x}{0} = \frac{18}{0}$; the solution of this equation is not 3 because the expressions on either side of the equals sign are undefined. So the statement is true for some numbers and not true for 0. The statement is sometimes true.

Determine whether the statement is always true, sometimes true, or never true.

1. Both sides of an equation can be multiplied by the same number without changing the solution of the equation.

2. For an equation of the form $ax = b$, $a \neq 0$, multiplying both sides of the equation by the reciprocal of a will result in an equation of the form $x = constant$.

3. The Multiplication Property of Equations is used to remove a term from one side of an equation.

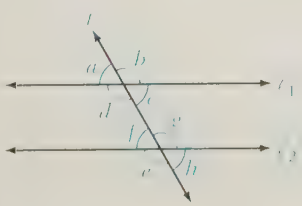
4. Adding -3 to each side of an equation yields the same result as subtracting 3 from each side of the equation.

5. An equation contains an equals sign.

6. The same variable term can be added to both sides of an equation without changing the solution of the equation.

7. An equation of the form $ax + b = c$ cannot be solved if a is a negative number.

8. The solution of the equation $\frac{x}{0} = 0$ is 0.



9. For the diagram at the left, $\angle b = \angle d = \angle e = \angle g$.

10. In solving an equation of the form $ax + b = cx + d$, subtracting cx from each side of the equation results in an equation with only one variable term in it.

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11. If a rope 8 meters long is cut into two pieces and one of the pieces has length x meters, then the length of the other piece can be represented as $(x - 8)$ meters.

12. An even integer is a multiple of 2.

13. If the first of three consecutive odd integers is n, then the second and third consecutive odd integers are represented as $n + 1$ and $n + 3$.

14. Suppose we are mixing two salt solutions. Then the variable Q in the percent mixture equation $Q = Ar$ represents the amount of salt in a solution.

15. If 100 oz of a silver alloy is 25% silver, then the alloy contains 25 oz of silver.

16. If we combine an alloy that costs \$8 an ounce with an alloy that costs \$5 an ounce, the cost of the resulting mixture will be greater than \$8 an ounce.

17. If we combine a 9% acid solution with a solution that is 4% acid, the resulting solution will be less than 4% acid.

18. If the speed of one train is 20 mph slower than that of a second train, then the speeds of the two trains can be represented as r and $20 - r$.

Projects and Group Activities

Nielsen Ratings

Nielsen Media Research surveys television viewers to determine the number of people watching particular shows. There are an estimated 97 million television households in the United States. Each **rating point** represents 1% of that number, or 970,000 households. Therefore, for example, if *60 Minutes* receives a rating of 10 points, then 10%, or (0.10)(97,000,000) = 9,700,000 households, watched that program.

POINT OF INTEREST

The top ranked programs in prime time for the week of July 28–August 3, 1997, ranked by Nielsen Media Research, were:

Seinfeld
20/20
E.R.
Suddenly Susan
60 Minutes
Dateline NBC (Monday)
Dateline NBC (Tuesday)
Friends
Touched by an Angel
Law & Order

A rating point does not mean that 970,000 people are watching a program. A rating point refers to the number of TV sets tuned to that program; there may be more than one person watching a television set in a household.

The Nielsen Company also describes a program's share of the market. **Share** is the percentage of television sets in use that are tuned to a program. Suppose the same week that *60 Minutes* received 10 rating points, the show received a share of 20%. This would mean that 20% of all households with a television *turned on* were turned to *60 Minutes*, while 10% of all households with a television were turned to the program.

1. If *20/20* received a Nielsen rating of 11.2 and a share of 23, how many TV households watched the program that week? How many TV households were watching television during that hour?

2. Suppose that *Dateline NBC* (Tuesday) received a rating of 9.6 and a share of 18. How many TV households were watching television during that hour? How many TV households watched the program that week?

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3. Suppose *Seinfeld* received a rating of 12.3 during a week that 17,100,000 people were watching the show. Find the average number of people per TV household who watched the program. Round to the nearest tenth.

The cost to advertise on a program is related to its Nielsen rating. The sponsor (the company paying for the advertisement) pays a certain number of dollars for each rating point a show receives.

4. Suppose a television network charges $25,000 per rating point for a 30-second commercial on a daytime talk show. Determine the cost for three 30-second commercials if the Nielsen rating of the show is 11.5.

Nielsen Media Research also tracks the exposure of advertisements. For example, it might be reported that commercials for McDonald's had 500,000,000 household exposures during a week when their advertisement was aired 90 times.

5. Information regarding household exposure of advertisements can be found in *USA Today* each Monday. For a recent week, find the information for the top four advertised brands. For each brand, calculate the average household exposure for each time the ad was aired.

Nielsen Media Research has a web site on the internet. You can locate the site under entertainment and television using a search engine. You will find the rating points and market share for the networks and cable stations, as well as rating points, share, and audience size for categories such as prime time and sports programming.

6. Find the top two prime-time television shows for last week. Calculate the number of TV households that watched each program. Compare these figures with the top two sports programs for last week.

Chapter Summary

Key Words

An *equation* expresses the equality of two mathematical expressions.

A *solution* of an equation is a number that, when substituted for the variable, results in a true equation.

To *solve an equation* means to find a solution of the equation. The goal is to rewrite the equation in the form *variable* = *constant*.

To *translate* a sentence into an equation requires recognition of the words or phrases that mean "equals." Some of these phrases are *equals, is, is equal to, amounts to,* and *represents.*

Cost is the price that a business pays for a product. *Selling price* is the price for which a business sells a product to a customer. *Markup* is the difference between selling price and cost. When the markup is expressed as a percent of the retailer's cost, it is called the *markup rate.*

Discount, or *markdown,* is the amount by which a retailer reduces the regular price of a product. The percent discount is called the *discount rate.*

The *perimeter* of a geometric figure is a measure of the distance around the figure.

An *isosceles triangle* has two sides of equal measure. The three sides of an *equilateral triangle* are of equal measure.

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An angle is measured in *degrees*. A 90° angle is a *right angle*. A 180° angle is a *straight angle*. One complete revolution is 360°. An *acute angle* is an angle whose measure is between 0° and 90°. An *obtuse angle* is an angle whose measure is between 90° and 180°.

Complementary angles are two angles whose measures have the sum 90°.

Supplementary angles are two angles whose measures have the sum 180°.

Parallel lines never meet; the distance between them is always the same.

Perpendicular lines are intersecting lines that form right angles.

Two angles that are on opposite sides of the intersection of two lines are *vertical angles;* vertical angles have the same measure. Two angles that share a common side are *adjacent angles;* adjacent angles of intersecting lines are supplementary angles.

A line that intersects two other lines at two different points is a *transversal.* If the lines cut by a transversal are parallel lines, equal angles are formed: *alternate interior angles, alternate exterior angles,* and *corresponding angles.*

Uniform motion means that an object, traveling at a constant speed, moves in a straight line.

Essential Rules

Addition Property of Equations

The same number can be added to each side of an equation without changing the solution of the equation.

If $a = b$, then $a + c = b + c$.

Multiplication Property of Equations

Each side of an equation can be multiplied by the same nonzero number without changing the solution of the equation.

If $a = b$ and $c \neq 0$, then $ac = bc$.

Basic Percent Equation

Percent · Base = Amount

$P \cdot B = A$

Consecutive Integers

$n, n + 1, n + 2, \ldots$

Consecutive Even Integers

$n, n + 2, n + 4, \ldots$

Consecutive Odd Integers

$n, n + 2, n + 4, \ldots$

Formulas for Perimeter

Triangle	$P = a + b + c$
Rectangle	$P = 2L + 2W$
Square	$P = 4s$

Sum of the Angles of a Triangle

The sum of the measures of the interior angles of a triangle is 180°.

The sum of an interior and corresponding exterior angle is 180°.

$\angle a + \angle b + \angle c = 180°$

Value Mixture Equation

Amount · Unit Cost = Value

$AC = V$

Percent Mixture Equation

Amount · Percent Concentration = Quantity

$Ar = Q$

Uniform Motion Equation

Rate · Time = Distance

$rt = d$

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Chapter Review

1. Solve: $x + 3 = 24$

2. Solve: $x + 5(3x - 20) = 10(x - 4)$

3. Solve: $5x - 6 = 29$

4. Is 3 a solution of $5x - 2 = 4x + 5$?

5. Solve: $\frac{3}{5}a = 12$

6. Solve: $6x + 3(2x - 1) = -27$

7. 30 is what percent of 12?

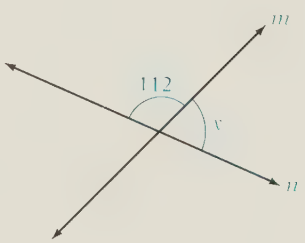
8. Solve: $5x + 3 = 10x - 17$

9. Solve: $7 - [4 + 2(x - 3)] = 11(x + 2)$

10. Solve: $-6x + 16 = -2x$

11. A furniture store uses a markup rate of 60%. The store sells a solid oak curio cabinet for \$1074. Find the cost of the curio cabinet. Use the formula $S = C + rC$, where S is the selling price, C is the cost, and r is the markup rate.

12. Find the measure of $\angle x$.



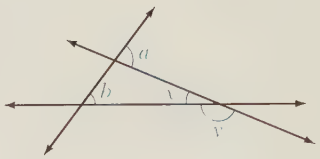
13. A lever is 12 ft long. At a distance of 2 ft from the fulcrum, a force of 120 lb is applied. How large a force must be applied to the other end so that the system will balance? Use the lever system equation $F_1x = F_2(d - x)$.

14. A bus traveled on a level road for 2 h at an average speed of 20 mph faster than it traveled on a winding road. The time spent on the winding road was 3 h. Find the average speed on the winding road if the total trip was 200 mi.

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15. A ceiling fan, which regularly sells for \$60, is on sale for \$40. Find the discount rate. Use the formula $S = R - rR$, where S is the sale price, R is the regular price, and r is the discount rate.

16. Given that $\angle a = 74°$ and $\angle b = 52°$, find the measures of angles x and y.



17. A health food store combined cranberry juice that cost \$1.79 per quart with apple juice that cost \$1.19 per quart. How many quarts of each were used to make 10 qt of cranapple juice costing \$1.61 per quart?

18. Four times the second of three consecutive integers equals the sum of the first and third integers. Find the integers.

19. One angle of a triangle is 15° more than the measure of the second angle. The third angle is 15° less than the measure of the second angle. Find the measure of each angle.

20. Translate "four less than the product of five and a number is sixteen" into an equation and solve.

21. The Empire State Building is 1472 ft tall. This is 514 ft less than twice the height of the Eiffel Tower. Find the height of the Eiffel Tower.

22. The length of a rectangle is four times the width of the rectangle. The perimeter is 200 ft. Find the length and width of the rectangle.

23. A jet plane traveling at 600 mph overtakes a propeller-driven plane that had a 2-hour head start. The propeller-driven plane is traveling at 200 mph. How far from the starting point does the jet overtake the propeller-driven plane?

24. The sum of two numbers is twenty-one. Three times the smaller number is two less than twice the larger number. Find the two numbers.

25. A dairy owner mixed 5 gal of cream containing 30% butterfat with 8 gal of milk containing 4% butterfat. What is the percent of butterfat in the resulting mixture?

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Chapter Test

1. Solve: $3x - 2 = 5x + 8$

2. Solve: $x - 3 = -8$

3. Solve: $3x - 5 = -14$

4. Solve: $4 - 2(3 - 2x) = 2(5 - x)$

5. Is -2 a solution of $x^2 - 3x = 2x - 6$?

6. Solve: $7 - 4x = -13$

7. What is 0.5% of 8?

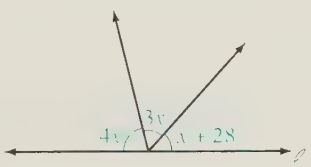
8. Solve: $5x - 2(4x - 3) = 6x + 9$

9. Solve: $5x + 3 - 7x = 2x - 5$

10. Solve: $\frac{3}{4}x = -9$

11. A baker wants to make a 15-pound blend of flour that costs \$.60 per pound. The blend is made using a rye flour that costs \$.70 per pound and a wheat flour that costs \$.40 per pound. How many pounds of each flour should be used?

12. Find x.



13. A television that regularly sells for \$450 is on sale for \$360. Find the discount rate. Use the formula $S = R - rR$, where S is the sale price, R is the regular price, and r is the discount rate.

14. A financial manager has determined that the cost per unit for a calculator is \$15 and that the fixed cost per month is \$2000. Find the number of calculators produced during a month in which the total cost was \$5000. Use the equation $T = U \cdot N + F$, where T is the total cost, U is the cost per unit, N is the number of units produced, and F is the fixed cost.

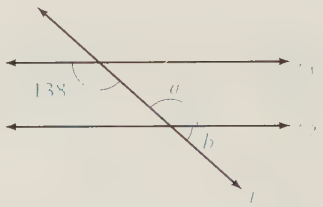
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15. In an isosceles triangle, two angles are equal. The third angle of the triangle is 30° less than one of the equal angles. Find the measure of one of the equal angles.

16. Find three consecutive even integers whose sum is 36.

17. How many gallons of water must be mixed with 5 gal of a 20% salt solution to make a 16% salt solution?

18. Given that $\ell_1 \parallel \ell_2$, find the measures of angles a and b.



19. Translate "the difference between three times a number and fifteen is twenty-seven" into an equation and solve.

20. A cross-country skier leaves a camp to explore a wilderness area. Two hours later a friend leaves the camp in a snowmobile, traveling 4 mph faster than the skier, and meets the skier 1 h later. Find the rate of the snowmobile.

21. A company makes 140 televisions per day. Three times the number of black-and-white TVs made equals 20 less than the number of color TVs made. Find the number of color TVs made each day.

22. The sum of two numbers is eighteen. The difference between four times the smaller number and seven is equal to the sum of two times the larger number and five. Find the two numbers.

23. As part of flight training, a student pilot was required to fly to an airport and then return. The average speed to the airport was 90 mph, and the average speed returning was 120 mph. Find the distance between the two airports if the total flying time was 7 h.

24. The perimeter of a triangle is 23 ft. One side is twice the second side. The third side is 3 ft more than the second side. Find the measure of each side.

25. A chemist mixes 100 g of water at 80°C with 50 g of water at 20°C. To find the final temperature of the water after mixing, use the equation $m_1(T_1 - T) = m_2(T - T_2)$, where m_1 is the quantity of water at the hotter temperature, T_1 is the temperature of the hotter water, m_2 is the quantity of water at the cooler temperature, T_2 is the temperature of the cooler water, and T is the final temperature of the water after mixing.

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Cumulative Review

1. Subtract: $-6 - (-20) - 8$

2. Multiply: $(-2)(-6)(-4)$

3. Simplify: $-\frac{5}{6} - \left(-\frac{7}{16}\right)$

4. Simplify: $-2\frac{1}{3} \div 1\frac{1}{6}$

5. Simplify: $-4^2 \cdot \left(-\frac{3}{2}\right)^3$

6. Simplify: $25 - 3\frac{(5-2)^2}{2^3+1} - (-2)$

7. Evaluate $3(a - c) - 2ab$ when $a = 2$, $b = 3$, and $c = -4$.

8. Simplify: $3x - 8x + (-12x)$

9. Simplify: $2a - (-3b) - 7a - 5b$

10. Simplify: $(16x)\left(\frac{1}{8}\right)$

11. Simplify: $-4(-9y)$

12. Simplify: $-2(-x^2 - 3x + 2)$

13. Simplify: $-2(x - 3) + 2(4 - x)$

14. Simplify: $-3[2x - 4(x - 3)] + 2$

15. Is -3 a solution of $x^2 + 6x + 9 = x + 3$?

16. Is $\frac{1}{2}$ a solution of $3 - 8x = 12x - 2$?

17. Find 32% of 60.

18. Solve: $\frac{3}{5}x = -15$

19. Solve: $7x - 8 = -29$

20. Solve: $13 - 9x = -14$

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21. Solve: $8x - 3(4x - 5) = -2x - 11$

22. Solve: $6 - 2(5x - 8) = 3x - 4$

23. Solve: $5x - 8 = 12x + 13$

24. Solve: $11 - 4x = 2x + 8$

25. A chemist mixes 300 g of water at 75°C with 100 g of water at 15°C. To find the final temperature of the water after mixing, use the equation $m_1(T_1 - T) = m_2(T - T_2)$, where m_1 is the quantity of water at the hotter temperature, T_1 is the temperature of the hotter water, m_2 is the quantity of water at the cooler temperature, T_2 is the temperature of the cooler water, and T is the final temperature of the water after mixing.

26. Translate "the difference between twelve and the product of five and a number is negative eighteen" into an equation and solve.

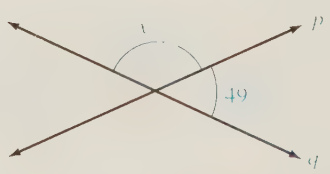
27. The area of a cement foundation of a house is 2000 ft^2. This is 200 ft^2 more than three times the area of the garage. Find the area of the garage.

28. How many pounds of an oat flour that costs \$.80 per pound must be mixed with 40 lb of a wheat flour that costs \$.50 per pound to make a blend that costs \$.60 per pound?

29. How many grams of pure gold must be added to 100 g of a 20% gold alloy to make an alloy that is 36% gold?

30. The perimeter of a rectangular office is 44 ft. The length of the office is 2 ft more than the width. Find the dimensions of the office.

31. Find the measure of $\angle x$.



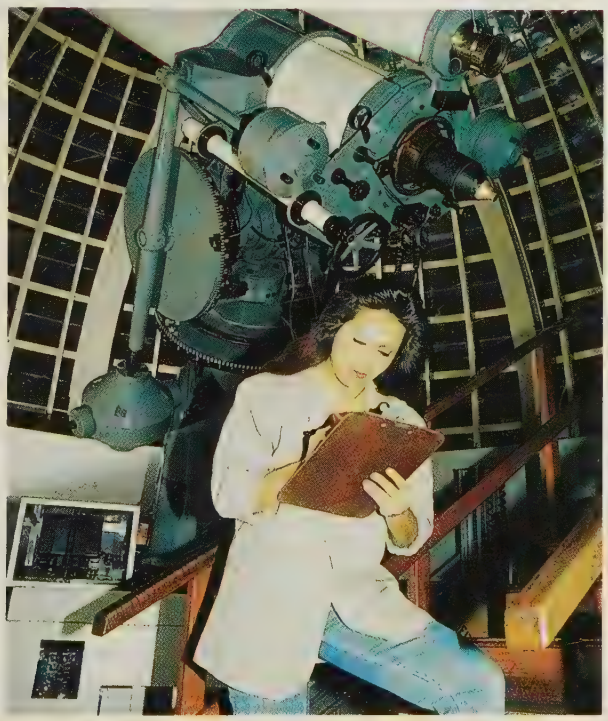
32. In an equilateral triangle, all three angles are equal. Find the measure of one of the angles of an equilateral triangle.

33. A sprinter ran to the end of a track at an average rate of 8 m/s and then jogged back to the starting point at an average rate of 3 m/s. The sprinter took 55 s to run to the end of the track and jog back. Find the length of the track.

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CHAPTER

4 Polynomials



Astronomers use mathematics and physics to explore the nature of our universe. Theories pertaining to the universe are represented as mathematical equations. Astronomers analyze these theories using computers and data collected from observations of objects in space. Among the mathematical concepts that an astronomer must be knowledgeable about is scientific notation, which is presented in Section 4 of this chapter.

Objectives

Section 4.1
To add polynomials
To subtract polynomials

Section 4.2
To multiply monomials
To simplify powers of monomials

Section 4.3
To multiply a polynomial by a monomial
To multiply two polynomials
To multiply two binomials
To multiply binomials that have special products
To solve application problems

Section 4.4
To divide monomials
To write a number in scientific notation

Section 4.5
To divide a polynomial by a monomial
To divide polynomials

Early Egyptian Arithmetic Operations

The early Egyptian arithmetic processes are recorded on the Rhind Papyrus, but the underlying principles are not included. Scholars of today can only guess how these early developments were discovered.

Egyptian hieroglyphics used a base-ten system of numbers in which a vertical line represented 1, a heel bone, ∩, represented 10, and a scroll, 𓍢, represented 100.

The symbols at the right represent the number 237. There are 7 vertical lines, 3 heel bones, and 2 scrolls. Thus the symbols at the right represent 7 + 30 + 200, or 237.



Addition in hieroglyphic notation does not require memorization of addition facts. Addition is done just by counting symbols.

The top example at the left shows that addition is a simple grouping operation. Write down the total of each kind of symbol. Then group 10 straight lines into one heel bone.

Subtraction in the hieroglyphic system is similar to making change. For example, what change do you get from a $1.00 bill when buying a $.55 item? 5 cannot be subtracted from 0, so a 10 is "borrowed" and 10 ones are added.

In the bottom example at the left, 5 cannot be subtracted from 3, so a 10 is "borrowed" and 10 ones are added.

Note that no zero is provided in this number system. That place value symbol is just not used. As shown at the left, the heel bone is not used when writing 208 because there are no 10's in 208.

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4.1 Addition and Subtraction of Polynomials

Objective A To add polynomials

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A monomial is a number, a variable, or a product of numbers and variables. For instance,

7	b	$\frac{2}{3}a$	$12xy^2$
A number	A variable	A product of a number and a variable	A product of a number and variables

TAKE NOTE

The expression $3\sqrt{x}$ is not a monomial because $\sqrt{x}$ cannot be written as a product of variables.

The expression $\frac{2x}{y}$ is not a monomial because it is a *quotient* of variables.

A **polynomial** is a variable expression in which the terms are monomials.

A polynomial of *one* term is a **monomial.** $-7x^2$ is a monomial.
A polynomial of *two* terms is a **binomial.** $4x + 2$ is a binomial.
A polynomial of *three* terms is a **trinomial.** $7x^2 + 5x - 7$ is a trinomial.

The terms of a polynomial in one variable are usually arranged so that the exponents of the variable decrease from left to right. This is called **descending order.**

$5x^3 - 4x^2 + 6x - 1$

$7z^4 + 4z^3 + z - 6$

$2y^4 + y^3 - 2y^2 + 4y - 1$

The **degree** of a polynomial in one variable is its largest exponent (on a variable). The degree of $4x^3 - 5x^2 + 7x - 8$ is 3; the degree of $2y^4 + y^2 - 1$ is 4.

Polynomials can be added, using either a horizontal or a vertical format, by combining like terms.

➡ Simplify $(3x^3 - 7x + 2) + (7x^2 + 2x - 7)$. Use a horizontal format.

$(3x^3 - 7x + 2) + (7x^2 + 2x - 7)$

$= 3x^3 + 7x^2 + (-7x + 2x) + (2 - 7)$ • Use the Commutative and Associative Properties of Addition to rearrange and group like terms.

$= 3x^3 + 7x^2 - 5x - 5$ • Then combine like terms.

➡ Simplify $(-4x^2 + 6x - 9) + (12 - 8x + 2x^3)$. Use a vertical format.

$$\begin{array}{r} -4x^2 + 6x - 9 \\ 2x^3 \qquad - 8x + 12 \\ \hline 2x^3 - 4x^2 - 2x + 3 \end{array}$$

• Arrange the terms of each polynomial in descending order with like terms in the same column.

• Combine the terms in each column.

Example 1

Use a horizontal format to simplify $(8x^2 - 4x - 9) + (2x^2 + 9x - 9)$.

Solution

$(8x^2 - 4x - 9) + (2x^2 + 9x - 9)$
$= (8x^2 + 2x^2) + (-4x + 9x) + (-9 - 9)$
$= 10x^2 + 5x - 18$

You Try It 1

Use a horizontal format to simplify $(-4x^3 + 2x^2 - 8) + (4x^3 + 6x^2 - 7x + 5)$.

Your solution

Solution on p. S8

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Example 2

Use a vertical format to simplify
$(-5x^3 + 4x^2 - 7x + 9) + (2x^3 + 5x - 11)$.

Solution

$$\begin{array}{r} -5x^3 + 4x^2 - 7x + 9 \\ 2x^3 \qquad + 5x - 11 \\ \hline -3x^3 + 4x^2 - 2x - 2 \end{array}$$

You Try It 2

Use a vertical format to simplify
$(6x^3 + 2x + 8) + (-9x^3 + 2x^2 - 12x - 8)$.

Your solution

Solution on p. S8

Objective B To subtract polynomials

14 CT

The **opposite** of the polynomial $(3x^2 - 7x + 8)$ is $-(3x^2 - 7x + 8)$.

To simplify the opposite of a polynomial, change the sign of each term inside the parentheses.

$$-(3x^2 - 7x + 8) = -3x^2 + 7x - 8$$

TAKE NOTE

This is the same definition used for subtraction of integers: subtraction is addition of the opposite.

Polynomials can be subtracted using either a horizontal or a vertical format. To subtract, add the opposite of the second polynomial to the first.

➡ Simplify $(4y^2 - 6y + 7) - (2y^3 - 5y - 4)$. Use a horizontal format.

$$\begin{aligned} &(4y^2 - 6y + 7) - (2y^3 - 5y - 4) \\ &\quad = (4y^2 - 6y + 7) + (-2y^3 + 5y + 4) \\ &\quad = -2y^3 + 4y^2 + (-6y + 5y) + (7 + 4) \\ &\quad = -2y^3 + 4y^2 - y + 11 \end{aligned}$$

- Add the opposite of the second polynomial to the first.
- Combine like terms.

➡ Simplify $(9 + 4y + 3y^3) - (2y^2 + 4y - 21)$. Use a vertical format.

The opposite of $2y^2 + 4y - 21$ is $-2y^2 - 4y + 21$.

$$\begin{array}{r} 3y^3 \qquad + 4y + 9 \\ -2y^2 - 4y + 21 \\ \hline 3y^3 - 2y^2 \qquad + 30 \end{array}$$

- Arrange the terms of each polynomial in descending order with like terms in the same column.
- Note that $4y - 4y = 0$, but 0 is not written.

Example 3

Use a horizontal format to simplify
$(7c^2 - 9c - 12) - (9c^2 + 5c - 8)$.

Solution

$$\begin{aligned} &(7c^2 - 9c - 12) - (9c^2 + 5c - 8) \\ &\quad = (7c^2 - 9c - 12) + (-9c^2 - 5c + 8) \\ &\quad = -2c^2 - 14c - 4 \end{aligned}$$

You Try It 3

Use a horizontal format to simplify
$(-4w^3 + 8w - 8) - (3w^3 - 4w^2 - 2w - 1)$.

Your solution

Example 4

Use a vertical format to simplify
$(3k^2 - 4k + 1) - (k^3 + 3k^2 - 6k - 8)$.

Solution

$$\begin{array}{r} 3k^2 - 4k + 1 \\ -k^3 - 3k^2 + 6k + 8 \\ \hline -k^3 \qquad + 2k + 9 \end{array}$$

- Add the opposite of $(k^3 + 3k^2 - 6k - 8)$ to the first polynomial.

You Try It 4

Use a vertical format to simplify
$(13y^3 - 6y - 7) - (4y^2 - 6y - 9)$.

Your solution

Solutions on p. S8

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4.1 Exercises

Objective A

State whether the expression is a monomial.

1. 17 **2.** $3x^4$ **3.** $\frac{17}{\sqrt{x}}$ **4.** xyz

5. $\frac{2}{3}y$ **6.** $\frac{xy}{z}$ **7.** $\sqrt{5}x$ **8.** πx

State whether the expression is a monomial, binomial, trinomial, or none of these.

9. $3x + 5$ **10.** $2y - 3\sqrt{y}$ **11.** $9x^2 - x - 1$ **12.** $x^2 + y^2$

13. $\frac{2}{x} - 3$ **14.** $\frac{ab}{4}$ **15.** $6x^2 + 7x$ **16.** $12a^4 - 3a + 2$

Simplify. Use a vertical format.

17. $(x^2 + 7x) + (-3x^2 - 4x)$ **18.** $(3y^2 - 2y) + (5y^2 + 6y)$

19. $(y^2 + 4y) + (-4y - 8)$ **20.** $(3x^2 + 9x) + (6x - 24)$

21. $(2x^2 + 6x + 12) + (3x^2 + x + 8)$ **22.** $(x^2 + x + 5) + (3x^2 - 10x + 4)$

23. $(-7x + x^3 + 4) + (2x^2 + x - 10)$ **24.** $(y^2 + 3y^3 + 1) + (-4y^3 - 6y - 3)$

25. $(2a^3 - 7a + 1) + (1 - 4a - 3a^2)$ **26.** $(5r^3 - 6r^2 + 3r) + (-3 - 2r + r^2)$

Simplify. Use a horizontal format.

27. $(4x^2 + 2x) + (x^2 + 6x)$ **28.** $(-3y^2 + y) + (4y^2 + 6y)$

29. $(4x^2 - 5xy) + (3x^2 + 6xy - 4y^2)$ **30.** $(2x^2 - 4y^2) + (6x^2 - 2xy + 4y^2)$

31. $(2a^2 - 7a + 10) + (a^2 + 4a + 7)$ **32.** $(-6x^2 + 7x + 3) + (3x^2 + x + 3)$

33. $(7x + 5x^3 - 7) + (10x^2 - 8x + 3)$ **34.** $(4y + 3y^3 + 9) + (2y^2 + 4y - 21)$

35. $(7 - 5r + 2r^2) + (3r^3 - 6r)$ **36.** $(14 + 4y + 3y^3) + (-4y^2 + 21)$

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Objective B

Simplify. Use a vertical format.

37. $(x^2 - 6x) - (x^2 - 10x)$

38. $(y^2 + 4y) - (y^2 + 10y)$

39. $(2y^2 - 4y) - (-y^2 + 2)$

40. $(-3a^2 - 2a) - (4a^2 - 4)$

41. $(x^2 - 2x + 1) - (x^2 + 5x + 8)$

42. $(3x^2 + 2x - 2) - (5x^2 - 5x + 6)$

43. $(4x^3 + 5x + 2) - (1 + 2x - 3x^2)$

44. $(5y^2 - y + 2) - (-3 + 3y - 2y^3)$

45. $(-2y + 6y^2 + 2y^3) - (4 + y^2 + y^3)$

46. $(4 - x - 2x^2) - (-2 + 3x - x^3)$

Simplify. Use a horizontal format.

47. $(y^2 - 10xy) - (2y^2 + 3xy)$

48. $(x^2 - 3xy) - (-2x^2 + xy)$

49. $(3x^2 + x - 3) - (4x + x^2 - 2)$

50. $(5y^2 - 2y + 1) - (-y - 2 - 3y^2)$

51. $(-2x^3 + x - 1) - (-x^2 + x - 3)$

52. $(2x^2 + 5x - 3) - (3x^3 + 2x - 5)$

53. $(1 - 2a + 4a^3) - (a^3 - 2a + 3)$

54. $(7 - 8b + b^2) - (4b^3 - 7b - 8)$

55. $(-1 - y + 4y^3) - (3 - 3y - 2y^2)$

56. $(-3 - 2x + 3x^2) - (4 - 2x^2 + 2x^3)$

APPLYING THE CONCEPTS

57. What polynomial must be added to $3x^2 - 6x + 9$ so that the sum is $4x^2 + 3x - 2$?

58. What polynomial must be subtracted from $2x^2 - x - 2$ so that the difference is $5x^2 + 3x + 1$?

59. In your own words, explain the terms monomial, binomial, trinomial, and polynomial. Give an example of each.

60. Is it possible to subtract two polynomials, each of degree 3, and have the difference be a polynomial of degree 2? If so, give an example. If not, explain why not.

61. Is it possible to add two polynomials, each of degree 3, and have the sum be a polynomial of degree 2? If so, give an example. If not, explain why not.

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4.2 Multiplication of Monomials

Objective A To multiply monomials

Recall that in an exponential expression such as x^6, x is the base and 6 is the exponent. The exponent indicates the number of times the base occurs as a factor.

The product of exponential expressions with the *same* base can be simplified by writing each expression in factored form and writing the result with an exponent.

$$x^3 \cdot x^2 = \overbrace{(x \cdot x \cdot x)}^{3 \text{ factors}} \cdot \overbrace{(x \cdot x)}^{2 \text{ factors}} = \underbrace{x^5}_{5 \text{ factors}}$$

Note that adding the exponents results in the same product.

$$x^3 \cdot x^2 = x^{3+2} = x^5$$

Rule for Multiplying Exponential Expressions

If m and n are positive integers, then $x^m \cdot x^n = x^{m+n}$.

➡ Simplify: $y^4 \cdot y \cdot y^3$

$y^4 \cdot y \cdot y^3 = y^{4+1+3}$
$= y^8$

- The bases are the same. Add the exponents. Recall that $y = y^1$.

➡ Simplify: $(-3a^4b^3)(2ab^4)$

$(-3a^4b^3)(2ab^4) = (-3 \cdot 2)(a^4 \cdot a)(b^3 \cdot b^4)$

- Use the Commutative and Associative Properties of Multiplication to rearrange and group factors.

$= -6(a^{4+1})(b^{3+4})$

- Multiply variables with the same base by adding their exponents.

$= -6a^5b^7$

- Simplify.

TAKE NOTE

The Rule for Multiplying Exponential Expressions requires the bases to be the same. The expression a^5b^7 cannot be simplified.

Example 1 Simplify: $(-5ab^3)(4a^5)$

Solution

$(-5ab^3)(4a^5) = (-5 \cdot 4)(a \cdot a^5)b^3$
$= -20a^6b^3$

You Try It 1 Simplify: $(8m^3n)(-3n^5)$

Your solution

Example 2 Simplify: $(6x^3y^2)(4x^4y^5)$

Solution

$(6x^3y^2)(4x^4y^5) = (6 \cdot 4)(x^3 \cdot x^4)(y^2 \cdot y^5)$
$= 24x^7y^7$

You Try It 2 Simplify: $(12p^4q^3)(-3p^5q^2)$

Your solution

Solutions on p. S8

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Objective B To simplify powers of monomials

14 15 CT

POINT OF INTEREST

One of the first symbolic representations of powers was given by Diophantus (c. 250 A.D.) in his book *Arithmetica*. He used Δ^Y for x^2 and κ^Y for x^3. The symbol Δ^Y was the first two letters of the Greek word *dunamis* meaning power; κ^Y was from the Greek word *kubos* meaning cube. He also combined these symbols to denote higher powers. For instance, $\Delta\kappa^Y$ was the symbol for x^5.

The power of a monomial can be simplified by writing the power in factored form and then using the Rule for Multiplying Exponential Expressions.

$(x^4)^3 = x^4 \cdot x^4 \cdot x^4$ $(a^2b^3)^2 = (a^2b^3)(a^2b^3)$ • Write in factored form.

$= x^{4+4+4}$ $= a^{2+2}b^{3+3}$ • Use the Rule for Multiplying Exponential Expressions.

$= x^{12}$ $= a^4b^6$

Note that multiplying each exponent inside the parentheses by the exponent outside the parentheses results in the same product.

$(x^4)^3 = x^{4\cdot3} = x^{12}$ $(a^2b^3)^2 = a^{2\cdot2}b^{3\cdot2} = a^4b^6$ • Multiply each exponent inside the parentheses by the exponent outside the parentheses.

Rule for Simplifying the Power of an Exponential Expression

If m and n are positive integers, then $(x^m)^n = x^{mn}$.

Rule for Simplifying Powers of Products

If m, n, and p are positive integers, then $(x^my^n)^p = x^{mp}y^{np}$.

➡ Simplify: $(5x^2y^3)^3$

$(5x^2y^3)^3 = 5^{1\cdot3}x^{2\cdot3}y^{3\cdot3}$ • Multiply each exponent inside the parentheses by the exponent outside the parentheses. Note that $5 = 5^1$.

$= 5^3x^6y^9$

$= 125x^6y^9$ • Evaluate 5^3.

Example 3 Simplify: $(-2p^3r)^4$

Solution $(-2p^3r)^4 = (-2)^{1\cdot4}p^{3\cdot4}r^{1\cdot4}$

$= (-2)^4p^{12}r^4 = 16p^{12}r^4$

You Try It 3 Simplify: $(-3a^4bc^2)^3$

Your solution

Example 4 Simplify: $(2a^2b)(2a^3b^2)^3$

Solution $(2a^2b)(2a^3b^2)^3$

$= (2a^2b)(2^{1\cdot3}a^{3\cdot3}b^{2\cdot3})$

$= (2a^2b)(2^3a^9b^6)$

$= (2a^2b)(8a^9b^6) = 16a^{11}b^7$

You Try It 4 Simplify: $(-xy^4)(-2x^3y^2)^2$

Your solution

Solutions on pp. S8–S9

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4.2 Exercises

Objective A

Simplify.

1. $(6x^2)(5x)$
2. $(-4y^3)(2y)$
3. $(7c^2)(-6c^4)$
4. $(-8z^5)(5z^8)$
5. $(-3a^3)(-3a^4)$
6. $(-5a^6)(-2a^5)$
7. $(x^2)(xy^4)$
8. $(x^2y^4)(xy^7)$
9. $(-2x^4)(5x^5y)$
10. $(-3a^3)(2a^2b^4)$
11. $(-4x^2y^4)(-3x^5y^4)$
12. $(-6a^2b^4)(-4ab^3)$
13. $(2xy)(-3x^2y^4)$
14. $(-3a^2b)(-2ab^3)$
15. $(x^2yz)(x^2y^4)$
16. $(-ab^2c)(a^2b^5)$
17. $(-a^2b^3)(-ab^2c^4)$
18. $(-x^2y^3z)(-x^3y^4)$
19. $(-5a^2b^2)(6a^3b^6)$
20. $(7xy^4)(-2xy^3)$
21. $(-6a^3)(-a^2b)$
22. $(-2a^2b^3)(-4ab^2)$
23. $(-5y^4z)(-8y^6z^5)$
24. $(3x^2y)(-4xy^2)$
25. $(x^2y)(yz)(xyz)$
26. $(xy^2z)(x^2y)(z^2y^2)$
27. $(3ab^2)(-2abc)(4ac^2)$
28. $(-2x^3y^2)(-3x^2z^2)(-5y^3z^3)$
29. $(4x^4z)(-yz^3)(-2x^3z^2)$
30. $(-a^3b^4)(-3a^4c^2)(4b^3c^4)$
31. $(-2x^2y^3)(3xy)(-5x^3y^4)$
32. $(4a^2b)(-3a^3b^4)(a^5b^2)$
33. $(3a^2b)(-6bc)(2ac^2)$

Objective B

Simplify.

34. $(z^4)^3$
35. $(x^3)^5$
36. $(y^4)^2$
37. $(x^7)^2$
38. $(-y^5)^3$
39. $(-x^2)^4$
40. $(-x^2)^3$
41. $(-y^3)^4$
42. $(-3y)^3$
43. $(-2x^2)^3$
44. $(a^3b^4)^3$
45. $(x^2y^3)^2$
46. $(2x^3y^4)^5$
47. $(3x^2y)^2$
48. $(-2ab^3)^4$

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49. $(-3x^3y^2)^5$ **50.** $(3b^2)(2a^3)^4$ **51.** $(-2x)(2x^3)^2$ **52.** $(2y)(-3y^4)^3$

53. $(3x^2y)(2x^2y^2)^3$ **54.** $(a^3b)^2(ab)^3$ **55.** $(ab^2)^2(ab)^2$ **56.** $(-x^2y^3)^2(-2x^3y)^3$

57. $(-2x)^3(-2x^3y)^3$ **58.** $(-3y)(-4x^2y^3)^3$ **59.** $(-2x)(-3xy^2)^2$ **60.** $(-3y)(-2x^2y)^3$

61. $(ab^2)(-2a^2b)^3$ **62.** $(a^2b^2)(-3ab^4)^2$ **63.** $(-2a^3)(3a^2b)^3$ **64.** $(-3b^2)(2ab^2)^3$

APPLYING THE CONCEPTS

Simplify.

65. $3x^2 + (3x)^2$ **66.** $4x^2 - (4x)^2$ **67.** $2x^6y^2 + (3x^2y)^2$ **68.** $(x^2y^2)^3 + (x^3y^3)^2$

69. $(2a^3b^2)^3 - 8a^9b^6$ **70.** $4y^2z^4 - (2yz^2)^2$ **71.** $(x^2y^4)^2 + (2xy^2)^4$ **72.** $(3a^3)^2 - 4a^6 + (2a^2)^3$

For Exercises 73 to 76, answer true or false. If the answer is false, correct the right-hand side of the equation.

73. $(-a)^5 = -a^5$ **74.** $(-b)^8 = b^8$ **75.** $(x^2)^5 = x^{2+5} = x^7$ **76.** $x^3 + x^3 = 2x^{3+3} = 2x^6$

77. Evaluate $(2^3)^2$ and $2^{(3^2)}$. Are the results the same? If not, which expression has the larger value?

78. If n is a positive integer and $x^n = y^n$, does $x = y$? Explain your answer.

79. The distance a rock will fall in t seconds is $16t^2$ (neglecting air resistance). Find other examples of quantities that can be expressed in terms of an exponential expression, and explain where the expression is used.

80. Explain in your own words how to multiply monomials.

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4.3 Multiplication of Polynomials

Objective A To multiply a polynomial by a monomial

To multiply a polynomial by a monomial, use the Distributive Property and the Rule for Multiplying Exponential Expressions.

Simplify: $-3a(4a^2 - 5a + 6)$

$$-3a(4a^2 - 5a + 6) = -3a(4a^2) - (-3a)(5a) + (-3a)(6)$$
$$= -12a^3 + 15a^2 - 18a$$

• Use the Distributive Property.

Example 1
Simplify: $(5x + 4)(-2x)$

Solution
$(5x + 4)(-2x) = -10x^2 - 8x$

You Try It 1
Simplify: $(-2y + 3)(-4y)$

Your solution

Example 2
Simplify: $2a^2b(4a^2 - 2ab + b^2)$

Solution
$2a^2b(4a^2 - 2ab + b^2)$
$= 8a^4b - 4a^3b^2 + 2a^2b^3$

You Try It 2
Simplify: $-a^2(3a^2 + 2a - 7)$

Your solution

Solutions on p. S9

Objective B To multiply two polynomials

Multiplication of two polynomials requires the repeated application of the Distributive Property.

$$(y - 2)(y^2 + 3y + 1) = (y - 2)(y^2) + (y - 2)(3y) + (y - 2)(1)$$
$$= y^3 - 2y^2 + 3y^2 - 6y + y - 2$$
$$= y^3 + y^2 - 5y - 2$$

A convenient method of multiplying two polynomials is to use a vertical format similar to that used for multiplication of whole numbers.

$$\begin{array}{r} y^2 + 3y + 1 \\ \underline{\qquad\quad y - 2} \\ -2y^2 - 6y - 2 \\ \underline{y^3 + 3y^2 + y\quad\;\;} \\ y^3 + y^2 - 5y - 2 \end{array}$$

$-2(y^2 + 3y + 1)$ • Multiply by -2.
$y(y^2 + 3y + 1)$ • Multiply by y.
• Add the terms in each column.

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➡ Simplify: $(2a^3 + a - 3)(a + 5)$

$$\begin{array}{rrrr} & 2a^3 & + a & - 3 \\ & & a & + 5 \\ \hline & 10a^3 & + 5a & - 15 \\ 2a^4 & + a^2 & - 3a & \\ \hline 2a^4 & + 10a^3 & + a^2 + 2a & - 15 \end{array}$$

- Note that spaces are provided in each product so that like terms are in the same column.
- Add the terms in each column.

Example 3

Simplify: $(2b^3 - b + 1)(2b + 3)$

Solution

$$\begin{array}{rrrr} & 2b^3 & - b & + 1 \\ & & 2b & + 3 \\ \hline & 6b^3 & - 3b & + 3 \\ 4b^4 & - 2b^2 & + 2b & \\ \hline 4b^4 & + 6b^3 - 2b^2 & - b & + 3 \end{array}$$

You Try It 3

Simplify: $(2y^3 + 2y^2 - 3)(3y - 1)$

Your solution

Solution on p. S9

Objective C To multiply two binomials

14 CT

It is frequently necessary to find the product of two binomials. The product can be found using a method called **FOIL**, which is based on the Distributive Property. The letters of FOIL stand for **F**irst, **O**uter, **I**nner, and **L**ast.

➡ Simplify: $(2x + 3)(x + 5)$

Multiply the First terms.	$(2x + 3)(x + 5)$	$2x \cdot x = 2x^2$
Multiply the Outer terms.	$(2x + 3)(x + 5)$	$2x \cdot 5 = 10x$
Multiply the Inner terms.	$(2x + 3)(x + 5)$	$3 \cdot x = 3x$
Multiply the Last terms.	$(2x + 3)(x + 5)$	$3 \cdot 5 = 15$
		F O I L
Add the products.	$(2x + 3)(x + 5)$	$= 2x^2 + 10x + 3x + 15$
Combine like terms.		$= 2x^2 + 13x + 15$

TAKE NOTE

FOIL is not really a different way of multiplying. It is based on the Distributive Property.

$(2x + 3)(x + 5)$
$= 2x(x + 5) + 3(x + 5)$
F O I L
$= 2x^2 + 10x + 3x + 15$
$= 2x^2 + 13x + 15$

➡ Simplify: $(4x - 3)(3x - 2)$

$$\begin{aligned} (4x - 3)(3x - 2) &= 4x(3x) + 4x(-2) + (-3)(3x) + (-3)(-2) \\ &= 12x^2 - 8x - 9x + 6 \\ &= 12x^2 - 17x + 6 \end{aligned}$$

➡ Simplify: $(3x - 2y)(x + 4y)$

$$\begin{aligned} (3x - 2y)(x + 4y) &= 3x(x) + 3x(4y) + (-2y)(x) + (-2y)(4y) \\ &= 3x^2 + 12xy - 2xy - 8y^2 \\ &= 3x^2 + 10xy - 8y^2 \end{aligned}$$

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Example 4
Simplify: $(2a - 1)(3a - 2)$

Solution
$(2a - 1)(3a - 2) = 6a^2 - 4a - 3a + 2$
$= 6a^2 - 7a + 2$

You Try It 4
Simplify: $(4y - 5)(2y - 3)$

Your solution

Example 5
Simplify: $(3x - 2)(4x + 3)$

Solution
$(3x - 2)(4x + 3) = 12x^2 + 9x - 8x - 6$
$= 12x^2 + x - 6$

You Try It 5
Simplify: $(3b + 2)(3b - 5)$

Your solution

Solutions on p. S9

Objective D To multiply binomials that have special products

14 CT

Using FOIL, it is possible to find a pattern for the product of the sum and difference of two terms and for the square of a binomial.

The Sum and Difference of Two Terms

$$(a + b)(a - b) = a^2 - ab + ab - b^2$$
$$= a^2 - b^2$$

Square of first term
Square of second term

The Square of a Bionomial

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

Square of first term
Twice the product of the two terms
Square of the last term

➡ Simplify: $(2x + 3)(2x - 3)$

$(2x + 3)(2x - 3) = (2x)^2 - 3^2$
$= 4x^2 - 9$

• This is the sum and difference of two terms.

➡ Simplify: $(3x - 2)^2$

$(3x - 2)^2 = (3x)^2 + 2(3x)(-2) + (-2)^2$
$= 9x^2 - 12x + 4$

• This is the square of a binomial.

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Example 6
Simplify: $(4z - 2w)(4z + 2w)$

Solution
$(4z - 2w)(4z + 2w) = 16z^2 - 4w^2$

You Try It 6
Simplify: $(2a + 5c)(2a - 5c)$

Your solution

Example 7
Simplify: $(2r - 3s)^2$

Solution
$(2r - 3s)^2 = 4r^2 - 12rs + 9s^2$

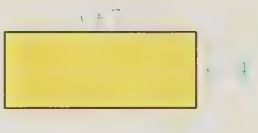
You Try It 7
Simplify: $(3x + 2y)^2$

Your solution

Solutions on p. S9

Objective E To solve application problems

Example 8
The length of a rectangle is $(x + 7)$ m. The width is $(x - 4)$ m. Find the area of the rectangle in terms of the variable x.

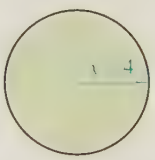


Strategy
To find the area, replace the variables L and W in the equation $A = L \cdot W$ by the given values and solve for A.

Solution
$A = L \cdot W$
$A = (x + 7)(x - 4)$
$A = x^2 - 4x + 7x - 28$
$A = x^2 + 3x - 28$

The area is $(x^2 + 3x - 28)$ m^2.

You Try It 8
The radius of a circle is $(x - 4)$ ft. Use the equation $A = \pi r^2$, where r is the radius, to find the area of the circle in terms of x. Leave the answer in terms of π.



Your strategy

Your solution

Solution on p. S9

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4.3 Exercises

Objective A

Simplify.

1. $x(x - 2)$ **2.** $y(3 - y)$ **3.** $-x(x + 7)$ **4.** $-y(7 - y)$

5. $3a^2(a - 2)$ **6.** $4b^2(b + 8)$ **7.** $-5x^2(x^2 - x)$ **8.** $-6y^2(y + 2y^2)$

9. $-x^3(3x^2 - 7)$ **10.** $-y^4(2y^2 - y^6)$ **11.** $2x(6x^2 - 3x)$ **12.** $3y(4y - y^2)$

13. $(2x - 4)3x$ **14.** $(3y - 2)y$ **15.** $(3x + 4)x$ **16.** $(2x + 1)2x$

17. $-xy(x^2 - y^2)$ **18.** $-x^2y(2xy - y^2)$ **19.** $x(2x^3 - 3x + 2)$ **20.** $y(-3y^2 - 2y + 6)$

21. $-a(-2a^2 - 3a - 2)$ **22.** $-b(5b^2 + 7b - 35)$ **23.** $x^2(3x^4 - 3x^2 - 2)$

24. $y^3(-4y^3 - 6y + 7)$ **25.** $2y^2(-3y^2 - 6y + 7)$ **26.** $4x^2(3x^2 - 2x + 6)$

27. $(a^2 + 3a - 4)(-2a)$ **28.** $(b^3 - 2b + 2)(-5b)$ **29.** $-3y^2(-2y^2 + y - 2)$

30. $-5x^2(3x^2 - 3x - 7)$ **31.** $xy(x^2 - 3xy + y^2)$ **32.** $ab(2a^2 - 4ab - 6b^2)$

Objective B

Simplify.

33. $(x^2 + 3x + 2)(x + 1)$ **34.** $(x^2 - 2x + 7)(x - 2)$ **35.** $(a^2 - 3a + 4)(a - 3)$

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36. $(x^2 - 3x + 5)(2x - 3)$ **37.** $(-2b^2 - 3b + 4)(b - 5)$ **38.** $(-a^2 + 3a - 2)(2a - 1)$

39. $(-2x^2 + 7x - 2)(3x - 5)$ **40.** $(-a^2 - 2a + 3)(2a - 1)$ **41.** $(x^2 + 5)(x - 3)$

42. $(y^2 - 2y)(2y + 5)$ **43.** $(x^3 - 3x + 2)(x - 4)$ **44.** $(y^3 + 4y^2 - 8)(2y - 1)$

45. $(5y^2 + 8y - 2)(3y - 8)$ **46.** $(3y^2 + 3y - 5)(4y - 3)$ **47.** $(5a^3 - 5a + 2)(a - 4)$

48. $(3b^3 - 5b^2 + 7)(6b - 1)$ **49.** $(y^3 + 2y^2 - 3y + 1)(y + 2)$ **50.** $(2a^3 - 3a^2 + 2a - 1)(2a - 3)$

Objective C

Simplify.

51. $(x + 1)(x + 3)$ **52.** $(y + 2)(y + 5)$ **53.** $(a - 3)(a + 4)$ **54.** $(b - 6)(b + 3)$

55. $(y + 3)(y - 8)$ **56.** $(x + 10)(x - 5)$ **57.** $(y - 7)(y - 3)$ **58.** $(a - 8)(a - 9)$

59. $(2x + 1)(x + 7)$ **60.** $(y + 2)(5y + 1)$ **61.** $(3x - 1)(x + 4)$ **62.** $(7x - 2)(x + 4)$

63. $(4x - 3)(x - 7)$ **64.** $(2x - 3)(4x - 7)$ **65.** $(3y - 8)(y + 2)$ **66.** $(5y - 9)(y + 5)$

67. $(3x + 7)(3x + 11)$ **68.** $(5a + 6)(6a + 5)$ **69.** $(7a - 16)(3a - 5)$ **70.** $(5a - 12)(3a - 7)$

71. $(3a - 2b)(2a - 7b)$ **72.** $(5a - b)(7a - b)$ **73.** $(a - 9b)(2a + 7b)$

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74. $(2a + 5b)(7a - 2b)$ **75.** $(10a - 3b)(10a - 7b)$ **76.** $(12a - 5b)(3a - 4b)$

77. $(5x + 12y)(3x + 4y)$ **78.** $(11x + 2y)(3x + 7y)$ **79.** $(2x + 15y)(7x - 4y)$

80. $(5x + 2y)(2x - 5y)$ **81.** $(8x - 3y)(7x - 5y)$ **82.** $(2x - 9y)(8x - 3y)$

Objective D

Simplify.

83. $(y - 5)(y + 5)$ **84.** $(y + 6)(y - 6)$ **85.** $(2x + 3)(2x - 3)$ **86.** $(4x - 7)(4x + 7)$

87. $(x + 1)^2$ **88.** $(y - 3)^2$ **89.** $(3a - 5)^2$ **90.** $(6x - 5)^2$

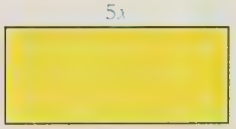
91. $(3x - 7)(3x + 7)$ **92.** $(9x - 2)(9x + 2)$ **93.** $(2a + b)^2$ **94.** $(x + 3y)^2$

95. $(x - 2y)^2$ **96.** $(2x - 3y)^2$ **97.** $(4 - 3y)(4 + 3y)$

98. $(4x - 9y)(4x + 9y)$ **99.** $(5x + 2y)^2$ **100.** $(2a - 9b)^2$

Objective E Application Problems

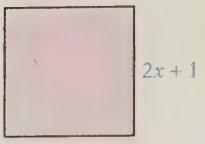
101. The length of a rectangle is $(5x)$ ft. The width is $(2x - 7)$ ft. Find the area of the rectangle in terms of the variable x.



102. The width of a rectangle is $(3x + 1)$ in. The length of the rectangle is twice the width. Find the area of the rectangle in terms of the variable x.

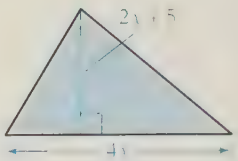
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103. The length of a side of a square is $(2x + 1)$ km. Find the area of the square in terms of the variable x.

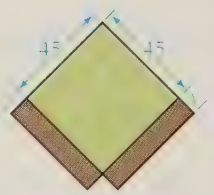


104. The radius of a circle is $(x + 4)$ cm. Find the area of the circle in terms of the variable x. Leave the answer in terms of π.

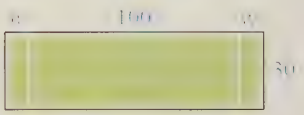
105. The base of a triangle is $(4x)$ m and the height is $(2x + 5)$ m. Find the area of the triangle in terms of the variable x.



106. A softball diamond has dimensions 45 ft by 45 ft. A base-path border x feet wide lies on both the first-base side and third-base side of the diamond. Express the total area of the softball diamond and the base paths in terms of the variable x.



107. An athletic field has dimensions 30 yd by 100 yd. An end zone that is w yards wide borders each end of the field. Express the total area of the field and the end zones in terms of the variable w.



APPLYING THE CONCEPTS

Simplify.

108. $(a + b)^2 - (a - b)^2$ **109.** $(x^2 + x - 3)^2$ **110.** $(a + 3)^3$

111. What polynomial has quotient $3x - 4$ when divided by $4x + 5$?

112. Add $x^2 + 2x - 3$ to the product of $2x - 5$ and $3x + 1$.

113. Subtract $4x^2 - x - 5$ from the product of $x^2 + x + 3$ and $x - 4$.

114. If a polynomial of degree 3 is multiplied by a polynomial of degree 2, what is the degree of the resulting polynomial?

115. Is it possible to multiply a polynomial of degree 2 by a polynomial of degree 2 and have the product be a polynomial of degree 3? If so, give an example. If not, explain why not.

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4.4 Integer Exponents and Scientific Notation

Objective A To divide monomials

The quotient of two exponential expressions with the same base can be simplified by writing each expression in factored form, dividing by the common factors, and then writing the result with an exponent.

$$\frac{x^5}{x^2} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot x \cdot x \cdot x}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = x^3$$

Note that subtracting the exponents gives the same result.

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

To divide two monomials with the same base, subtract the exponents of the like bases.

➡ Simplify: $\frac{a^7}{a^3}$

$$\frac{a^7}{a^3} = a^{7-3}$$ • The bases are the same. Subtract the exponents.

$$= a^4$$

➡ Simplify: $\frac{r^8t^6}{r^7t}$

$$\frac{r^8t^6}{r^7t} = r^{8-7}t^{6-1}$$ • Subtract the exponents of the like bases.

$$= rt^5$$

➡ Simplify: $\frac{p^7}{z^4}$

Because the bases are not the same, $\frac{p^7}{z^4}$ is already in simplest form.

Consider the expression $\frac{x^4}{x^4}$, $x \neq 0$. This expression can be simplified, as shown below, by subtracting exponents or by dividing by common factors.

$$\frac{x^4}{x^4} = x^{4-4} = x^0 \qquad \frac{x^4}{x^4} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = 1$$

The equations $\frac{x^4}{x^4} = x^0$ and $\frac{x^4}{x^4} = 1$ suggest the following definition of x^0.

Definition of Zero as an Exponent

If $x \neq 0$, then $x^0 = 1$. The expression 0^0 is not defined.

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➡ Simplify: $(12a^3)^0$, $a \neq 0$

$(12a^3)^0 = 1$ • Any nonzero expression to the zero power is 1.

➡ Simplify: $-(4x^3y^7)^0$

$-(4x^3y^7)^0 = -(1) = -1$

POINT OF INTEREST

In the 15th century, the expression $12^{2\overline{m}}$ was used to mean $12x^{-2}$. The use of $\overline{m}$ reflected an Italian influence. In Italy, m was used for minus and p was used for plus. It was understood that $2\overline{m}$ referred to an unnamed variable. Issac Newton, in the 17th century, advocated the current use of a negative exponent.

Consider the expression $\frac{x^4}{x^6}$, $x \neq 0$. This expression can be simplified, as shown below, by subtracting exponents or by dividing by common factors.

$$\frac{x^4}{x^6} = x^{-2} = \frac{1}{x^2} \qquad \frac{x^4}{x^6} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot x \cdot x} = \frac{1}{x^2}$$

The equations $\frac{x^4}{x^6} = x^{-2}$ and $\frac{x^4}{x^6} = \frac{1}{x^2}$ suggest that $x^{-2} = \frac{1}{x^2}$.

Definition of a Negative Exponent

If $x \neq 0$ and n is a positive integer, then

$$x^{-n} = \frac{1}{x^n} \quad \text{and} \quad \frac{1}{x^{-n}} = x^n$$

An exponential expression is in simplest form when it is written with only positive exponents.

TAKE NOTE

Note from the example at the right that 2^{-4} is a *positive* number. A negative exponent does not change the sign of a number.

➡ Evaluate 2^{-4}.

$2^{-4} = \frac{1}{2^4}$ • Use the Definition of a Negative Exponent.

$= \frac{1}{16}$ • Evaluate the expression.

TAKE NOTE

The exponent on n is -5 (*negative* 5). The n^{-5} is written in the denominator as n^5. The exponent on 3 is 1 (*positive* 1). The 3 remains in the numerator.

➡ Simplify: $3n^{-5}$

$3n^{-5} = 3 \cdot \frac{1}{n^5} = \frac{3}{n^5}$ • Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

➡ Simplify: $\frac{2}{5a^{-4}}$

$\frac{2}{5a^{-4}} = \frac{2}{5} \cdot \frac{1}{a^{-4}} = \frac{2}{5} \cdot a^4 = \frac{2a^4}{5}$ • Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

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The expression $\left(\frac{x^4}{y^3}\right)^2$, $y \neq 0$, can be simplified by squaring $\frac{x^4}{y^3}$ or by multiplying each exponent in the quotient by the exponent outside the parentheses.

$$\left(\frac{x^4}{y^3}\right)^2 = \left(\frac{x^4}{y^3}\right)\left(\frac{x^4}{y^3}\right) = \frac{x^4 \cdot x^4}{y^3 \cdot y^3} = \frac{x^{4+4}}{y^{3+3}} = \frac{x^8}{y^6} \qquad \left(\frac{x^4}{y^3}\right)^2 = \frac{x^{4\cdot 2}}{y^{3\cdot 2}} = \frac{x^8}{y^6}$$

Rule for Simplifying Powers of Quotients

If m, n, and q are integers and $y \neq 0$, then $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

➡ Simplify: $\left(\frac{a^3}{b^2}\right)^{-2}$

$$\left(\frac{a^3}{b^2}\right)^{-2} = \frac{a^{3(-2)}}{b^{2(-2)}}$$

- Use the Rule for Simplifying Powers of Quotients.

$$= \frac{a^{-6}}{b^{-4}} = \frac{b^4}{a^6}$$

- Use the Definition of a Negative Exponent to write the expression with positive exponents.

The example above suggests the following rule.

Rule for Negative Exponents on Fractional Expressions

If $a \neq 0$, $b \neq 0$, and n is a positive integer, then

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Now that zero as an exponent and negative exponents have been defined, a rule for dividing exponential expressions can be stated.

Rule for Dividing Exponential Expressions

If m and n are integers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

➡ Evaluate $\frac{5^{-2}}{5}$.

$$\frac{5^{-2}}{5} = 5^{-2-1} = 5^{-3}$$

- Use the Rule for Dividing Exponential Expressions.

$$= \frac{1}{5^3} = \frac{1}{125}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent. Then evaluate.

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➡ Simplify: $\frac{x^4}{x^9}$

$$\frac{x^4}{x^9} = x^{4-9}$$ • Use the Rule for Dividing Exponential Expressions.

$$= x^{-5}$$ • Subtract the exponents.

$$= \frac{1}{x^5}$$ • Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

The rules for simplifying exponential expressions and powers of exponential expressions are true for all integers. These rules are restated here along with the rules for dividing exponential expressions.

Rules of Exponents

If m, n, and p are integers, then

$x^m \cdot x^n = x^{m+n}$ $\quad (x^m)^n = x^{mn}$ $\quad (x^m y^n)^p = x^{mp} y^{np}$

$\frac{x^m}{x^n} = x^{m-n}, x \neq 0$ $\quad \left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}, y \neq 0$ $\quad x^{-n} = \frac{1}{x^n}, x \neq 0$

$x^0 = 1, x \neq 0$

➡ Simplify: $(3ab^{-4})(-2a^{-3}b^7)$

$$(3ab^{-4})(-2a^{-3}b^7) = [3 \cdot (-2)](a^{1+(-3)}b^{-4+7})$$ • When multiplying expressions, add the exponents on like bases.

$$= -6a^{-2}b^3$$

$$= \frac{6b^3}{a^2}$$

➡ Simplify: $\left[\frac{6m^2n^3}{8m^7n^2}\right]^{-3}$

$$\left[\frac{6m^2n^3}{8m^7n^2}\right]^{-3} = \left[\frac{3m^{2-7}n^{3-2}}{4}\right]^{-3}$$ • Simplify inside the brackets.

$$= \left[\frac{3m^{-5}n}{4}\right]^{-3}$$ • Subtract the exponents.

$$= \frac{3^{-3}m^{15}n^{-3}}{4^{-3}}$$ • Use the Rule for Simplifying Powers of Quotients.

$$= \frac{4^3m^{15}}{3^3n^3} = \frac{64m^{15}}{27n^3}$$ • Use the Definition of a Negative Exponent to rewrite the expression with positive exponents. Then simplify.

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Simplify: $\frac{4a^{-2}b^5}{6a^5b^2}$

$$\frac{4a^{-2}b^5}{6a^5b^2} = \frac{2a^{-2}b^5}{3a^5b^2}$$

- Divide the coefficients by their common factor.

$$= \frac{2a^{-2-5}b^{5-2}}{3}$$

- Use the Rule for Dividing Exponential Expressions.

$$= \frac{2a^{-7}b^3}{3} = \frac{2b^3}{3a^7}$$

- Use the Definition of a Negative Exponent to rewrite the expression with positive exponents.

Example 1 Simplify: $(-2x)(3x^{-2})^{-3}$

Solution $(-2x)(3x^{-2})^{-3} = (-2x)(3^{-3}x^6)$

$$= \frac{-2x^{1+6}}{3^3}$$

$$= \frac{-2x^7}{27}$$

You Try It 1 Simplify: $(-2x^2)(x^{-3}y^{-4})^{-2}$

Your solution

Example 2 Simplify: $\frac{(2r^2t^{-1})^{-3}}{(r^{-3}t^4)^2}$

Solution $\frac{(2r^2t^{-1})^{-3}}{(r^{-3}t^4)^2} = \frac{2^{-3}r^{-6}t^3}{r^{-6}t^8}$

$$= 2^{-3}r^{-6-(-6)}t^{3-8}$$

$$= 2^{-3}r^0t^{-5}$$

$$= \frac{1}{2^3t^5}$$

$$= \frac{1}{8t^5}$$

You Try It 2 Simplify: $\frac{(6a^{-2}b^3)^{-1}}{(4a^3b^{-2})^{-2}}$

Your solution

Example 3 Simplify: $\left[\frac{4a^{-2}b^3}{6a^4b^{-2}}\right]^{-3}$

Solution $\left[\frac{4a^{-2}b^3}{6a^4b^{-2}}\right]^{-3} = \left[\frac{2a^{-6}b^5}{3}\right]^{-3}$

$$= \frac{2^{-3}a^{18}b^{-15}}{3^{-3}}$$

$$= \frac{27a^{18}}{8b^{15}}$$

You Try It 3 Simplify: $\left[\frac{6r^3s^{-3}}{9r^3s^{-1}}\right]^{-2}$

Your solution

Solutions on p. S9

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Objective B To write a number in scientific notation

Very large and very small numbers abound in the natural sciences. For example, the mass of an electron is 0.000000000000000000000000000000911 kg. Numbers such as this are difficult to read, so a more convenient system called **scientific notation** is used. In scientific notation, a number is expressed as the product of two factors, one a number between 1 and 10, and the other a power of 10.

POINT OF INTEREST

An electron microscope uses wavelengths that are approximately 4×10^{-12} meters to make images of viruses.

The human eye can detect wavelengths between 4.3×10^{-7} meters and 6.9×10^{-7} meters. Although these are very short, they are approximately 10^5 times longer than the waves used in an electron miscroscope.

To express a number in scientific notation, write it in the form $a \times 10^n$, where a is a number between 1 and 10, and n is an integer.

For numbers greater than or equal to 10, move the decimal point to the right of the first digit. The exponent n is positive and equal to the number of places the decimal point has been moved.

$240{,}000 = 2.4 \times 10^5$

$93{,}000{,}000 = 9.3 \times 10^7$

For numbers less than 1, move the decimal point to the right of the first nonzero digit. The exponent n is negative. The absolute value of the exponent is equal to the number of places the decimal point has been moved.

$0.0003 = 3.0 \times 10^{-4}$

$0.0000832 = 8.32 \times 10^{-5}$

Changing a number written in scientific notation to decimal notation also requires moving the decimal point.

When the exponent is positive, move the decimal point to the right the same number of places as the exponent.

$3.45 \times 10^6 = 3{,}450{,}000$

$2.3 \times 10^8 = 230{,}000{,}000$

When the exponent is negative, move the decimal point to the left the same number of places as the absolute value of the exponent.

$8.1 \times 10^{-3} = 0.0081$

$6.34 \times 10^{-7} = 0.000000634$

Example 4 Write the number 824,300,000 in scientific notation.

Solution $824{,}300{,}000 = 8.243 \times 10^8$

You Try It 4 Write the number 0.000000961 in scientific notation.

Your solution

Example 5 Write the number 6.8×10^{-10} in decimal notation.

Solution $6.8 \times 10^{-10} = 0.00000000068$

You Try It 5 Write the number 7.329×10^6 in decimal notation.

Your solution

Solutions on p. S9

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4.4 Exercises

Objective A

Evaluate.

1. 5^{-2}
2. 3^{-3}
3. $\frac{1}{8^{-2}}$
4. $\frac{1}{12^{-1}}$
5. $\frac{3^{-2}}{3}$
6. $\frac{5^{-3}}{5}$
7. $\frac{2^{-2}}{2^{-3}}$
8. $\frac{3^2}{3^2}$

Simplify.

9. x^{3}
10. y^{-10}
11. $\frac{1}{a^{5}}$
12. $\frac{1}{b^{-4}}$
13. $4x^{-1}$
14. $-6y^{-1}$
15. $\frac{2}{3}z^{-2}$
16. $\frac{4}{5}a^{-3}$
17. $\frac{5}{b^{-4}}$
18. $\frac{-3}{y^{-5}}$
19. $\frac{1}{3x^{-2}}$
20. $\frac{2}{5c^{-5}}$
21. $(ab^{3})^{5}$
22. $(32x^{3}y^{-5})^{-1}$
23. $-(3p^{2}q^{-5})^{-3}$
24. $-\left(\frac{2}{3}xy\right)^{-4}$
25. $\frac{z^{-7}}{z^{5}}$
26. $\frac{z^{3}}{z^{-2}}$
27. $\frac{a^{-4}}{a^{-5}}$
28. $\frac{c^{12}}{c^{-5}}$
29. $\frac{p^{-7}}{p}$
30. $\frac{w^{-5}}{w}$
31. $\frac{4x^{-8}}{2x^{-5}}$
32. $\frac{12z^{-7}}{4z^{-3}}$
33. $\frac{22k^{-5}}{11k^{-5}}$
34. $\frac{14m^{-11}}{7m^{-9}}$
35. $\frac{m^{-3}n^{-7}}{m^{-4}n^{-5}}$
36. $\frac{y^{-5}z^{-6}}{yz^{-3}}$
37. $\frac{6r^{-4}}{4r^{-2}}$
38. $\frac{8x^{-5}}{12x^{-5}}$
39. $\frac{-16a^{-7}}{24a^{-5}}$
40. $\frac{-18b^{-5}}{27b^{-4}}$
41. $\frac{y^{-3}}{y^{-4}}$
42. $\frac{z^{-4}}{z^{-5}}$
43. $\frac{a^{-5}}{a}$
44. $\frac{m}{m^{-1}}$

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45. $\dfrac{4x^2}{12x^5}$ **46.** $\dfrac{6y^8}{8y^9}$ **47.** $\dfrac{-12x}{-18x^6}$ **48.** $\dfrac{-24c^2}{-36c^{11}}$

49. $\dfrac{x^6y^5}{x^8y}$ **50.** $\dfrac{a^3b^2}{a^2b^3}$ **51.** $\dfrac{2m^6n^2}{5m^9n^{10}}$ **52.** $\dfrac{5r^3t^7}{6r^5t^7}$

53. $\dfrac{pq^3}{p^4q^4}$ **54.** $\dfrac{a^4b^5}{a^5b^6}$ **55.** $\dfrac{3x^4y^5}{6x^4y^8}$ **56.** $\dfrac{14a^3b^6}{21a^5b^6}$

57. $\dfrac{14x^4y^6z^2}{16x^3y^9z}$ **58.** $\dfrac{24a^2b^7c^9}{36a^7b^5c}$ **59.** $\dfrac{15mn^9p^3}{30m^4n^9p}$ **60.** $\dfrac{25x^4y^7z^2}{20x^5y^9z^{11}}$

61. $(-2xy^{-2})^3$ **62.** $(-3x^{-1}y^2)^2$ **63.** $(3x^{-1}y^{-2})^2$

64. $(5xy^{-3})^{-2}$ **65.** $(2x^{-1})(x^{-3})$ **66.** $(-2x^{-5})x^7$

67. $(-5a^2)(a^{-5})^2$ **68.** $(2a^{-3})(a^7b^{-1})^3$ **69.** $(-2ab^{-2})(4a^{-2}b)^{-2}$

70. $(3ab^{-2})(2a^{-1}b)^{-3}$ **71.** $(-5x^{-2}y)(-2x^{-2}y^2)$ **72.** $\dfrac{a^{-3}b^{-4}}{a^2b^2}$

73. $\dfrac{3x^{-2}y^2}{6xy^2}$ **74.** $\dfrac{2x^{-2}y}{8xy}$ **75.** $\dfrac{3x^{-2}y}{xy}$ **76.** $\dfrac{2x^{-1}y^4}{x^2y^3}$

77. $\dfrac{2x^{-1}y^{-4}}{4xy^2}$ **78.** $\dfrac{(x^{-1}y)^2}{xy^2}$ **79.** $\dfrac{(x^{-2}y)^2}{x^2y^3}$ **80.** $\dfrac{(x^{-3}y^{-2})^2}{x^6y^8}$

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81. $\dfrac{(a^{-2}y^3)^{-3}}{a^2y}$ **82.** $\dfrac{12a^2b^3}{-27a^2b^2}$ **83.** $\dfrac{-16xy^4}{96x^4y^4}$ **84.** $\dfrac{-8x^2y^4}{44y^2z^5}$

85. $\dfrac{22a^2b^4}{-132b^3c^2}$ **86.** $\dfrac{-(8a^2b^4)^3}{64a^3b^8}$ **87.** $\dfrac{-(14ab^4)^2}{28a^4b^2}$ **88.** $\dfrac{(2a^{-2}b^3)^{\ 2}}{(4a^2b^{-4})^{-1}}$

89. $\dfrac{(3^{-1}r^4s^{-3})^{-2}}{(6r^2s^{-1}t^{-2})^2}$ **90.** $\left(\dfrac{6x^{-4}yz^{-1}}{14xy^{-4}z^2}\right)^{-3}$ **91.** $\left(\dfrac{15m^3n^{-2}p^{-1}}{25m^{-2}n^{-4}}\right)^{-3}$ **92.** $\left(\dfrac{18a^4b^{-2}c^4}{12ab^{-3}d^2}\right)^{-2}$

Objective B

Write in scientific notation.

93. 0.00000000324 **94.** 0.00000012 **95.** 0.000000000000000003

96. 1,800,000,000 **97.** 32,000,000,000,000,000 **98.** 76,700,000,000,000

99. 0.0000000000000000000122 **100.** 0.00137 **101.** 547,000,000

Write in decimal notation.

102. 2.3×10^{-12} **103.** 1.67×10^{-4} **104.** 2×10^{15}

105. 6.8×10^{7} **106.** 9×10^{-21} **107.** 3.05×10^{-5}

108. 9.05×10^{11} **109.** 1.02×10^{-9} **110.** 7.2×10^{-3}

111. Avogadro's number is used in chemistry, and its value is approximately 602,300,000,000,000,000,000,000. Express this number in scientific notation.

112. 5,980,000,000,000,000,000,000,000 kg is the aproximate mass of the earth. Write the mass of the earth in scientific notation.



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113. The length of an infrared light wave is approximately 0.0000037 m. Write this number in scientific notation.

114. Light travels approximately 16,000,000,000 mi in one day. Write this number in scientific notation.

115. One unit used to measure the speed of a computer is the picosecond. One picosecond is 0.000000001 s. Write this number in scientific notation.

116. One light-year is the distance traveled by light in 1 year. One light-year is 5,880,000,000,000 mi. Write this number in scientific notation.

117. The electric charge on an electron is 0.00000000000000000016 coulomb. Write this number in scientific notation.

118. Approximately 35 teragrams (3.5×10^{13} g) of sulfur in the atmosphere is converted to sulfate each year. Write this number in decimal notation.

APPLYING THE CONCEPTS

119. Evaluate 2^x when $x = -2, -1, 0, 1,$ and 2.

120. Evaluate 3^x when $x = -2, -1, 0, 1,$ and 2.

121. Evaluate 2^{-x} when $x = -2, -1, 0, 1,$ and 2.

122. Evaluate 3^{-x} when $x = -2, -1, 0, 1,$ and 2.

Determine whether each equation for Exercises 123 to 125 is true or false. If the equation is false, change the right-hand side of the equation to make a true equation.

123. $(2a)^{-3} = \dfrac{2}{a^3}$

124. $\dfrac{x^{-3}}{y^{-3}} = \left(\dfrac{x}{y}\right)^{-3}$

125. $(2 + 3)^{-1} = 2^{-1} + 3^{-1}$

126. Simplify: $\left(\dfrac{6x^4yz^3}{2x^2y^3}\right)\left(\dfrac{2x^2z^3}{4y^2z}\right) \div \left(\dfrac{6x^2y^3}{x^4y^2z}\right)$

127. If x is a nonzero real number, is x^{-2} always positive, always negative, or positive or negative depending on whether x is positive or negative? Explain your answer.

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4.5 Division of Polynomials

Objective A To divide a polynomial by a monomial

To divide a polynomial by a monomial, divide each term in the numerator by the denominator and write the sum of the quotients.

➡ Simplify: $\frac{6x^3 - 3x^2 + 9x}{3x}$

$$\frac{6x^3 - 3x^2 + 9x}{3x} = \frac{6x^3}{3x} - \frac{3x^2}{3x} + \frac{9x}{3x}$$

- Divide each term of the polynomial by the monomial.

$$= 2x^2 - x + 3$$

- Simplify each expression.

Example 1 Simplify: $\frac{12x^2y - 6xy + 4x^2}{2xy}$

Solution

$$\frac{12x^2y - 6xy + 4x^2}{2xy} = \frac{12x^2y}{2xy} - \frac{6xy}{2xy} + \frac{4x^2}{2xy}$$

You Try It 1 Simplify: $\frac{24x^3 - 12x^2 + 6x}{4x}$

Your solution

Solution on p. 193

Objective B To divide polynomials

The procedure for dividing two polynomials is similar to the one for dividing whole numbers. The same equation used to check division of whole numbers is used to check polynomial division.

(Quotient × divisor) + remainder = dividend

➡ Simplify: $(x^2 - 5x - 8) \div (x - 3)$

Step 1

$$x - 3\overline{)x^2 - 5x - 8}$$

$$\underline{x^2 - 3x}$$

$$-2x - 8$$

- Think $x \cdot x = \frac{x^2}{x} = x$
- Multiply $x(x - 3) = x^2 - 3x$
- Subtract $x^2 - 5x - (x^2 - 3x) = -2x$. Bring down the 8.

Step 2

$$x - 2$$
$$x - 3\overline{)x^2 - 5x - 8}$$
$$\underline{x^2 - 3x}$$
$$-2x - 8$$
$$\underline{-2x + 6}$$
$$-2$$

- Think $x \cdot -2x = \frac{-2x}{x} = -2$
- Multiply $-2(x - 3) = -2x + 6$
- Subtract $-2x - 2 - (-2x + 6) = -2x - 2 + 2x - 6 = -2$

Check: $(x - 2)(x - 3) - 2 = x^2 - 5x + 6 - 2 = x^2 - 5x - 8$

$(x^2 - 5x - 8) \div (x - 3) =$

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If a term is missing from the dividend, a zero can be inserted for that term. This helps keep like terms in the same column.

Simplify: $(6x + 26 + 2x^3) \div (2 + x)$

$(2x^3 + 6x + 26) \div (x + 2)$

$$\begin{array}{r} 2x^2 - 4x + 14 \\ x + 2\overline{)2x^3 + 0 + 6x + 26} \\ \underline{2x^3 + 4x^2} \\ -4x^2 + 6x \\ \underline{-4x^2 - 8x} \\ 14x + 26 \\ \underline{14x + 28} \\ -2 \end{array}$$

- Arrange the terms of each polynomial in descending order.
- There is no x^2 term in $2x^3 + 6x + 26$. Insert a zero for the missing term.

Check:

$(2x^2 - 4x + 14)(x + 2) + (-2) = (2x^3 + 6x + 28) + (-2) = 2x^3 + 6x + 26$

$(2x^3 + 6x + 26) \div (x + 2) = 2x^2 - 4x + 14 - \frac{2}{x + 2}$

Example 2

Simplify: $(8x^2 + 4x^3 + x - 4) \div (2x + 3)$

Solution

$$\begin{array}{r} 2x^2 + x - 1 \\ 2x + 3\overline{)4x^3 + 8x^2 + x - 4} \\ \underline{4x^3 + 6x^2} \\ 2x^2 + x \\ \underline{2x^2 + 3x} \\ -2x - 4 \\ \underline{-2x - 3} \\ -1 \end{array}$$

- Write the dividend in descending powers of x.

$(4x^3 + 8x^2 + x - 4) \div (2x + 3)$

$= 2x^2 + x - 1 \quad \frac{-1}{2x + 3}$

You Try It 2

Simplify: $(2x^3 + x^2 - 8x - 3) \div (2x - 3)$

Your solution

Example 3

Simplify: $(x^2 - 1) \div (x + 1)$

Solution

$$\begin{array}{r} x - 1 \\ x + 1\overline{)x^2 + 0 - 1} \\ \underline{x^2 + x} \\ -x - 1 \\ \underline{-x - 1} \\ 0 \end{array}$$

- Insert a zero for the missing term.

$(x^2 - 1) \div (x + 1) = x - 1$

You Try It 3

Simplify: $(x^3 - 2x + 1) \div (x - 1)$

Your solution

Solutions on p. S9

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4.5 Exercises

Objective A

Simplify.

1. $\dfrac{10a - 25}{5}$

2. $\dfrac{16b - 40}{8}$

3. $\dfrac{6y^2 + 4y}{y}$

4. $\dfrac{4b^3 - 3b}{b}$

5. $\dfrac{3x^2 - 6x}{3x}$

6. $\dfrac{10y^2 - 6y}{2y}$

7. $\dfrac{5x^2 - 10x}{-5x}$

8. $\dfrac{3y^2 - 27y}{-3y}$

9. $\dfrac{x^3 + 3x^2 - 5x}{x}$

10. $\dfrac{a^3 - 5a^2 + 7a}{a}$

11. $\dfrac{x^6 - 3x^4 - x^2}{x^2}$

12. $\dfrac{a^8 - 5a^5 - 3a^3}{a^2}$

13. $\dfrac{5x^2y^2 + 10xy}{5xy}$

14. $\dfrac{8x^2y^2 - 24xy}{8xy}$

15. $\dfrac{9y^6 - 15y^3}{-3y^3}$

16. $\dfrac{4x^4 - 6x^2}{-2x^2}$

17. $\dfrac{3x^2 - 2x + 1}{x}$

18. $\dfrac{8y^2 + 2y - 3}{y}$

19. $\dfrac{-3x^2 + 7x - 6}{x}$

20. $\dfrac{2y^2 - 6y + 9}{y}$

21. $\dfrac{16a^2b - 20ab + 24ab^2}{4ab}$

22. $\dfrac{22a^2b - 11ab - 33ab^2}{11ab}$

23. $\dfrac{9x^2y + 6xy - 3xy^2}{xy}$

24. $\dfrac{5a^2b - 15ab + 30ab^2}{5ab}$

Objective B

Simplify.

25. $(b^2 - 14b + 49) \div (b - 7)$

26. $(x^2 - x - 6) \div (x - 3)$

27. $(y^2 + 2y - 35) \div (y + 7)$

28. $(2x^2 + 5x + 2) \div (x + 2)$

29. $(2y^2 - 13y + 21) \div (y - 3)$

30. $(4x^2 - 16) \div (2x + 4)$

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31. $(2y^2 + 7) \div (y - 3)$

32. $(x^2 + 1) \div (x - 1)$

33. $(x^2 + 4) \div (x + 2)$

34. $(6x^2 - 7x) \div (3x - 2)$

35. $(6y^2 + 2y) \div (2y + 4)$

36. $(5x^2 + 7x) \div (x - 1)$

37. $(6x^2 - 5) \div (x + 2)$

38. $(a^2 + 5a + 10) \div (a + 2)$

39. $(b^2 - 8b - 9) \div (b - 3)$

40. $(2y^2 - 9y + 8) \div (2y + 3)$

41. $(3x^2 + 5x - 4) \div (x - 4)$

42. $(8x + 3 + 4x^2) \div (2x - 1)$

43. $(10 + 21y + 10y^2) \div (2y + 3)$

44. $(15a^2 - 8a - 8) \div (3a + 2)$

45. $(12a^2 - 25a - 7) \div (3a - 7)$

46. $(5 - 23x + 12x^2) \div (4x - 1)$

47. $(24 + 6a^2 + 25a) \div (3a - 1)$

48. $(x^3 + 3x^2 + 5x + 3) \div (x + 1)$

49. $(x^3 - 6x^2 + 7x - 2) \div (x - 1)$

50. $(x^4 - x^2 - 6) \div (x^2 + 2)$

51. $(x^4 + 3x^2 - 10) \div (x^2 - 2)$

APPLYING THE CONCEPTS

52. In your own words, explain how to divide exponential expressions.



53. The product of a monomial and $4b$ is $12ab^2$. Find the monomial.

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Focus on Problem Solving

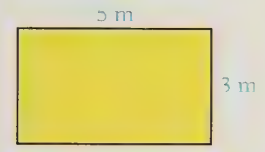
Dimensional Analysis

In solving application problems, it may be useful to include the units in order to organize the problem so that the answer is in the proper units. Using units to organize and check the correctness of an application is called **dimensional analysis.** We use the operations of multiplying units and dividing units in applying dimensional analysis to application problems.

The Rule for Multiplying Exponential Expressions states that we multiply two expressions with the same base by adding the exponents.

$$x^4 \cdot x^6 = x^{4+6} =$$

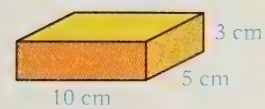
In calculations that involve quantities, the units are operated on algebraically.



➡ A rectangle measures 3 m by 5 m. Find the area of the rectangle.

$A = LW = (3 \text{ m})(5 \text{ m}) = (3 \cdot 5)(\text{m} \cdot \text{m}) = 15 \text{ m}^2$

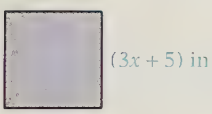
The area of the rectangle is 15 m^2 (square meters).



➡ A box measures 10 cm by 5 cm by 3 cm. Find the volume of the box.

$V = LWH = (10 \text{ cm})(5 \text{ cm})(3 \text{ cm}) = (10 \cdot 5 \cdot 3)(\text{cm} \cdot \text{cm} \cdot \text{cm}) = 150 \text{ cm}^3$

The volume of the box is 150 cm^3 (cubic centimeters).



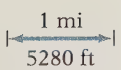
➡ Find the area of a square whose side measures $(3x + 5)$ in.

$A = s^2 = [(3x + 5) \text{ in.}]^2 = (3x + 5)^2 \text{ in}^2 = (9x^2 + 30x + 25) \text{ in}^2$

The area of the square is $(9x^2 + 30x + 25)$ in^2 (square inches).

Dimensional analysis is used in the conversion of units.

The following example converts the unit miles to feet. The equivalent measures 1 mi = 5280 ft are used to form the following rates, which are called conversion factors: $\frac{1 \text{ mi}}{5280 \text{ ft}}$ and $\frac{5280 \text{ ft}}{1 \text{ mi}}$. Because 1 mi = 5280 ft, both of the conversion factors $\frac{1 \text{ mi}}{5280 \text{ ft}}$ and $\frac{5280 \text{ ft}}{1 \text{ mi}}$ are equal to 1.



To convert 3 mi to feet, multiply 3 mi by the conversion factor $\frac{5280 \text{ ft}}{1 \text{ mi}}$.



$$3 \text{ mi} = 3 \text{ mi} \cdot 1 = \frac{3 \text{ mi}}{1} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = \frac{3 \cancel{\text{mi}} \cdot 5280 \text{ ft}}{1 \cancel{\text{mi}}} = 3 \cdot 5280 \text{ ft} = 15{,}840 \text{ ft}$$

There are two important points in the above illustration. First, you can think of dividing the numerator and denominator by the common unit "mile" just as you would divide the numerator and denominator of a fraction by a common factor. Second, the conversion factor $\frac{5280 \text{ ft}}{1 \text{ mi}}$ is equal to 1, and multiplying an expression by 1 does not change the value of the expression.

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In the application problem that follows, the units are kept in the problem while the problem is worked.

In 1980, a horse named Fiddle Isle ran a 1.5-mile race in 2.38 min. Find Fiddle Isle's average speed for that race in miles per hour. Round to the nearest tenth.

Strategy To find the average speed, use the formula $r = \frac{d}{t}$, where r is the speed, d is the distance, and t is the time. Use the conversion factor $\frac{60 \text{ min}}{1 \text{ h}}$.

Solution

$$r = \frac{d}{t} = \frac{1.5 \text{ mi}}{2.38 \text{ min}} = \frac{1.5 \text{ mi}}{2.38 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}}$$

$$= \frac{90 \text{ mi}}{2.38 \text{ h}} \approx 37.8 \text{ mph}$$

Fiddle Isle's average speed was 37.8 mph.

Try each of the following problems. Round to the nearest tenth.

1. Convert 88 ft/s to miles per hour.

2. Convert 8 m/s to kilometers per hour (1 km = 1000 m).

3. A carpet is to be placed in a meeting hall that is 36 ft wide and 80 ft long. At \$21.50 per square yard, how much will it cost to carpet the meeting hall?

4. A carpet is to be placed in a room that is 20 ft wide and 30 ft long. At \$22.25 per square yard, how much will it cost to carpet the area?

5. Find the number of gallons of water in a fish tank that is 36 in. long and 24 in. wide and is filled to a depth of 16 in. (1 gal = 231 in^3).

6. Find the number of gallons of water in a fish tank that is 24 in. long and 18 in. wide and is filled to a depth of 12 in. (1 gal = 231 in^3).

7. A $\frac{1}{4}$-acre commercial lot is on sale for \$2.15 per square foot. Find the sale price of the commercial lot (1 acre = 43,560 ft^2).

8. A 0.75-acre industrial parcel was sold for \$98,010. Find the parcel's price per square foot (1 acre = 43,560 ft^2).

9. A new driveway will require 800 ft^3 of concrete. Concrete is ordered by the cubic yard. How much concrete should be ordered?

10. A piston-engined dragster traveled 440 yd in 4.936 s at Ennis, Texas, on October 9, 1988. Find the average speed of the dragster in miles per hour.

11. The Marianas Trench in the Pacific Ocean is the deepest part of the ocean. The depth is 6.85 mi. The speed of sound under water is 4700 ft/s. Find the time it takes sound to travel from the surface to the bottom of the Marianas Trench and back.

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Projects and Group Activities

Pascal's Triangle

Simplifying the power of a binomial is called *expanding the binomial*. The expansion of the first three powers of a binomial is shown below.

$$(a + b)^1 = a + b$$

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a + b)^3 = (a + b)^2(a + b) = (a^2 + 2ab + b^2)(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$$

POINT OF INTEREST

Pascal did not invent the triangle of numbers known as Pascal's Triangle. It was known to mathematicians in China probably as early as 1050 A.D. But Pascal's *Traite du triangle arithmetique* (*Treatise Concerning the Arithmetical Triangle*) brought together all the different aspects of the numbers for the first time.

Find $(a + b)^4$. [*Hint*: $(a + b)^4 = (a + b)^3(a + b)$]

Find $(a + b)^5$. [*Hint*: $(a + b)^5 = (a + b)^4(a + b)$]

If we continue in this way, the results for $(a + b)^6$ are

$$(a + b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

Now expand $(a + b)^8$. Before you begin, see if you can find a pattern that will assist in writing the expansion of $(a + b)^8$ without having to multiply it out. Here are some hints.

1. Write out the variable terms of each binomial expansion from $(a + b)^1$ through $(a + b)^6$. Observe how the exponents on the variables change.
2. Write out the coefficients of all the terms without the variable parts. It will be helpful to make a triangular arrangement as shown at the left. Note that each row begins and ends with a 1. Also note in the two shaded regions that any number in a row is the sum of the two closest numbers above it. For instance, $1 + 5 = 6$ and $6 + 4 = 10$.

```
          1     1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
1   6  15  20  15   6   1
```

The triangle of numbers shown at the left is called Pascal's Triangle. To find the expansion of $(a + b)^8$, you need to find the eighth row of Pascal's Triangle. First find row seven. Then find row eight and use the patterns you have observed to write the expansion $(a + b)^8$.

Pascal's Triangle has been the subject of extensive analysis, and many patterns have been found. See if you can find some of them.

Chapter Summary

Key Words

A *monomial* is a number, a variable, or a product of a number and variables.

A *polynomial* is a variable expression in which the terms are monomials. A polynomial of one term is a *monomial*. A polynomial of two terms is a *binomial*. A polynomial of three terms is a *trinomial*.

The *degree of a polynomial* in one variable is the greatest of the degrees of any of its terms.

The *opposite of a polynomial* is the polynomial with the sign of every term changed.

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A number written in *scientific notation* is a number written in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Essential Rules

Rule for Multiplying Exponential Expressions

$$x^m \cdot x^n = x^{m+n}$$

Rule for Simplifying the Power of an Exponential Expression

$$(x^m)^n = x^{mn}$$

Rule for Simplifying Powers of Products

$$(x^m y^n)^p = x^{mp} y^{np}$$

Definition of Zero as an Exponent

For $x \neq 0$, $x^0 = 1$.

Definition of a Negative Exponent

For $x \neq 0$, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

Rule for Simplifying Powers of Quotients

For $y \neq 0$, $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

Rule for Negative Exponents on Fractional Expressions

For $a \neq 0, b \neq 0$, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

Rule for Dividing Exponential Expressions

For $x \neq 0$, $\frac{x^m}{x^n} = x^{m-n}$.

The Sum and Difference of Two Terms

$$(a + b)(a - b) = a^2 - b^2$$

The Square of a Binomial

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$

Addition of Polynomials

To add polynomials, add the coefficients of the like terms.

Subtraction of Polynomials

To subtract two polynomials, add the opposite of the second polynomial to the first.

The FOIL Method

Add the products of the **F**irst terms, the **O**uter terms, the **I**nner terms, and the **L**ast terms.

Scientific Notation

To express a number in scientific notation, write it in the form $a \times 10^n$, where a is a number between 1 and 10, and n is an integer. If the number is greater than 10, the exponent on 10 will be positive. If the number is less than 1, the exponent on 10 will be negative.

$367{,}000{,}000 = 3.67 \times 10^8$

$0.0000059 = 5.9 \times 10^{-6}$

To change a number written in scientific notation to decimal notation, move the decimal point to the right if the exponent on 10 is positive and to the left if the exponent on 10 is negative. Move the decimal point the same number of places as the absolute value of the exponent on 10.

$2.418 \times 10^7 = 24{,}180{,}000$

$9.06 \times 10^{-5} = 0.0000906$

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Chapter Review

1. Simplify: $(2b - 3)(4b + 5)$

2. Simplify: $(12y^2 + 17y - 4) + (9y^2 - 13y + 3)$

3. Simplify: $(b^3 - 2b^2 - 33b - 7) \div (b - 7)$

4. Simplify: $(xy^5z^3)(x^3y^3z)$

5. Simplify: $-2x(4x^2 + 7x - 9)$

6. Simplify: $\frac{(4a^{-2}b^{-3})^2}{(2a^{-1}b^{-2})^4}$

7. Simplify: $(2^3)^2$

8. Simplify: $(5x^2 - 2x - 1) - (3x^2 - 5x + 7)$

9. Simplify: $\frac{8x^{12}}{12x^9}$

10. Simplify: $(5y - 7)^2$

11. Simplify: $(5a^7b^6)^2(4ab)$

12. Simplify: $\frac{12b^7 + 36b^5 - 3b^3}{3b^3}$

13. Simplify: $(13y^3 - 7y - 2) - (12y^2 - 2y - 1)$

14. Simplify: $(5a^2 + 6a - 11) + (5a^2 + 6a - 11)$

15. Simplify: $(3y^2 + 4y - 7)(2y + 3)$

16. Simplify: $\frac{-18a^6b}{27a^3b^4}$

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17. Simplify: $2ab^3(4a^2 - 2ab + 3b^2)$

18. Simplify: $(6y^2 + 2y + 7) - (8y^2 + y + 12)$

19. Simplify: $(-3x^2y^3)^2$

20. Simplify: $(6b^3 - 2b^2 - 5)(2b^2 - 1)$

21. Simplify: $(2x^3 + 7x^2 + x) + (2x^2 - 4x - 12)$

22. Simplify: $\frac{16y^2 - 32y}{-4y}$

23. Simplify: $(a + 7)(a - 7)$

24. Simplify: $(2a^{12}b^3)(-9b^2c^6)(3ac)$

25. Simplify: $(6y^2 - 35y + 36) \div (3y - 4)$

26. Simplify: $(-3x^{-2}y^{-3})^{-2}$

27. Simplify: $(5xy^2)(-4x^2y^3)$

28. Simplify: $(5a - 7)(2a + 9)$

29. Write 0.000000127 in scientific notation.

30. Write 3.2×10^{-12} in decimal notation.

31. The length of a Ping-Pong table is 1 ft less than twice the width of the table. Let w represent the width of the Ping-Pong table. Express the area of the table in terms of the variable w.

32. The side of a checkerboard is $(3x - 2)$ in. Express the area of the checkerboard in terms of the variable x.

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Chapter Test

1. Simplify: $2x(2x^2 - 3x)$

2. Simplify: $\frac{12x^3 - 3x^2 + 9}{3x^2}$

3. Simplify: $\frac{12x^2}{-3x^8}$

4. Simplify: $(-2xy^2)(3x^2y^4)$

5. Simplify: $(x^2 + 1) \div (x + 1)$

6. Simplify: $(x - 3)(x^2 - 4x + 5)$

7. Simplify: $(-2a^2b)^3$

8. Simplify: $\frac{(3x^{-2}y^3)^3}{3x^4y^{-1}}$

9. Simplify: $(a - 2b)(a + 5b)$

10. Simplify: $\frac{16x^5 - 8x^3 + 20x}{4x}$

11. Simplify: $(x^2 + 6x - 7) \div (x - 1)$

12. Simplify: $-3y^2(-2y^2 + 3y - 6)$

13. Simplify: $(-2x^3 + x^2 - 7)(2x - 3)$

14. Simplify: $(4y - 3)(4y + 3)$

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15. Simplify: $(ab^2)(a^3b^5)$

16. Simplify: $\frac{2a^{-1}b}{2^{-2}a\ ^2b^{-3}}$

17. Simplify: $\frac{20a - 35}{5}$

18. Simplify: $(3a^2 - 2a - 7) - (5a^3 + 2a - 10)$

19. Simplify: $(2x - 5)^2$

20. Simplify: $(4x^2 - 7) \div (2x - 3)$

21. Simplify: $\frac{-(2x^2y)^3}{4x^3y^3}$

22. Simplify: $(2x - 7y)(5x - 4y)$

23. Simplify: $(3x^3 - 2x^2 - 4) + (8x^2 - 8x + 7)$

24. Write 0.00000000302 in scientific notation.

25. The radius of a circle is $(x - 5)$ m. Use the equation $A = \pi r^2$, where r is the radius, to find the area of the circle in terms of the variable x. Leave the answer in terms of π.

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Cumulative Review

1. Simplify: $\frac{3}{16} - \left(-\frac{5}{8}\right) - \frac{7}{9}$

2. Evaluate $-3^2 \cdot \left(\frac{2}{3}\right)^3 \cdot \left(-\frac{5}{8}\right)$.

3. Simplify: $\left(-\frac{1}{2}\right)^3 \div \left(\frac{3}{8} - \frac{5}{6}\right) + 2$

4. Evaluate $\frac{b - (a - b)^2}{b^2}$ when $a = -2$ and $b = 3$.

5. Simplify: $-2x - (-xy) + 7x - 4xy$

6. Simplify: $(12x)\left(-\frac{3}{4}\right)$

7. Simplify: $-2[3x - 2(4 - 3x) + 2]$

8. Solve: $12 = -\frac{3}{4}x$

9. Solve: $2x - 9 = 3x + 7$

10. Solve: $2 - 3(4 - x) = 2x + 5$

11. 35.2 is what percent of 160?

12. Simplify: $(4b^3 - 7b^2 - 7) + (3b^2 - 8b + 3)$

13. Simplify: $(3y^3 - 5y + 8) - (-2y^2 + 5y + 8)$

14. Simplify: $(a^3b^5)^3$

15. Simplify: $(4xy^3)(-2x^2y^3)$

16. Simplify: $-2y^2(-3y^2 - 4y + 8)$

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17. Simplify: $(2a - 7)(5a^2 - 2a + 3)$

18. Simplify: $(3b - 2)(5b - 7)$

19. Simplify: $\frac{(-2a^2b^3)^2}{8a^4b^8}$

20. Simplify: $(a^2 - 4a - 21) \div (a + 3)$

21. Write 6.09×10^{-5} in decimal notation.

22. Translate "the difference between eight times a number and twice the number is eighteen" into an equation and solve.

23. Fifty ounces of orange juice are added to 200 oz of a fruit punch that is 10% orange juice. What is the percent concentration of orange juice in the resulting mixture?

24. A car traveling at 50 mph overtakes a cyclist who, riding at 10 mph, has had a two hour head start. How far from the starting point does the car overtake the cyclist?

25. The width of a rectangle is 40% of the length. The perimeter of the rectangle is 42 m. Find the length and width of the rectangle.

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CHAPTER

5 Factoring



Technical writers translate technological information into easily understood language for use in training manuals, public relations brochures, and the like. The computer industry uses software technical writers to explain the programs that computers run. Generally these technical writers need to learn from programmers how things work before they write the descriptions. It is common for software technical writers to have a liberal arts degree with some courses in mathematics and science, which enable them to better understand technical instructions.

Objectives

Section 5.1

To factor a monomial from a polynomial

To factor by grouping

Section 5.2

To factor a trinomial of the form $x^2 + bx + c$

To factor completely

Section 5.3

To factor a trinomial of the form $ax^2 + bx + c$ by using trial factors

To factor a trinomial of the form $ax^2 + bx + c$ by grouping

Section 5.4

To factor the difference of two squares and perfect-square trinomials

To factor completely

Section 5.5

To solve equations by factoring

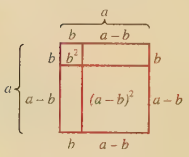
To solve application problems

Algebra from Geometry

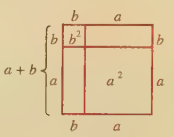
The early Babylonians made substantial progress in both algebra and geometry. Often the progress they made in algebra was based on geometric concepts.

Here are some geometric proofs of algebraic identities the Babylonians understood.

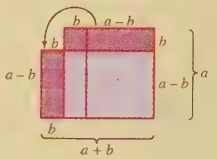




$(a-b)^2 = a^2 - 2ab + b^2$



$(a+b)^2 = a^2 + 2ab + b^2$



$(a-b)(a+b) = a^2 - b^2$

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5.1 Common Factors

Objective A To factor a monomial from a polynomial

The **greatest common factor (GCF)** of two or more monomials is the product of the GCF of the coefficients and the common variable factors.

$6x^3y = 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y$
$8x^2y^2 = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y$
$\text{GCF} = 2 \cdot x \cdot x \cdot y = 2x^2y$

Note that the exponent of each variable in the GCF is the same as the *smallest* exponent of that variable in either of the monomials.

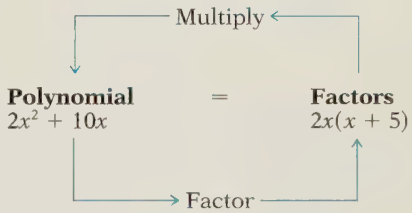
The GCF of $6x^3y$ and $8x^2y^2$ is $2x^2y$.

➡ Find the GCF of $12a^4b$ and $18a^2b^2c$.

The common variable factors are a^2 and b. c is not a common variable factor.

$12a^4b = 2 \cdot 2 \cdot 3 \cdot a^4 \cdot b$
$18a^2b^2c = 2 \cdot 3 \cdot 3 \cdot a^2 \cdot b^2 \cdot c$
$\text{GCF} = 2 \cdot 3 \cdot a^2 \cdot b = 6a^2b$

To **factor** a polynomial means to write the polynomial as a product of other polynomials.



In the example above, $2x$ is the GCF of the terms $2x^2$ and $10x$. It is a **common factor** of the terms.

➡ Factor: $5x^3 - 35x^2 + 10x$

Find the GCF of the terms of the polynomial.

$5x^3 = 5 \cdot x^3$
$35x^2 = 5 \cdot 7 \cdot x^2$
$10x = 2 \cdot 5 \cdot x$

The GCF is $5x$.

Rewrite the polynomial, expressing each term as a product with the GCF as one of the factors.

$5x^3 - 35x^2 + 10x = 5x(x^2) + 5x(-7x) + 5x(2)$
$= 5x(x^2 - 7x + 2)$

• Use the Distributive Property to write the polynomial as a product of factors.

TAKE NOTE

At the right, the factors in parentheses are determined by dividing each term of the trinomial by the GCF, $5x$.

$\frac{5x^3}{5x} = x^2$,
$\frac{-35x^2}{5x} = -7x$, and
$\frac{10x}{5x} = 2$

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Factor: $21x^2y^3 - 6xy^5 + 15x^4y^2$

Find the GCF of the terms of the polynomial.

$21x^2y^3 = 3 \cdot 7 \cdot x^2 \cdot y^3$

$6xy^5 = 2 \cdot 3 \cdot x \cdot y^5$

$15x^4y^2 = 3 \cdot 5 \cdot x^4 \cdot y^2$

The GCF is $3xy^2$.

Rewrite the polynomial, expressing each term as a product with the GCF as one of the factors.

$$21x^2y^3 - 6xy^5 + 15x^4y^2 = 3xy^2(7xy) + 3xy^2(-2y^3) + 3xy^2(5x^3)$$
$$= 3xy^2(7xy - 2y^3 + 5x^3)$$

• Use the Distributive Property to write the polynomial as a product of factors.

Example 1

Factor: $8x^2 + 2xy$

Solution

The GCF is $2x$.

$8x^2 + 2xy = 2x(4x) + 2x(y) = 2x(4x + y)$

You Try It 1

Factor: $14a^2 - 21a^4b$

Your solution

Example 2

Factor: $n^3 - 5n^2 + 2n$

Solution

The GCF is n.

$$n^3 - 5n^2 + 2n = n(n^2) + n(-5n) + n(2)$$
$$= n(n^2 - 5n + 2)$$

You Try It 2

Factor: $27b^2 + 18b + 9$

Your solution

Example 3

Factor: $16x^2y + 8x^4y^2 - 12x^4y^5$

Solution

The GCF is $4x^2y$.

$$16x^2y + 8x^4y^2 - 12x^4y^5$$
$$= 4x^2y(4) + 4x^2y(2x^2y) + 4x^2y(-3x^2y^4)$$
$$= 4x^2y(4 + 2x^2y - 3x^2y^4)$$

You Try It 3

Factor: $6x^4y^2 - 9x^3y^2 + 12x^2y^4$

Your solution

Solutions on p. S10

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Objective B To factor by grouping

In the examples at the right, the binomials in parentheses are called **binomial factors.**

$2a(a + b)^2$
$3xy(x - y)$

The Distributive Property is used to factor a common binomial factor from an expression.

The common binomial factor of the expression $6x(x - 3) + y^2(x - 3)$ is $(x - 3)$. To factor that expression, use the Distributive Property to write the expression as a product of factors.

$$6x(x - 3) + y^2(x - 3) = (x - 3)(6x + y^2)$$

Consider the following simplification of $-(a - b)$.

$$-(a - b) = -1(a - b) = -a + b = b - a$$

Thus, $b - a = -(a - b)$

This equation is sometimes used to factor a common binomial from an expression.

➡ Factor: $2x(x - y) + 5(y - x)$

$$2x(x - y) + 5(y - x) = 2x(x - y) - 5(x - y)$$
$$= (x - y)(2x - 5)$$

• $5(y - x) = 5[(-1)(x - y)]$
$= -5(x - y)$

Some polynomials can be factored by grouping terms in such a way that a common binomial factor is found.

➡ Factor: $ax + bx - ay - by$

$$ax + bx - ay - by = (ax + bx) - (ay + by)$$

• Group the first two terms and the last two terms. Note that $-ay - by = -(ay + by)$.

$$= x(a + b) - y(a + b)$$
$$= (a + b)(x - y)$$

• Factor the GCF, $(a + b)$, from each group.

➡ Factor: $6x^2 - 9x - 4xy + 6y$

$$6x^2 - 9x - 4xy + 6y = (6x^2 - 9x) - (4xy - 6y)$$

• Group the first two terms and the last two terms. Note that $-4xy + 6y = -(4xy - 6y)$.

$$= 3x(2x - 3) - 2y(2x - 3)$$
$$= (2x - 3)(3x - 2y)$$

• Factor the GCF, $(2x - 3)$, from each group.

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Example 4
Factor: $4x(3x - 2) - 7(3x - 2)$

Solution
$4x(3x - 2) - 7(3x - 2)$

$= (3x - 2)(4x - 7)$

You Try It 4
Factor: $2y(5x - 2) - 3(2 - 5x)$

Your solution

Example 5
Factor: $9x^2 - 15x - 6xy + 10y$

Solution
$9x^2 - 15x - 6xy + 10y$

$= (9x^2 - 15x) - (6xy - 10y)$

$= 3x(3x - 5) - 2y(3x - 5)$

$= (3x - 5)(3x - 2y)$

You Try It 5
Factor: $a^2 - 3a + 2ab - 6b$

Your solution

Example 6
Factor: $3x^2y - 4x - 15xy + 20$

Solution
$3x^2y - 4x - 15xy + 20$

$= (3x^2y - 4x) - (15xy - 20)$

$= x(3xy - 4) - 5(3xy - 4)$

$= (3xy - 4)(x - 5)$

You Try It 6
Factor: $2mn^2 - n + 8mn - 4$

Your solution

Example 7
Factor: $4ab - 6 + 3b - 2ab^2$

Solution
$4ab - 6 + 3b - 2ab^2$

$= (4ab - 6) + (3b - 2ab^2)$

$= 2(2ab - 3) + b(3 - 2ab)$

$= 2(2ab - 3) - b(2ab - 3)$

$= (2ab - 3)(2 - b)$

You Try It 7
Factor: $2xy - 6y - 12 + 4x$

Your solution

Solutions on p. S10

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5.1 Exercises

Objective A

Factor.

1. $5a + 5$ **2.** $7b - 7$ **3.** $16 - 8a^2$ **4.** $12 + 12y^2$ **5.** $8x + 12$

6. $16a - 24$ **7.** $30a - 6$ **8.** $20b + 5$ **9.** $7x^2 - 3x$ **10.** $12y^2 - 5y$

11. $3a^2 + 5a^5$ **12.** $9x - 5x^2$ **13.** $14y^2 + 11y$ **14.** $6b^3 - 5b^2$ **15.** $2x^4 - 4x$

16. $3y^4 - 9y$ **17.** $10x^4 - 12x^2$ **18.** $12a^5 - 32a^2$ **19.** $8a^8 - 4a^5$ **20.** $16y^4 - 8y^7$

21. $x^2y^2 - xy$ **22.** $a^2b^2 + ab$ **23.** $3x^2y^4 - 6xy$ **24.** $12a^2b^5 - 9ab$ **25.** $x^2y - xy^3$

26. $3x^3 + 6x^2 + 9x$ **27.** $5y^3 - 20y^2 + 10y$ **28.** $2x^4 - 4x^3 + 6x^2$ **29.** $3y^4 - 9y^3 - 6y^2$

30. $2x^3 + 6x^2 - 14x$ **31.** $3y^3 - 9y^2 + 24y$ **32.** $2y^5 - 3y^4 + 7y^3$ **33.** $6a^5 - 3a^3 - 2a^2$

34. $x^3y - 3x^2y^2 + 7xy^3$ **35.** $2a^2b - 5a^2b^2 + 7ab^2$ **36.** $5y^3 + 10y^2 - 25y$

37. $4b^5 + 6b^3 - 12b$ **38.** $3a^2b^2 - 9ab^2 + 15b^2$ **39.** $8x^2y^2 - 4x^2y + x^2$

Objective B

Factor.

40. $x(b + 4) + 3(b + 4)$ **41.** $y(a + z) + 7(a + z)$ **42.** $a(y - x) - b(y - x)$

43. $3r(a - b) + s(a - b)$ **44.** $x(x - 2) + y(2 - x)$ **45.** $t(m - 7) + 7(7 - m)$

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46. $2x(7 + b) - y(b + 7)$

47. $2y(4a - b) - (b - 4a)$

48. $8c(2m - 3n) + (3n - 2m)$

49. $x^2 + 2x + 2xy + 4y$

50. $x^2 - 3x + 4ax - 12a$

51. $p^2 - 2p - 3rp + 6r$

52. $t^2 + 4t - st - 4s$

53. $ab + 6b - 4a - 24$

54. $xy - 5y - 2x + 10$

55. $2z^2 - z + 2yz - y$

56. $2y^2 - 10y + 7xy - 35x$

57. $8v^2 - 12vy + 14v - 21y$

58. $21x^2 + 6xy - 49x - 14y$

59. $2x^2 - 5x - 6xy + 15y$

60. $4a^2 + 5ab - 10b - 8a$

61. $3y^2 - 6y - ay + 2a$

62. $2ra + a^2 - 2r - a$

63. $3xy - y^2 - y + 3x$

64. $2ab - 3b^2 - 3b + 2a$

65. $3st + t^2 - 2t - 6s$

66. $4x^2 + 3xy - 12y - 16x$

APPLYING THE CONCEPTS

67. Factor: **a.** $2x^2 + 6x + 5x + 15$ **b.** $2x^2 + 5x + 6x + 15$

68. Look at parts a and b of Exercise 67. Do different groupings of the terms in a polynomial affect the binomial factors?

A natural number is a perfect number if it is the sum of all its factors less than itself. For example, 6 is a perfect number because all the factors of 6 that are less than 6 are 1, 2, and 3, and $1 + 2 + 3 = 6$.

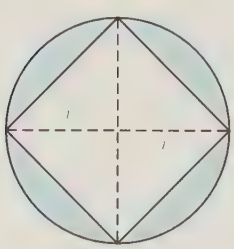
69. Find the one perfect number between 20 and 30.

70. Find the one perfect number between 490 and 500.

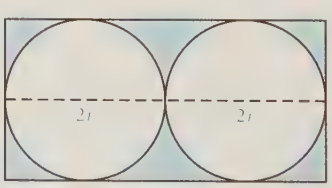
71. In the equation $P = 2L + 2W$, what is the effect on P when the quantity $L + W$ doubles?

72. Write an expression in factored form for each of the shaded portions in the following diagrams. (Use the equation for the area of a rectangle $A = bh$ and the equation for the area of a circle $A = \pi r^2$.)

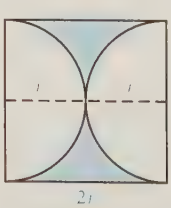
a.



b.



c.



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5.2 Factoring Polynomials of the Form $x^2 + bx + c$

Objective A To factor a trinomial of the form $x^2 + bx + c$

Trinomials of the form $x^2 + bx + c$, where b and c are integers, are shown at the right.

$x^2 + 8x + 12;\ b = 8,\ c = 12$
$x^2 - 7x + 12;\ b = -7,\ c = 12$
$x^2 - 2x - 15;\ b = -2,\ c = -15$

To factor a trinomial of this form means to express the trinomial as the product of two binomials.

Trinomials expressed as the product of binomials are shown at the right.

$x^2 + 8x + 12 = (x + 6)(x + 2)$
$x^2 - 7x + 12 = (x - 3)(x - 4)$
$x^2 - 2x - 15 = (x + 3)(x - 5)$

The method by which factors of a trinomial are found is based on FOIL. Consider the following binomial products, noting the relationship between the constant terms of the binomials and the terms of the trinomials.

Signs in the binomials are the same.

$(x + 6)(x + 2) = x^2 + 2x + 6x + (6)(2) = x^2 + 8x + 12$
Sum of 6 and 2
Product of 6 and 2

$(x - 3)(x - 4) = x^2 - 4x - 3x + (-3)(-4) = x^2 - 7x + 12$
Sum of -3 and -4
Product of -3 and -4

Signs in the binomials are opposite.

$(x + 3)(x - 5) = x^2 - 5x + 3x + (3)(-5) = x^2 - 2x - 15$
Sum of 3 and -5
Product of 3 and -5

$(x - 4)(x + 6) = x^2 + 6x - 4x + (-4)(6) = x^2 + 2x - 24$
Sum of -4 and 6
Product of -4 and 6

IMPORTANT RELATIONSHIPS

1. When the constant term of the trinomial is positive, the constant terms of the binomials have the same sign. They are both positive when the coefficient of the x term in the trinomial is positive. They are both negative when the coefficient of the x term in the trinomial is negative.

2. When the constant term of the trinomial is negative, the constant terms of the binomials have opposite signs.

3. In the trinomial, the coefficient of x is the sum of the constant terms of the binomials.

4. In the trinomial, the constant term is the product of the constant terms of the binomials.

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➡ Factor: $x^2 - 7x + 10$

Because the constant term is positive and the coefficient of x is negative, the binomial constants will be negative. Find two negative factors of 10 whose sum is -7. The results can be recorded in a table.

Negative Factors of 10	*Sum*
$-1, -10$	-11
$-2, -5$	-7

• These are the correct factors.

$x^2 - 7x + 10 = (x - 2)(x - 5)$ • Write the trinomial as a product of its factors.

You can check the proposed factorization by multiplying the two binomials.

TAKE NOTE

Always check your proposed factorization to ensure accuracy.

Check: $(x - 2)(x - 5) = x^2 - 5x - 2x + 10$
$= x^2 - 7x + 10$

➡ Factor: $x^2 - 9x - 36$

The constant term is negative. The binomial constants will have opposite signs. Find two factors of -36 whose sum is -9.

Factors of -36	*Sum*
$+1, -36$	-35
$-1, +36$	35
$+2, -18$	-16
$-2, +18$	16
$+3, -12$	-9

• Once the correct factors are found, it is not necessary to try the remaining factors.

$x^2 - 9x - 36 = (x + 3)(x - 12)$ • Write the trinomial as a product of its factors.

➡ Factor: $x^2 + 7x + 8$

Because the constant term is positive and the coefficient of x is positive, the binomial constants will be positive. Find two positive factors of 8 whose sum is 7.

Positive Factors of 8	*Sum*
$1, 8$	9
$2, 4$	6

There are no positive integer factors of 8 whose sum is 7. The trinomial $x^2 + 7x + 8$ is said to be **nonfactorable over the integers.**

Just as 17 is a prime number, $x^2 + 7x + 8$ is a **prime polynomial.** Binomials of the form $x - a$ and $x + a$ are also prime polynomials.

Example 1
Factor: $x^2 - 8x + 15$

Solution
Find two negative factors of 15 whose sum is -8.

Factors	*Sum*
$-1, -15$	-16
$-3, -5$	-8

$x^2 - 8x + 15 = (x - 3)(x - 5)$

You Try It 1
Factor: $x^2 + 9x + 20$

Your solution

Solution on p. S10

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Example 2
Factor: $x^2 + 6x - 27$

Solution
Find two factors of -27 whose sum is 6.

Factors	*Sum*
$+1, -27$	-26
$-1, +27$	26
$+3, -9$	-6
$-3, +9$	6

$x^2 + 6x - 27 = (x - 3)(x + 9)$

You Try It 2
Factor: $x^2 + 7x - 18$

Your solution

Solution on p. S10

Objective B To factor completely

16 CT

A polynomial is factored completely when it is written as a product of factors that are nonfactorable over the integers.

TAKE NOTE

The first step in *any* factoring problem is to determine whether the terms of the polynomial have a **common factor.** If they do, factor it out first.

➡ Factor: $4y^3 - 4y^2 - 24y$

$4y^3 - 4y^2 - 24y = 4y(y^2) - 4y(y) - 4y(6)$ • The GCF is $4y$.

$= 4y(y^2 - y - 6)$ • Use the Distributive Property to factor out the GCF.

$= 4y(y + 2)(y - 3)$ • Factor $y^2 - y - 6$. The two factors of -6 whose sum is -1 are 2 and -3.

It is always possible to check the proposed factorization by multiplying the polynomials. Here is the check for the last example.

Check: $4y(y + 2)(y - 3) = 4y(y^2 - 3y + 2y - 6)$
$= 4y(y^2 - y - 6)$
$= 4y^3 - 4y^2 - 24y$ • This is the original polynomial.

➡ Factor: $5x^2 + 60xy + 100y^2$

$5x^2 + 60xy + 100y^2 = 5(x^2) + 5(12xy) + 5(20y^2)$ • The GCF is 5.

$= 5(x^2 + 12xy + 20y^2)$ • Use the Distributive Property to factor out the GCF.

$= 5(x + 2y)(x + 10y)$ • Factor $x^2 + 12xy + 20y^2$. The two factors of 20 whose sum is 12 are 2 and 10.

TAKE NOTE

$2y$ and $10y$ are placed in the binomials. This is necessary so that the middle term contains xy and the last term contains y^2.

Note that $2y$ and $10y$ were placed in the binomials. The following check shows that was necessary.

Check: $5(x + 2y)(x + 10y) = 5(x^2 + 10xy + 2xy + 20y^2)$
$= 5(x^2 + 12xy + 20y^2)$
$= 5x^2 + 60xy + 100y^2$ • This is the original polynomial.

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TAKE NOTE

When the coefficient of the highest power in a polynomial is negative, consider factoring out a negative GCF. Example 3 is another example of this technique.

Factor: $15 - 2x - x^2$

Because the coefficient of x^2 is -1, factor -1 from the trinomial and then write the resulting trinomial in descending order.

$15 - 2x - x^2 = -(x^2 + 2x - 15)$ • $15 - 2x - x^2 = -1(-15 + 2x + x^2)$ $= -(x^2 + 2x - 15)$

$= -(x + 5)(x - 3)$ • Factor $x^2 + 2x - 15$. The two factors of -15 whose sum is 2 are 5 and -3.

Check: $-(x + 5)(x - 3) = -(x^2 + 2x - 15)$

$= -x^2 - 2x + 15$

$= 15 - 2x - x^2$ • This is the original polynomial.

Example 3

Factor: $-3x^3 + 9x^2 + 12x$

Solution

The GCF is $-3x$.

$-3x^3 + 9x^2 + 12x = -3x(x^2 - 3x - 4)$

Factor the trinomial $x^2 - 3x - 4$. Find two factors of -4 whose sum is -3.

Factors	*Sum*
$-2, +2$	0
$+1, -4$	-3

$-3x^3 + 9x^2 + 12x = -3x(x + 1)(x - 4)$

You Try It 3

Factor: $-2x^3 + 14x^2 - 12x$

Your solution

Example 4

Factor: $4x^2 - 40xy + 84y^2$

Solution

The GCF is 4.

$4x^2 - 40xy + 84y^2 = 4(x^2 - 10xy + 21y^2)$

Factor the trinomial $x^2 - 10xy + 21y^2$. Find two negative factors of 21 whose sum is -10.

Factors	*Sum*
$-1, -21$	-22
$-3, -7$	-10

$4x^2 - 40xy + 84y^2 = 4(x \quad 3y)(x \quad 7y)$

You Try It 4

Factor: $3x^2 - 9xy - 12y^2$

Your solution

Solutions on p. S10

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5.2 Exercises

Objective A

Factor.

1. $x^2 + 3x + 2$ **2.** $x^2 + 5x + 6$ **3.** $x^2 - x - 2$ **4.** $x^2 + x - 6$

5. $a^2 + a - 12$ **6.** $a^2 - 2a - 35$ **7.** $a^2 - 3a + 2$ **8.** $a^2 - 5a + 4$

9. $a^2 + a - 2$ **10.** $a^2 - 2a - 3$ **11.** $b^2 - 6b + 9$ **12.** $b^2 + 8b + 16$

13. $b^2 + 7b - 8$ **14.** $y^2 - y - 6$ **15.** $y^2 + 6y - 55$ **16.** $z^2 - 4z - 45$

17. $y^2 - 5y + 6$ **18.** $y^2 - 8y + 15$ **19.** $z^2 - 14z + 45$ **20.** $z^2 - 14z + 49$

21. $z^2 - 12z - 160$ **22.** $p^2 + 2p - 35$ **23.** $p^2 + 12p + 27$ **24.** $p^2 - 6p + 8$

25. $x^2 + 20x + 100$ **26.** $x^2 + 18x + 81$ **27.** $b^2 + 9b + 20$ **28.** $b^2 + 13b + 40$

29. $x^2 - 11x - 42$ **30.** $x^2 + 9x - 70$ **31.** $b^2 - b - 20$ **32.** $b^2 + 3b - 40$

33. $y^2 - 14y - 51$ **34.** $y^2 - y - 72$ **35.** $p^2 - 4p - 21$ **36.** $p^2 + 16p + 39$

37. $y^2 - 8y + 32$ **38.** $y^2 - 9y + 81$ **39.** $x^2 - 20x + 75$ **40.** $x^2 - 12x + 11$

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41. $p^2 + 24p + 63$

42. $x^2 - 15x + 56$

43. $x^2 + 21x + 38$

44. $x^2 + x - 56$

45. $x^2 + 5x - 36$

46. $a^2 - 21a - 72$

47. $a^2 - 7a - 44$

48. $a^2 - 15a + 36$

49. $a^2 - 21a + 54$

50. $z^2 - 9z - 136$

51. $z^2 + 14z - 147$

52. $c^2 - c - 90$

53. $c^2 - 3c - 180$

54. $z^2 + 15z + 44$

55. $p^2 + 24p + 135$

56. $c^2 + 19c + 34$

57. $c^2 + 11c + 18$

58. $x^2 - 4x - 96$

59. $x^2 + 10x - 75$

60. $x^2 - 22x + 112$

61. $x^2 + 21x - 100$

62. $b^2 + 8b - 105$

63. $b^2 - 22b + 72$

64. $a^2 - 9a - 36$

65. $a^2 + 42a - 135$

66. $b^2 - 23b + 102$

67. $b^2 - 25b + 126$

68. $a^2 + 27a + 72$

69. $z^2 + 24z + 144$

70. $x^2 + 25x + 156$

71. $x^2 - 29x + 100$

72. $x^2 - 10x - 96$

73. $x^2 + 9x - 112$

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Objective B

Factor.

74. $2x^2 + 6x + 4$

75. $3x^2 + 15x + 18$

76. $18 + 7x - x^2$

77. $12 - 4x - x^2$

78. $ab^2 + 2ab - 15a$

79. $ab^2 + 7ab - 8a$

80. $xy^2 - 5xy + 6x$

81. $xy^2 + 8xy + 15x$

82. $z^3 - 7z^2 + 12z$

83. $-2a^3 - 6a^2 - 4a$

84. $-3y^3 + 15y^2 - 18y$

85. $4y^3 + 12y^2 - 72y$

86. $3x^2 + 3x - 36$

87. $2x^3 - 2x^2 + 4x$

88. $5z^2 - 15z - 140$

89. $6z^2 + 12z - 90$

90. $2a^3 + 8a^2 - 64a$

91. $3a^3 - 9a^2 - 54a$

92. $x^2 - 5xy + 6y^2$

93. $x^2 + 4xy - 21y^2$

94. $a^2 - 9ab + 20b^2$

95. $a^2 - 15ab + 50b^2$

96. $x^2 - 3xy - 28y^2$

97. $s^2 + 2st - 48t^2$

98. $y^2 - 15yz - 41z^2$

99. $y^2 + 85xy + 36z^2$

100. $z^4 - 12z^3 + 35z^2$

101. $z^4 + 2z^3 - 80z^2$

102. $b^4 - 22b^3 + 120b^2$

103. $b^4 - 3b^3 - 10b^2$

104. $2y^4 - 26y^3 - 96y^2$

105. $3y^4 + 54y^3 + 135y^2$

106. $-x^4 - 7x^3 + 8x^2$

107. $-x^4 + 11x^3 + 12x^2$

108. $4x^2y + 20xy - 56y$

109. $3x^2y - 6xy - 45y$

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110. $c^3 + 18c^2 - 40c$

111. $-3x^3 + 36x^2 - 81x$

112. $-4x^3 - 4x^2 + 24x$

113. $x^2 - 8xy + 15y^2$

114. $y^2 - 7xy - 8x^2$

115. $a^2 - 13ab + 42b^2$

116. $y^2 + 4yz - 21z^2$

117. $y^2 + 8yz + 7z^2$

118. $y^2 - 16yz + 15z^2$

119. $3x^2y + 60xy - 63y$

120. $4x^2y - 68xy - 72y$

121. $3x^3 + 3x^2 - 36x$

122. $4x^3 + 12x^2 - 160x$

123. $4z^3 + 32z^2 - 132z$

124. $5z^3 - 50z^2 - 120z$

125. $4x^3 + 8x^2 - 12x$

126. $5x^3 + 30x^2 + 40x$

127. $5p^2 + 25p - 420$

128. $4p^2 - 28p - 480$

129. $p^4 + 9p^3 - 36p^2$

130. $p^4 + p^3 - 56p^2$

131. $t^2 - 12ts + 35s^2$

132. $a^2 - 10ab + 25b^2$

133. $a^2 - 8ab - 33b^2$

APPLYING THE CONCEPTS

Factor.

134. $2 + c^2 + 9c$

135. $x^2y - 54y - 3xy$

136. $45a^2 + a^2b^2 - 14a^2b$

Find all integers k such that the trinomial can be factored over the integers.

137. $x^2 + kx + 35$

138. $x^2 + kx + 18$

139. $x^2 + kx + 21$

Determine the positive integer values of k for which the following polynomials are factorable over the integers.

140. $y^2 + 4y + k$

141. $z^2 + 7z + k$

142. $a^2 - 6a + k$

143. $c^2 - 7c + k$

144. $x^2 - 3x + k$

145. $y^2 + 5y + k$

146. In Exercises 140 to 145, there was the stated requirement that $k > 0$. If k is allowed to be any integer, how many different values of k are possible for each polynomial?

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5.3 Factoring Polynomials of the Form $ax^2 + bx + c$

Objective A To factor a trinomial of the form $ax^2 + bx + c$ by using trial factors

Trinomials of the form $ax^2 + bx + c$, where a, b, and c are integers, are shown at the right.

$3x^2 - x + 4$; $a = 3$, $b = -1$, $c = 4$
$6x^2 + 2x - 3$; $a = 6$, $b = 2$, $c = -3$

These trinomials differ from those in the previous section in that the coefficient of x^2 is not 1. There are various methods of factoring these trinomials. The method described in this objective is factoring polynomials using trial factors.

To reduce the number of trial factors that must be considered, remember the following:

1. Use the signs of the constant term and the coefficient of x in the trinomial to determine the signs of the binomial factors. If the constant term is positive, the signs of the binomial factors will be the same as the sign of the coefficient of x in the trinomial. If the sign of the constant term is negative, the constant terms in the binomials have opposite signs.
2. If the terms of the trinomial do not have a common factor, then the terms of neither of the binomial factors will have a common factor.

➡ Factor: $2x^2 - 7x + 3$

The terms have no common factor. The constant term is positive. The coefficient of x is negative. The binomial constants will be negative.

Positive Factors of 2 (coefficient of x^2)	*Negative Factors of 3* (constant term)
1, 2	$-1, -3$

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $-7x$, of the trinomial.

Trial Factors	*Middle Term*
$(x - 1)(2x - 3)$	$-3x - 2x = -5x$
$(x - 3)(2x - 1)$	$-x - 6x = -7x$

Write the factors of the trinomial. $2x^2 - 7x + 3 = (x - 3)(2x - 1)$

➡ Factor: $6x^2 - 13x + 6$

The terms have no common factor. The constant term is positive. The coefficient of x is negative. The binomial constants will be negative.

Positive Factors of 6 (coefficient of x^2)	*Negative Factors of 6* (constant term)
2, 3	$-2, -3$
1, 6	$-1, -6$

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $-13x$, of the trinomial.

Trial Factors	*Middle Term*
$(2x - 3)(3x - 2)$	$-4x - 9x = -13x$
$(x - 6)(6x - 1)$	$-x - 36x = -37x$

Write the factors of the trinomial. $6x^2 - 13x + 6 = (2x - 3)(3x - 2)$

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➡ Factor: $6x^3 + 14x^2 - 12x$.

Factor the GCF, $2x$, from the terms.

$6x^3 + 14x^2 - 12x = 2x(3x^2 + 7x - 6)$

Factor the trinomial. The constant term is negative. The binomial constants will have opposite signs.

Positive Factors of 3	*Factors of −6*
1, 3	−1, 6
	1, −6
	−2, 3
	2, −3

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $7x$, of the trinomial.

It is not necessary to test trial factors that have a common factor.

Trial Factors	*Middle Term*
$(x - 1)(3x + 6)$	Common factor
$(x + 6)(3x - 1)$	$-x + 18x = 17x$
$(x + 1)(3x - 6)$	Common factor
$(x - 6)(3x + 1)$	$x - 18x = -17x$
$(x - 2)(3x + 3)$	Common factor
$(x + 3)(3x - 2)$	$-2x + 9x = 7x$
$(x + 2)(3x - 3)$	Common factor
$(x - 3)(3x + 2)$	$2x - 9x = -7x$

Write the factors of the trinomial. $6x^3 + 14x^2 - 12x = 2x(x + 3)(3x - 2)$

For this example, all the trial factors were listed. Once the correct factors have been found, the remaining trial factors can be omitted. For the examples and solutions in this text, all trial factors except those that have a common factor will be listed.

Example 1

Factor: $3x^2 + x - 2$

Solution

Positive factors of 3: 1, 3

Factors of −2: 1, −2; −1, 2

Trial Factors	*Middle Term*
$(x + 1)(3x - 2)$	$-2x + 3x = x$
$(x - 2)(3x + 1)$	$x - 6x = -5x$
$(x - 1)(3x + 2)$	$2x - 3x = -x$
$(x + 2)(3x - 1)$	$-x + 6x = 5x$

$3x^2 + x - 2 = (x + 1)(3x - 2)$

You Try It 1

Factor: $2x^2 - x - 3$

Your solution

Example 2

Factor: $-12x^3 - 32x^2 + 12x$

Solution

The GCF is $-4x$.

$-12x^3 - 32x^2 + 12x = -4x(3x^2 + 8x - 3)$

Factor the trinomial.

Positive factors of 3: 1, 3

Factors of −3: 1, −3; −1, 3

Trial Factors	*Middle Term*
$(x - 3)(3x + 1)$	$x - 9x = -8x$
$(x + 3)(3x - 1)$	$-x + 9x = 8x$

$-12x^3 - 32x^2 + 12x = -4x(x + 3)(3x - 1)$

You Try It 2

Factor: $-45y^3 + 12y^2 + 12y$

Your solution

Solutions on pp. S10–S11

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Objective B To factor a trinomial of the form $ax^2 + bx + c$ by grouping



In the previous objective, trinomials of the form $ax^2 + bx + c$ were factored by using trial factors. In this objective, these trinomials will be factored by grouping.

To factor $ax^2 + bx + c$, first find two factors of $a \cdot c$ whose sum is b. Then use factoring by grouping to write the factorization of the trinomial.

➡ Factor: $2x^2 + 13x + 15$

Find two positive factors of 30 $(2 \cdot 15)$ whose sum is 13.

Positive Factors of 30	*Sum*
1, 30	31
2, 15	17
3, 10	13

• When the required sum has been found, the remaining factors need not be checked.

$$2x^2 + 13x + 15 = 2x^2 + 3x + 10x + 15$$

• Use the factors of 30 whose sum is 13 to write $13x$ as $3x + 10x$.

$$= (2x^2 + 3x) + (10x + 15)$$

• Factor by grouping.

$$= x(2x + 3) + 5(2x + 3)$$
$$= (2x + 3)(x + 5)$$

Check your answer. $(2x + 3)(x + 5) = 2x^2 + 10x + 3x + 15$
$= 2x^2 + 13x + 15$

➡ Factor: $6x^2 - 11x - 10$

Find two factors of -60 $[6 \cdot (-10)]$ whose sum is -11.

Factors of −60	*Sum*
1, −60	−59
−1, 60	59
2, −30	−28
−2, 30	28
3, −20	−17
−3, 20	17
4, −15	11

$$6x^2 - 11x - 10 = 6x^2 + 4x - 15x - 10$$

• Use the factors of -60 whose sum is -11 to write $-11x$ as $4x - 15x$.

$$= (6x^2 + 4x) - (15x + 10)$$

• Factor by grouping. Recall $-15x - 10 = -(15x + 10)$.

$$= 2x(3x + 2) - 5(3x + 2)$$
$$= (3x + 2)(2x - 5)$$

Check your answer. $(3x + 2)(2x - 5) = 6x^2 - 15x + 4x - 10$
$= 6x^2 - 11x - 10$

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Factor: $3x^2 - 2x - 4$

Find two factors of -12 $[3 \cdot (-4)]$ whose sum is -2.

Factors of −12	*Sum*
1, −12	−11
−1, 12	11
2, −6	−4
−2, 6	4
3, −4	−1
−3, 4	1

TAKE NOTE

$3x^2 - 2x - 4$ is a prime polynomial.

Because no integer factors of -12 have a sum of -2, $3x^2 - 2x - 4$ is nonfactorable over the integers.

Example 3

Factor: $2x^2 + 19x - 10$

Solution

Factors of −20 [2(−10)]	*Sum*
−1, 20	19

$$\begin{aligned} 2x^2 + 19x - 10 &= 2x^2 - x + 20x - 10 \\ &= (2x^2 - x) + (20x - 10) \\ &= x(2x - 1) + 10(2x - 1) \\ &= (2x - 1)(x + 10) \end{aligned}$$

You Try It 3

Factor: $2a^2 + 13a - 7$

Your solution

Example 4

Factor: $24x^2y - 76xy + 40y$

Solution

The GCF is $4y$.

$24x^2y - 76xy + 40y = 4y(6x^2 - 19x + 10)$

Negative Factors of 60 [6(10)]	*Sum*
−1, −60	−61
−2, −30	−32
−3, −20	−23
−4, −15	−19

$$\begin{aligned} 6x^2 - 19x + 10 &= 6x^2 - 4x - 15x + 10 \\ &= (6x^2 - 4x) - (15x - 10) \\ &= 2x(3x - 2) - 5(3x - 2) \\ &= (3x - 2)(2x - 5) \end{aligned}$$

$$\begin{aligned} 24x^2y - 76xy + 40y &= 4y(6x^2 - 19x + 10) \\ &= 4y(3x - 2)(2x - 5) \end{aligned}$$

You Try It 4

Factor: $15x^3 + 40x^2 - 80x$

Your solution

Solutions on p. S11

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5.3 Exercises

Objective A

Factor by using trial factors.

1. $2x^2 + 3x + 1$

2. $5x^2 + 6x + 1$

3. $2y^2 + 7y + 3$

4. $3y^2 + 7y + 2$

5. $2a^2 - 3a + 1$

6. $3a^2 - 4a + 1$

7. $2b^2 - 11b + 5$

8. $3b^2 - 13b + 4$

9. $2x^2 + x - 1$

10. $4x^2 - 3x - 1$

11. $2x^2 - 5x - 3$

12. $3x^2 + 5x - 2$

13. $2t^2 - t - 10$

14. $2t^2 + 5t - 12$

15. $3p^2 - 16p + 5$

16. $6p^2 + 5p + 1$

17. $12y^2 - 7y + 1$

18. $6y^2 - 5y + 1$

19. $6z^2 - 7z + 3$

20. $9z^2 + 3z + 2$

21. $6t^2 - 11t + 4$

22. $10t^2 + 11t + 3$

23. $8x^2 + 33x + 4$

24. $7x^2 + 50x + 7$

25. $5x^2 - 62x - 7$

26. $9x^2 - 13x - 4$

27. $12y^2 + 19y + 5$

28. $5y^2 - 22y + 8$

29. $7a^2 + 47a - 14$

30. $11a^2 - 54a - 5$

31. $3b^2 - 16b + 16$

32. $6b^2 - 19b + 15$

33. $2z^2 - 27z - 14$

34. $4z^2 + 5z - 6$

35. $3p^2 + 22p - 16$

36. $7p^2 + 19p + 10$

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37. $4x^2 + 6x + 2$ **38.** $12x^2 + 33x - 9$ **39.** $15y^2 - 50y + 35$ **40.** $30y^2 + 10y - 20$

41. $2x^3 - 11x^2 + 5x$ **42.** $2x^3 - 3x^2 - 5x$ **43.** $3a^2b - 16ab + 16b$ **44.** $2a^2b - ab - 21b$

45. $3z^2 + 95z + 10$ **46.** $8z^2 - 36z + 1$ **47.** $36x - 3x^2 - 3x^3$ **48.** $-2x^3 + 2x^2 + 4x$

49. $80y^2 - 36y + 4$ **50.** $24y^2 - 24y - 18$ **51.** $8z^3 + 14z^2 + 3z$ **52.** $6z^3 - 23z^2 + 20z$

53. $6x^2y - 11xy - 10y$ **54.** $8x^2y - 27xy + 9y$ **55.** $10t^2 - 5t - 50$

56. $16t^2 + 40t - 96$ **57.** $3p^3 - 16p^2 + 5p$ **58.** $6p^3 + 5p^2 + p$

59. $26z^2 + 98z - 24$ **60.** $30z^2 - 87z + 30$ **61.** $10y^3 - 44y^2 + 16y$

62. $14y^3 + 94y^2 - 28y$ **63.** $4yz^3 + 5yz^2 - 6yz$ **64.** $12a^3 + 14a^2 - 48a$

65. $42a^3 + 45a^2 - 27a$ **66.** $36p^2 - 9p^3 - p^4$ **67.** $9x^2y - 30xy^2 + 25y^3$

68. $8x^2y - 38xy^2 + 35y^3$ **69.** $9x^3y - 24x^2y^2 + 16xy^3$ **70.** $9x^3y + 12x^2y + 4xy$

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Objective B

Factor by grouping.

71. $6x^2 - 17x + 12$ **72.** $15x^2 - 19x + 6$ **73.** $5b^2 + 33b - 14$ **74.** $8x^2 - 30x + 25$

75. $6a^2 + 7a - 24$ **76.** $14a^2 + 15a - 9$ **77.** $4z^2 + 11z + 6$ **78.** $6z^2 - 25z + 14$

79. $22p^2 + 51p - 10$ **80.** $14p^2 - 41p + 15$ **81.** $8y^2 + 17y + 9$ **82.** $12y^2 - 145y + 12$

83. $18t^2 - 9t - 5$ **84.** $12t^2 + 28t - 5$ **85.** $6b^2 + 71b - 12$ **86.** $8b^2 + 65b + 8$

87. $9x^2 + 12x + 4$ **88.** $25x^2 - 30x + 9$ **89.** $6b^2 - 13b + 6$ **90.** $20b^2 + 37b + 15$

91. $33b^2 + 34b - 35$ **92.** $15b^2 - 43b + 22$ **93.** $18y^2 - 39y + 20$ **94.** $24y^2 + 41y + 12$

95. $15a^2 + 26a - 21$ **96.** $6a^2 + 23a + 21$ **97.** $8y^2 - 26y + 15$ **98.** $18y^2 - 27y + 4$

99. $8z^2 + 2z - 15$ **100.** $10z^2 + 3z - 4$ **101.** $15x^2 - 82x + 24$ **102.** $13z^2 + 49z - 8$

103. $10z^2 - 29z + 10$ **104.** $15z^2 - 44z + 32$ **105.** $36z^2 + 72z + 35$ **106.** $16z^2 + 8z - 35$

107. $3x^2 + xy - 2y^2$ **108.** $6x^2 + 10xy + 4y^2$ **109.** $3a^2 + 5ab - 2b^2$ **110.** $2a^2 - 9ab + 9b^2$

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111. $4y^2 - 11yz + 6z^2$ **112.** $2y^2 + 7yz + 5z^2$ **113.** $28 + 3z - z^2$ **114.** $15 - 2z - z^2$

115. $8 - 7x - x^2$ **116.** $12 + 11x - x^2$ **117.** $9x^2 + 33x - 60$ **118.** $16x^2 - 16x - 12$

119. $24x^2 - 52x + 24$ **120.** $60x^2 + 95x + 20$ **121.** $35a^4 + 9a^3 - 2a^2$

122. $15a^4 + 26a^3 + 7a^2$ **123.** $15b^2 - 115b + 70$ **124.** $25b^2 + 35b - 30$

125. $3x^2 - 26xy + 35y^2$ **126.** $4x^2 + 16xy + 15y^2$ **127.** $216y^2 - 3y - 3$

128. $360y^2 + 4y - 4$ **129.** $21 - 20x - x^2$ **130.** $18 + 17x - x^2$

APPLYING THE CONCEPTS

131. In your own words, explain how the signs of the last terms of the two binomial factors of a trinomial are determined.

Factor.

132. $(x + 1)^2 - (x + 1) - 6$ **133.** $(x - 2)^2 + 3(x - 2) + 2$ **134.** $(y + 3)^2 - 5(y + 3) + 6$

135. $2(y + 2)^2 - (y + 2) - 3$ **136.** $3(a + 2)^2 - (a + 2) - 4$ **137.** $4(y - 1)^2 - 7(y - 1) - 2$

Find all integers k such that the trinomial can be factored over the integers.

138. $2x^2 + kx + 3$ **139.** $2x^2 + kx - 3$ **140.** $3x^2 + kx + 2$

141. $3x^2 + kx - 2$ **142.** $2x^2 + kx + 5$ **143.** $2x^2 + kx - 5$

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5.4 Special Factoring

Objective A To factor the difference of two squares and perfect-square trinomials

Recall that the product of the sum and difference of the same terms equals the square of the first term minus the square of the second term.

Sum and difference of two terms		Difference of two squares
$(a + b)(a - b)$	$=$	$a^2 - b^2$

This suggests that the difference of two squares can be factored as

$$a^2 - b^2 = (a + b)(a - b)$$

Note that the polynomial $x^2 + y^2$ is the *sum* of two squares. The sum of two squares is nonfactorable over the integers.

Factor: $x^2 - 16$

$$x^2 - 16 = (x)^2 - (4)^2$$
$$= (x - 4)(x + 4)$$

Write $x^2 - 16$ as the difference of two squares. Then factor.

Check: $(x - 4)(x + 4) = x^2 + 4x - 4x - 16$
$= x^2 - 16$

Factor: $8x^3 - 18x$

$$8x^3 - 18x = 2x(4x^2 - 9)$$
$$= 2x[(2x)^2 - 3^2]$$
$$= 2x(2x - 3)(2x + 3)$$

The GCF is $2x$.

Factor the difference of two squares.

You should check the factorization.

Factor: $x^2 - 10$

Because 10 cannot be written as the square of an integer, $x^2 - 10$ is nonfactorable over the integers.

Recall from an earlier discussion the pattern for finding the square of a binomial. The result is a **perfect-square trinomial.**

The Square of a Binomial

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

Square of first term
Twice the product of the two terms
Square of the last term

This pattern is used to factor a perfect-square trinomial.

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➡ Factor: $4x^2 - 20x + 25$

Because the first and last terms are squares [$(2x)^2 = 4x^2$; $5^2 = 25$], try to factor this as the square of a binomial. Check the factorization.

$4x^2 - 20x + 25 = (2x - 5)^2$

Check: $(2x - 5)^2 = (2x)^2 + 2(2x)(-5) + 5^2$
$= 4x^2 - 20x + 25$ • The factorization is correct.

$4x^2 - 20x + 25 = (2x - 5)^2$

➡ Factor: $4x^2 + 37x + 9$

Because the first and last terms are squares [$(2x)^2 = 4x^2$; $3^2 = 9$], try to factor this as the square of a binomial. Check the proposed factorization.

$4x^2 + 37x + 9 = (2x + 3)^2$

Check: $(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2$
$= 4x^2 + 12x + 9$

Because $4x^2 + 12x + 9 \neq 4x^2 + 37x + 9$, the proposed factorization is not correct. In this case, the polynomial is not a perfect-square trinomial. It may, however, still factor. In fact, $4x^2 + 37x + 9 = (4x + 1)(x + 9)$.

Example 1
Factor: $16x^2 - y^2$

Solution
$16x^2 - y^2 = (4x)^2 - y^2 = (4x + y)(4x - y)$

You Try It 1
Factor: $25a^2 - b^2$

Your solution

Example 2
Factor: $z^4 - 16$

Solution
$z^4 - 16 = (z^2)^2 - 4^2 = (z^2 + 4)(z^2 - 4)$
$= (z^2 + 4)(z - 2)(z + 2)$

You Try It 2
Factor: $n^4 - 81$

Your solution

Example 3
Factor: $9x^2 - 30x + 25$

Solution
$9x^2 = (3x)^2$, $25 = (5)^2$
$9x^2 - 30x + 25 = (3x - 5)^2$

Check: $(3x - 5)^2 = (3x)^2 + 2(3x)(-5) + 5^2$
$= 9x^2 - 30x + 25$

You Try It 3
Factor: $16y^2 + 8y + 1$

Your solution

Solutions on p. S11

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Example 4
Factor: $9x^2 + 40x + 16$

Solution
Because $9x^2 = (3x)^2$, $16 = 4^2$, and $40x \neq 2(3x)(4)$, the trinomial is not a perfect-square trinomial.

Try to factor by another method.

$9x^2 + 40x + 16 = (9x + 4)(x + 4)$

You Try It 4
Factor: $x^2 + 15x + 36$

Your solution

Example 5
Factor: $(r + 2)^2 - 4$

Solution
$$\begin{aligned}(r + 2)^2 - 4 &= (r + 2)^2 - 2^2\\ &= (r + 2 - 2)(r + 2 + 2)\\ &= r(r + 4)\end{aligned}$$

You Try It 5
Factor: $(x^2 - 6x + 9) - y^2$

Your solution

Solutions on p. S11

Objective B To factor completely

When factoring a polynomial completely, ask the following questions about the polynomial.

1. Is there a common factor? If so, factor out the common factor.
2. Is the polynomial the difference of two perfect squares? If so, factor.
3. Is the polynomial a perfect-square trinomial? If so, factor.
4. Is the polynomial a trinomial that is the product of two binomials? If so, factor.
5. Does the polynomial contain four terms? If so, try factoring by grouping.
6. Is each binomial factor a prime polynomial over the integers? If not, factor.

➡ Factor: $z^3 + 4z^2 - 9z - 36$

$$\begin{aligned}z^3 + 4z^2 - 9z - 36 &= (z^3 + 4z^2) - (9z + 36)\\ &= z^2(z + 4) - 9(z + 4)\\ &= (z + 4)(z^2 - 9)\\ &= (z + 4)(z + 3)(z - 3)\end{aligned}$$

- Factor by grouping. Recall $-9z - 36 = -(9z + 36)$.
- $z^3 + 4z^2 = z^2(z + 4)$ $9z + 36 = 9(z + 4)$
- Factor out the common binomial factor $(z + 4)$.
- Factor the difference of squares.

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Example 6
Factor: $3x^2 - 48$

Solution
The GCF is 3.

$$3x^2 - 48 = 3(x^2 - 16)$$
$$= 3(x + 4)(x - 4)$$

$$3x^2 - 48 = 3(x + 4)(x - 4)$$

You Try It 6
Factor: $12x^3 - 75x$

Your solution

Example 7
Factor: $x^3 - 3x^2 - 4x + 12$

Solution
Factor by grouping.

$$x^3 - 3x^2 - 4x + 12 = (x^3 - 3x^2) - (4x - 12)$$
$$= x^2(x - 3) - 4(x - 3)$$
$$= (x - 3)(x^2 - 4)$$
$$= (x - 3)(x + 2)(x - 2)$$

$$x^3 - 3x^2 - 4x + 12 = (x - 3)(x + 2)(x - 2)$$

You Try It 7
Factor: $a^2b - 7a^2 - b + 7$

Your solution

Example 8
Factor: $4x^2y^2 + 12xy^2 + 9y^2$

Solution
The GCF is y^2.

$$4x^2y^2 + 12xy^2 + 9y^2 = y^2(4x^2 + 12x + 9)$$
$$= y^2(2x + 3)^2$$

$$4x^2y^2 + 12xy^2 + 9y^2 = y^2(2x + 3)^2$$

You Try It 8
Factor: $4x^3 + 28x^2 - 120x$

Your solution

Solutions on p. S11

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5.4 Exercises

Objective A

Factor.

1. $x^2 - 4$ **2.** $x^2 - 9$ **3.** $a^2 - 81$ **4.** $a^2 - 49$

5. $y^2 + 2y + 1$ **6.** $y^2 + 14y + 49$ **7.** $a^2 - 2a + 1$ **8.** $x^2 - 12x + 36$

9. $4x^2 - 1$ **10.** $9x^2 - 16$ **11.** $x^6 - 9$ **12.** $y^{12} - 4$

13. $x^2 + 8x - 16$ **14.** $z^2 - 18z - 81$ **15.** $x^2 + 2xy + y^2$ **16.** $x^2 + 6xy + 9y^2$

17. $4a^2 + 4a + 1$ **18.** $25x^2 + 10x + 1$ **19.** $9x^2 - 1$ **20.** $1 - 49x^2$

21. $1 - 64x^2$ **22.** $t^2 + 36$ **23.** $x^2 + 64$ **24.** $64a^2 - 16a + 1$

25. $9a^2 + 6a + 1$ **26.** $x^4 - y^2$ **27.** $b^4 - 16a^2$ **28.** $16b^2 + 8b + 1$

29. $4a^2 - 20a + 25$ **30.** $4b^2 + 28b + 49$ **31.** $9a^2 - 42a + 49$ **32.** $9x^2 - 16y^2$

33. $25z^2 - y^2$ **34.** $x^2y^2 - 4$ **35.** $a^2b^2 - 25$ **36.** $16 - x^2y^2$

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37. $25x^2 - 1$

38. $25a^2 + 30ab + 9b^2$

39. $4a^2 - 12ab + 9b^2$

40. $49x^2 + 28xy + 4y^2$

41. $4y^2 - 36yz + 81z^2$

42. $64y^2 - 48yz + 9z^2$

43. $\frac{1}{x^2} - 4$

44. $\frac{9}{a^2} - 16$

45. $9a^2b^2 - 6ab + 1$

46. $16x^2y^2 - 24xy + 9$

Objective B

Factor.

47. $8y^2 - 2$

48. $12n^2 - 48$

49. $3a^3 + 6a^2 + 3a$

50. $4rs^2 - 4rs + r$

51. $m^4 - 256$

52. $81 - t^4$

53. $9x^2 + 13x + 4$

54. $x^2 + 10x + 16$

55. $16y^4 + 48y^3 + 36y^2$

56. $36c^4 - 48c^3 + 16c^2$

57. $y^8 - 81$

58. $32s^4 - 2$

59. $25 - 20p + 4p^2$

60. $9 + 24a + 16a^2$

61. $(4x - 3)^2 - y^2$

62. $(2x + 5)^2 - 25$

63. $(x^2 - 4x + 4) - y^2$

64. $(4x^2 + 12x + 9) - 4y^2$

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65. $5x^2 - 5$

66. $2x^2 - 18$

67. $x^3 + 4x^2 + 4x$

68. $y^3 - 10y^2 + 25y$

69. $x^4 + 2x^3 - 35x^2$

70. $a^4 - 11a^3 + 24a^2$

71. $5b^2 + 75b + 180$

72. $6y^2 - 48y + 72$

73. $3a^2 + 36a + 10$

74. $5a^2 - 30a + 4$

75. $2x^2y + 16xy - 66y$

76. $3a^2b + 21ab - 54b$

77. $x^3 - 6x^2 - 5x$

78. $b^3 - 8b^2 - 7b$

79. $3y^2 - 36$

80. $3y^2 - 147$

81. $20a^2 + 12a + 1$

82. $12a^2 - 36a + 27$

83. $x^2y^2 - 7xy^2 - 8y^2$

84. $a^2b^2 + 3a^2b - 88a^2$

85. $10a^2 - 5ab - 15b^2$

86. $16x^2 - 32xy + 12y^2$

87. $50 - 2x^2$

88. $72 - 2x^2$

89. $a^2b^2 - 10ab^2 + 25b^2$

90. $a^2b^2 + 6ab^2 + 9b^2$

91. $12a^3b - a^2b^2 - ab^3$

92. $2x^3y - 7x^2y^2 + 6xy^3$

93. $12a^3 - 12a^2 + 3a$

94. $18a^3 + 24a^2 + 8a$

95. $243 + 3a^2$

96. $75 + 27y^2$

97. $12a^3 - 46a^2 + 40a$

98. $24x^3 - 66x^2 + 15x$

99. $4a^3 + 20a^2 + 25a$

100. $2a^3 - 8a^2b + 8ab^2$

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101. $27a^2b - 18ab + 3b$

102. $a^2b^2 - 6ab^2 + 9b^2$

103. $48 - 12x - 6x^2$

104. $21x^2 - 11x^3 - 2x^4$

105. $x^4 - x^2y^2$

106. $b^4 - a^2b^2$

107. $18a^3 + 24a^2 + 8a$

108. $32xy^2 - 48xy + 18x$

109. $2b + ab - 6a^2b$

110. $15y^2 - 2xy^2 - x^2y^2$

111. $4x^4 - 38x^3 + 48x^2$

112. $3x^2 - 27y^2$

113. $x^4 - 25x^2$

114. $y^3 - 9y$

115. $a^4 - 16$

116. $15x^4y^2 - 13x^3y^3 - 20x^2y^4$

117. $45y^2 - 42y^3 - 24y^4$

118. $a(2x - 2) + b(2x - 2)$

119. $4a(x - 3) - 2b(x - 3)$

120. $x^2(x - 2) - (x - 2)$

121. $y^2(a - b) - (a - b)$

122. $a(x^2 - 4) + b(x^2 - 4)$

123. $x(a^2 - b^2) - y(a^2 - b^2)$

124. $4(x - 5) - x^2(x - 5)$

APPLYING THE CONCEPTS

Find all integers k such that the trinomial is a perfect-square trinomial.

125. $4x^2 - kx + 9$

126. $x^2 + 6x + k$

127. $64x^2 + kxy + y^2$

128. $x^2 - 2x + k$

129. $25x^2 - kx + 1$

130. $x^2 + 10x + k$

131. Select any odd integer greater than 1, square it, and then subtract 1. Is the result evenly divisible by 8? Prove that this procedure always produces a number divisible by 8. (*Suggestion:* Any odd integer greater than 1 can be expressed as $2n + 1$, where n is a natural number.)

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5.5 Solving Equations

Objective A To solve equations by factoring

The Multiplication Property of Zero states that the product of a number and zero is zero. This property is stated below.

If a is a real number, then $a \cdot 0 = 0 \cdot a = 0$.

Now consider $x \cdot y = 0$. For this to be a true equation, then either $x = 0$ or $y = 0$.

Principle of Zero Products

If the product of two factors is zero, then at least one of the factors must be zero.

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

The Principle of Zero Products is used to solve some equations.

Solve: $(x - 2)(x - 3) = 0$

$(x - 2)(x - 3) = 0$

$x - 2 = 0 \quad x - 3 = 0$

- Let each factor equal zero (the Principle of Zero Products).

$x = 2 \qquad x = 3$

- Rewrite each equation in the form *variable = constant*.

Check:

$$\begin{array}{r|l} (x-2)(x-3) = 0 & \\ \hline (2-2)(2-3) & 0 \\ 0(-1) & 0 \\ 0 = 0 & \end{array}$$

A true equation

$$\begin{array}{r|l} (x-2)(x-3) = 0 & \\ \hline (3-2)(3-3) & 0 \\ (1)(0) & 0 \\ 0 = 0 & \end{array}$$

A true equation

The solutions are 2 and 3.

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, is a **quadratic equation.** A quadratic equation is in **standard form** when the polynomial is in descending order and equal to zero. The quadratic equations at the right are in standard form.

$3x^2 + 2x + 1 = 0$

$4x^2 - 3x + 2 = 0$

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➡ Solve: $2x^2 + x = 6$

$$2x^2 + x = 6$$
$$2x^2 + x - 6 = 0$$
$$(2x - 3)(x + 2) = 0$$
$$2x - 3 = 0 \qquad x + 2 = 0$$
$$2x = 3 \qquad x = -2$$
$$x = \frac{3}{2}$$

- Write the equation in standard form.
- Factor.
- Use the Principle of Zero Products.
- Rewrite each equation in the form *variable = constant.*

Check: $\frac{3}{2}$ and -2 check as solutions.

The solutions are $\frac{3}{2}$ and -2.

Example 1
Solve: $x(x - 3) = 0$

Solution
$x(x - 3) = 0$

$x = 0 \qquad x - 3 = 0$
$\qquad\qquad x = 3$

The solutions are 0 and 3.

You Try It 1
Solve: $2x(x + 7) = 0$

Your solution

Example 2
Solve: $2x^2 - 50 = 0$

Solution
$2x^2 - 50 = 0$
$2(x^2 - 25) = 0$
$2(x + 5)(x - 5) = 0$
$x + 5 = 0 \qquad x - 5 = 0$
$x = -5 \qquad x = 5$

The solutions are -5 and 5.

You Try It 2
Solve: $4x^2 - 9 = 0$

Your solution

Example 3
Solve: $(x - 3)(x - 10) = -10$

Solution
$(x - 3)(x - 10) = -10$
$x^2 - 13x + 30 = -10$
$x^2 - 13x + 40 = 0$
$(x - 8)(x - 5) = 0$
$x - 8 = 0 \qquad x - 5 = 0$
$x = 8 \qquad x = 5$

- Multiply $(x - 3)(x - 10)$.
- Add 10 to each side of the equation. The equation is now in standard form.

The solutions are 8 and 5.

You Try It 3
Solve: $(x + 2)(x - 7) = 52$

Your solution

Solutions on pp. S11–S12

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Objective B To solve application problems

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Example 4

The sum of the squares of two consecutive positive even integers is equal to 100. Find the two integers.

Strategy

First positive even integer: n
Second positive even integer: $n + 2$

The sum of the square of the first positive even integer and the square of the second positive even integer is 100.

Solution

$$n^2 + (n + 2)^2 = 100$$
$$n^2 + n^2 + 4n + 4 = 100$$
$$2n^2 + 4n + 4 = 100$$
$$2n^2 + 4n - 96 = 0$$
$$2(n^2 + 2n - 48) = 0$$
$$2(n - 6)(n + 8) = 0$$

$n - 6 = 0$ $\qquad$ $n + 8 = 0$
$n = 6$ $\qquad$ $n = -8$

Because -8 is not a positive even integer, it is not a solution.

$n = 6$
$n + 2 = 6 + 2 = 8$

The two integers are 6 and 8.

You Try It 4

The sum of the squares of two consecutive positive integers is 61. Find the two integers.

Your strategy

Your solution

Solution on p. S12

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Example 5

A stone is thrown into a well with an initial speed of 4 ft/s. The well is 420 ft deep. How many seconds later will the stone hit the bottom of the well? Use the equation $d = vt + 16t^2$, where d is the distance in feet, v is the initial speed in feet per second, and t is the time in seconds.

Strategy

To find the time for the stone to drop to the bottom of the well, replace the variables d and v by their given values and solve for t.

Solution

$$d = vt + 16t^2$$
$$420 = 4t + 16t^2$$
$$0 = -420 + 4t + 16t^2$$
$$0 = 16t^2 + 4t - 420$$
$$0 = 4(4t^2 + t - 105)$$
$$0 = 4(4t + 21)(t - 5)$$

$$4t + 21 = 0 \qquad t - 5 = 0$$
$$4t = -21 \qquad t = 5$$
$$t = -\frac{21}{4}$$

Because the time cannot be a negative number, $-\frac{21}{4}$ is not a solution.

The time is 5 s.

You Try It 5

The length of a rectangle is 4 in. longer than twice the width. The area of the rectangle is 96 in^2. Find the length and width of the rectangle.

Your strategy

Your solution

Solution on p. S12

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5.5 Exercises

Objective A

Solve.

1. $(y + 3)(y + 2) = 0$ **2.** $(y - 3)(y - 5) = 0$ **3.** $(z - 7)(z - 3) = 0$ **4.** $(z + 8)(z - 9) = 0$

5. $x(x - 5) = 0$ **6.** $x(x + 2) = 0$ **7.** $a(a - 9) = 0$ **8.** $a(a + 12) = 0$

9. $y(2y + 3) = 0$ **10.** $t(4t - 7) = 0$ **11.** $2a(3a - 2) = 0$ **12.** $4b(2b + 5) = 0$

13. $(b + 2)(b - 5) = 0$ **14.** $(b - 8)(b + 3) = 0$ **15.** $x^2 - 81 = 0$ **16.** $x^2 - 121 = 0$

17. $4x^2 - 49 = 0$ **18.** $16x^2 - 1 = 0$ **19.** $9x^2 - 1 = 0$ **20.** $16x^2 - 49 = 0$

21. $x^2 + 6x + 8 = 0$ **22.** $x^2 - 8x + 15 = 0$ **23.** $z^2 + 5z - 14 = 0$ **24.** $z^2 + z - 72 = 0$

25. $2a^2 - 9a - 5 = 0$ **26.** $3a^2 + 14a + 8 = 0$ **27.** $6z^2 + 5z + 1 = 0$ **28.** $6y^2 - 19y + 15 = 0$

29. $x^2 - 3x = 0$ **30.** $a^2 - 5a = 0$ **31.** $x^2 - 7x = 0$ **32.** $2a^2 - 8a = 0$

33. $a^2 + 5a = -4$ **34.** $a^2 - 5a = 24$ **35.** $y^2 - 5y = -6$ **36.** $y^2 - 7y = 8$

37. $2t^2 + 7t = 4$ **38.** $3t^2 + t = 10$ **39.** $3t^2 - 13t = -4$ **40.** $5t^2 - 16t = -12$

41. $x(x - 12) = -27$ **42.** $x(x - 11) = 12$ **43.** $y(y - 7) = 18$ **44.** $y(y + 8) = -15$

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45. $p(p + 3) = -2$ **46.** $p(p - 1) = 20$ **47.** $y(y + 4) = 45$ **48.** $y(y - 8) = -15$

49. $x(x + 3) = 28$ **50.** $p(p - 14) = 15$ **51.** $(x + 8)(x - 3) = -30$ **52.** $(x + 4)(x - 1) = 14$

53. $(z - 5)(z + 4) = 52$ **54.** $(z - 8)(z + 4) = -35$ **55.** $(z - 6)(z + 1) = -10$

56. $(a + 3)(a + 4) = 72$ **57.** $(a - 4)(a + 7) = -18$ **58.** $(2x + 5)(x + 1) = -1$

Objective B Application Problems

Solve.

59. The square of a positive number is six more than five times the positive number. Find the number.

60. The square of a negative number is fifteen more than twice the negative number. Find the number.

61. The sum of two numbers is six. The sum of the squares of the two numbers is twenty. Find the two numbers.

62. The sum of two numbers is eight. The sum of the squares of the two numbers is thirty-four. Find the two numbers.

63. The sum of the squares of two consecutive positive integers is forty-one. Find the two integers.

64. The sum of the squares of two consecutive positive even integers is one hundred. Find the two integers.

65. The sum of two numbers is ten. The product of the two numbers is twenty-one. Find the two numbers.

66. The sum of two numbers is thirteen. The product of the two numbers is forty. Find the two numbers.

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The formula $S = \frac{n^2 + n}{2}$ gives the sum, S, of the first n natural numbers. Use this formula for Exercises 67 and 68.

67. How many consecutive natural numbers beginning with 1 will give a sum of 78?

68. How many consecutive natural numbers beginning with 1 will give a sum of 171?

The formula $N = \frac{t^2 - t}{2}$ gives the number, N, of football games that must be scheduled in a league with t teams if each team is to play every other team once. Use this formula for Exercises 69 and 70.

69. How many teams are in a league that schedules 15 games in such a way that each team plays every other team once?

70. How many teams are in a league that schedules 45 games in such a way that each team plays every other team once?

The distance, s, in feet, that an object will fall (neglecting air resistance) in t seconds is given by $s = vt + 16t^2$, where v is the initial velocity of the object in feet per second. Use this formula for Exercises 71 and 72.

71. An object is released from the top of a building 192 ft high. The initial velocity is 16 ft/s, and air resistance is neglected. How many seconds later will the object hit the ground?

72. An object is released from the top of a building 320 ft high. The initial velocity is 16 ft/s, and air resistance is neglected. How many seconds later will the object hit the ground?

The height, h, in feet, an object will attain (neglecting air resistance) in t seconds is given by $h = vt - 16t^2$, where v is the initial velocity of the object in feet per second. Use this formula for Exercises 73 and 74.

73. A golf ball is thrown onto a cement surface and rebounds straight up. The initial velocity of the rebound is 60 ft/s. How many seconds later will the golf ball return to the ground?



74. A foul ball leaves a bat and travels straight up with an initial velocity of 64 ft/s. How many seconds later will the ball be 64 ft above the ground?

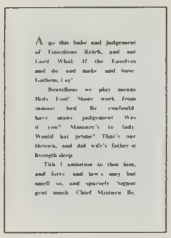
75. The length of a rectangle is 5 in. more than twice its width. Its area is 75 in^2. Find the length and width of the rectangle.

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76. The width of a rectangle is 5 ft less than the length. The area of the rectangle is 176 ft^2. Find the length and width of the rectangle.

77. The height of a triangle is 4 m more than twice the length of the base. The area of the triangle is 35 m^2. Find the height of the triangle.

78. The length of each side of a square is extended 5 in. The area of the resulting square is 64 in^2. Find the length of a side of the original square.



79. The page of a book measures 6 in. by 9 in. A uniform border around the page leaves 28 in^2 for type. What are the dimensions of the type area?

80. A small garden measures 8 ft by 10 ft. A uniform border around the garden increases the total area to 143 ft^2. What is the width of the border?

81. The radius of a circle is increased by 3 in.; this increases the area by 100 in^2. Find the radius of the original circle. Round to the nearest hundredth.

82. A circle has a radius of 10 in. Find the increase in area that occurs when the radius is increased by 2 in. Round to the nearest hundredth.

APPLYING THE CONCEPTS

83. In your own words, what is the Principle of Zero Products?

84. Explain the error made in solving the equation at the right. Solve the equation correctly.

$$(x + 2)(x - 3) = 6$$
$$x + 2 = 6 \quad x - 3 = 6$$
$$x = 4 \qquad x = 9$$

85. Explain the error made in solving the equation at the right. Solve the equation correctly.

$$x^2 = x$$
$$\frac{x^2}{x} = \frac{x}{x}$$
$$x = 1$$

86. Find $3n^2$ if $n(n + 5) = -4$.

87. Find $2n^2$ if $n(n + 3) = 4$.

Solve.

88. $2y(y + 4) = -5(y + 3)$

89. $(b + 5)^2 = 16$

90. $p^3 = 9p^2$

91. $(x + 3)(2x - 1) = (3 - x)(5 - 3x)$

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Focus on Problem Solving

Making a Table

Sometimes a table can be used in organizing information so that it is in a useful form. In the chapter Solving Equations, we used tables in the applications to organize the data. Tables are also useful in applications that require you to find all possible combinations to a given situation.

A basketball player scored 11 points in a game. The player can score 1 point for making a free throw, 2 points for making a field goal within the three-point line, and 3 points for making a field goal outside the three-point line. Find the possible combinations in which the player can score the 11 points.

The following table lists the possible combinations of scoring the 11 points.

Free throws	0	2	1	3	5	0	2	4	6	8	1	3	5	7	9	11
2-point field goal	1	0	2	1	0	4	3	2	1	0	5	4	3	2	1	0
3-point field goal	3	3	2	2	2	1	1	1	1	1	0	0	0	0	0	0
Total Points	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11

There are 16 possible ways in which the basketball player could have scored 11 points.

1. A football team scores 17 points. A touchdown counts 6 points, an extra point scores 1 point, a field goal scores 3 points, and a safety scores 2 points. Find the possible combinations in which the team can score 17 points. Remember that the number of extra points cannot exceed the number of touchdowns scored.

2. Repeat Exercise 1. Assume that no safety was scored.

3. Repeat Exercise 1. Assume that no safety was scored and that the team scored two field goals.

4. Find the number of possible combinations of nickels, dimes, and quarters when receiving $.85 in change.

5. Repeat Exercise 4. Assume no combination contains coins that could be exchanged for a larger coin. That is, the combination of three quarters and two nickels would not be allowed as the two nickels could be exchanged for a dime.

6. Find the number of possible combinations of $1, $5, $10, and $20 bills when receiving $33 in change.

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Projects and Group Activities

Evaluating Polynomials Using a Calculator



One way to evaluate a polynomial is first to express the polynomial in a form that suggests a sequence of steps on a calculator. To illustrate this method, consider the polynomial $4x^2 - 5x + 2$. First the polynomial is rewritten as

$$4x^2 - 5x + 2 = (4x - 5)x + 2$$

To evaluate the polynomial, work through the rewritten expression from left to right, substituting the appropriate value for x.

Here are some examples.

Evaluate $5x^2 - 2x + 4$ when $x = 3$.

Rewrite the polynomial. $5x^2 - 2x + 4 = (5x - 2)x + 4$

Replace x in the rewritten expression by the given value. $(5 \cdot 3 - 2) \cdot 3 + 4$

Work through the expression from left to right. [(] 5 [×] 3 [−] 2 [)] [×] 3 [+] 4 [=]

The result in the display should be 43.

Evaluate $2x^3 - 4x^2 + 7x - 12$ when $x = 4$.

Rewrite the polynomial. $2x^3 - 4x^2 + 7x - 12 = [(2x - 4)x + 7]x - 12$

Replace x in the rewritten expression by the given value. $[(2 \cdot 4 - 4) \cdot 4 + 7] \cdot 4 - 12$

Work through the expression from left to right.

[(] [(] 2 [×] 4 [−] 4 [)] [×] 4 [+] 7 [)] [×] 4 [−] 12 [=]

The result in the display should be 80.

Evaluate $4x^2 - 3x + 5$ when $x = -2$.

Rewrite the polynomial. $4x^2 - 3x + 5 = (4x - 3)x + 5$

Replace x in the given expression by the given value. $[4 \cdot (-2) - 3] \cdot (-2) + 5$

Work through the expression from left to right.

[(] 4 [×] 2 [+/−] [−] 3 [)] [×] 2 [+/−] [+] 5 [=]

The result in the display should be 27.

Here are some practice exercises. Evaluate for the given value.

1. $2x^2 - 3x + 7; x = 4$

2. $3x^2 + 7x - 12; x = -3$

3. $3x^3 - 2x^2 + 6x - 8; x = 3$

4. $2x^3 + 4x^2 - x - 2; x = -2$

5. $x^4 - 3x^3 + 6x^2 + 5x - 1; x = 2$

6. $2x^3 - 4x + 8; x = 2$
Hint: $2x^3 - 4x + 8 = 2x^3 + 0x^2 - 4x + 8$

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Search the World Wide Web

At the address http://www.utm.edu/research/primes/mersenne.shtml#hist, you can find the history, theorems, and lists of Mersenne Primes. When $2^n - 1$ is prime, it is said to be a Mersenne Prime. An interesting note is that the 25th and 26th Mersenne Primes were found by high school students Laura Nickel and Curt Noll.

Would you like to find the answer to questions such as these?

1. What good are perfect numbers?
2. Is zero prime, composite, or neither?
3. What is the difference between zero and nothing?
4. When something is divided by zero, why is the answer undefined?
5. What is infinity plus one?
6. Is infinity positive or negative?

These and many other questions are answered at the address

http://forum.swarthmore.edu/dr.math/problems/purpose_zero.html

The answers to these questions are presented in a clever and interesting fashion. The answer to the question "What is the purpose of the number zero?" is given below.

> The invention of zero was one of the most important breakthroughs in the history of civilization. More important, in my opinion, than the invention of the wheel. I think that it's a fairly deep concept.
>
> One crucial purpose that zero holds is as a placeholder in our system of notation. When we write the number 408, we're really using a shorthand notation. What we really mean by 408 is "4 times 100, plus 0 times 10, plus 8 times 1." Without the number zero, we wouldn't be able to tell the numbers 408, 48, 480, 408,000, and 4800 apart. So yes, zero is important.
>
> Another crucial role that zero plays in mathematics is that of an "additive identity element." What this means is that when you add zero to any number, you get the number that you started with. For instance, $5 + 0 = 5$. That may seem obvious and trivial, but it's actually quite important to have such a number. For instance, when you're manipulating some numerical quantity and you want to change its form but not its value, you might add some fancy version of zero to it, like this:
>
> $$\begin{aligned} x^2 + y^2 &= x^2 + y^2 + 2xy - 2xy \\ &= x^2 + 2xy + y^2 - 2xy \\ &= (x + y)^2 - 2xy \end{aligned}$$
>
> Now if we wanted to, we could use this as a proof that $(x + y)^2$ is always greater than $2xy$; the expression we started with was positive, so the one we ended up with must be positive, too. Therefore, subtracting $2xy$ from $(x + y)^2$ must leave us with a positive number. Neat stuff.*

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The internet is a good source for the history of and interesting facts about mathematical concepts. You can use the address

http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Prime_numbers.html

to find the history of prime numbers. This Web address also contains a list of unsolved problems on prime numbers.

There is an on-line mathematics dictionary at the address

http://www.mathpro.com/math/glossary/glossary.html

This site contains several hundred mathematical definitions as well as an on-line glossary of technical notation for unfamiliar mathematical notation.

Chapter Summary

Key Words

The *greatest common factor* (GCF) of two or more monomials is the product of the GCF of the coefficients and the common variable factors.

To *factor* a polynomial means to write the polynomial as a product of other polynomials.

To *factor* a trinomial of the form $ax^2 + bx + c$ means to express the trinomial as the product of two binomials.

A *polynomial* that does not factor using only integers is *nonfactorable over the integers.*

An equation of the form $ax^2 + bx + c = 0$ is a *quadratic equation*. A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero. The quadratic equation $ax^2 + bx + c = 0$ is in standard form.

Essential Rules

Sum and Difference of Two Terms = Difference of Two Squares

$$(a + b)(a - b) = a^2 - b^2$$

Square of a binomial = Perfect-Square Trinomial

$$(a + b)^2 = a^2 + 2ab + b^2$$

Principle of Zero Products

If the product of two factors is zero, then at least one of the factors must be zero.

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

General Factoring Strategy

1. Is there a common factor? If so, factor out the common factor.
2. Is the polynomial the difference of two perfect squares? If so, factor.
3. Is the polynomial a perfect-square trinomial? If so, factor.
4. Is the polynomial a trinomial that is the product of two binomials? If so, factor.
5. Does the polynomial contain four terms? If so, try factoring by grouping.
6. Is each binomial factor a prime polynomial over the integers? If not, factor.

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Chapter Review

1. Factor: $b^2 - 13b + 30$

2. Factor: $4x(x - 3) - 5(3 - x)$

3. Factor $2x^2 - 5x + 6$ by using trial factors.

4. Factor: $5x^3 + 10x^2 + 35x$

5. Factor: $14y^9 - 49y^6 + 7y^3$

6. Factor: $y^2 + 5y - 36$

7. Factor $6x^2 - 29x + 28$ by using trial factors.

8. Factor: $12a^2b + 3ab^2$

9. Factor: $a^6 - 100$

10. Factor: $n^4 - 2n^3 - 3n^2$

11. Factor $12y^2 + 16y - 3$ by using trial factors.

12. Factor: $12b^3 - 58b^2 + 56b$

13. Factor: $9y^4 - 25z^2$

14. Factor: $c^2 + 8c + 12$

15. Factor $18a^2 - 3a - 10$ by grouping.

16. Solve: $4x^2 + 27x = 7$

17. Factor: $4x^3 - 20x^2 - 24x$

18. Factor: $3a^2 - 15a - 42$

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19. Factor $2a^2 - 19a - 60$ by grouping.

20. Solve: $(x + 1)(x - 5) = 16$

21. Factor: $21ax - 35bx - 10by + 6ay$

22. Factor: $a^2b^2 - 1$

23. Factor: $10x^2 + 25x + 4xy + 10y$

24. Factor: $5x^2 - 5x - 30$

25. Factor: $3x^2 + 36x + 108$

26. Factor $3x^2 - 17x + 10$ by grouping.

27. The length of a hockey field is 20 yd less than twice the width of the hockey field. The area of the hockey field is 6000 yd^2. Find the length and width of the hockey field.

28. The size, S, of an image from a slide projector depends on the distance, d, of the screen from the projector and is given by $S = d^2$. Find the distance between the projector and the screen when the size of the picture is 400 ft^2.

29. A rectangular photograph has dimensions 15 in. by 12 in. A picture frame around the photograph increases the total area to 270 in^2. What is the width of the frame?

30. The length of each side of a square garden plot is extended 4 ft. The area of the resulting square is 576 ft^2. Find the length of a side of the original garden plot.

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Chapter Test

1. Factor: $ab + 6a - 3b - 18$

2. Factor: $2y^4 - 14y^3 - 16y^2$

3. Factor $8x^2 + 20x - 48$ by grouping.

4. Factor $6x^2 + 19x + 8$ by using trial factors.

5. Factor: $a^2 - 19a + 48$

6. Factor: $6x^3 - 8x^2 + 10x$

7. Factor: $x^2 + 2x - 15$

8. Solve: $4x^2 - 1 = 0$

9. Factor: $5x^2 - 45x - 15$

10. Factor: $p^2 + 12p + 36$

11. Solve: $x(x - 8) = -15$

12. Factor: $3x^2 + 12xy + 12y^2$

13. Factor: $b^2 - 16$

14. Factor $6x^2y^2 + 9xy^2 + 3y^2$ by grouping.

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15. Factor: $p^2 + 5p + 6$

16. Factor: $a(x - 2) + b(x - 2)$

17. Factor: $x(p + 1) - (p + 1)$

18. Factor: $3a^2 - 75$

19. Factor $2x^2 + 4x - 5$ by using trial factors.

20. Factor: $x^2 - 9x - 36$

21. Factor: $4a^2 - 12ab + 9b^2$

22. Factor: $4x^2 - 49y^2$

23. Solve: $(2a - 3)(a + 7) = 0$

24. The sum of two numbers is ten. The sum of the squares of the two numbers is fifty-eight. Find the two numbers.

25. The length of a rectangle is 3 cm longer than twice its width. The area of the rectangle is 90 cm^2. Find the length and width of the rectangle.

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Cumulative Review

1. Subtract: $-2 - (-3) - 5 - (-11)$

2. Simplify: $(3 - 7)^2 \div (-2) - 3 \cdot (-4)$

3. Evaluate $-2a^2 \div (2b) - c$ when $a = -4$, $b = 2$, and $c = -1$.

4. Simplify: $-\frac{3}{4}(-20x^2)$

5. Simplify: $-2[4x - 2(3 - 2x) - 8x]$

6. Solve: $-\frac{5}{7}x = -\frac{10}{21}$

7. Solve: $3x - 2 = 12 - 5x$

8. Solve: $-2 + 4[3x - 2(4 - x) - 3] = 4x + 2$

9. 120% of what number is 54?

10. Simplify: $(-3a^3b^2)^2$

11. Simplify: $(x + 2)(x^2 - 5x + 4)$

12. Simplify: $(8x^2 + 4x - 3) \div (2x - 3)$

13. Simplify: $(x^{-4}y^3)^2$

14. Factor: $3a - 3b - ax + bx$

15. Factor: $15xy^2 - 20xy^4$

16. Factor: $x^2 - 5xy - 14y^2$

17. Factor: $p^2 - 9p - 10$

18. Factor: $18a^3 + 57a^2 + 30a$

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19. Factor: $36a^2 - 49b^2$

20. Factor: $4x^2 + 28xy + 49y^2$

21. Factor: $9x^2 + 15x - 14$

22. Factor: $18x^2 - 48xy + 32y^2$

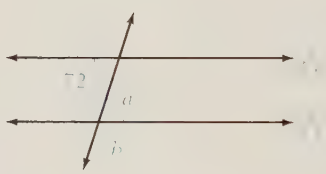
23. Factor: $3y(x - 3) - 2(x - 3)$

24. Solve: $3x^2 + 19x - 14 = 0$

25. A board 10 ft long is cut into two pieces. Four times the length of the shorter piece is 2 ft less than three times the length of the longer piece. Find the length of each piece.

26. A stereo that regularly sells for \$165 is on sale for \$99. Find the discount rate. Use the formula $S = R - rR$.

27. Given that lines ℓ_1 and ℓ_2 are parallel, find the measures of angles a and b.



28. A family drove to a resort at an average speed of 42 mph and later returned over the same road at an average speed of 56 mph. Find the distance to the resort if the total driving time was 7 h.

29. Find three consecutive even integers such that five times the middle integer is twelve more than twice the sum of the first and third.

30. The length of the base of a triangle is three times the height. The area of the triangle is 24 in^2. Find the length of the base of the triangle.

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CHAPTER

6 Rational Expressions



Carpentry requires the application of geometric concepts. A cabinet, for instance, has sides that must lie in parallel planes and doors that are in the shape of rectangles. A carpenter also applies the geometric principles of congruence and similarity; for example, the drawers in a dresser may be the same size and shape, or they may be the same shape but different sizes. Similar objects are a topic in Section 6 of this chapter.

Objectives

Section 6.1

To simplify a rational expression

To multiply rational expressions

To divide rational expressions

Section 6.2

To find the least common multiple (LCM) of two or more polynomials

To express two fractions in terms of the LCM of their denominators

Section 6.3

To add or subtract rational expressions with the same denominators

To add or subtract rational expressions with different denominators

Section 6.4

To simplify a complex fraction

Section 6.5

To solve an equation containing fractions

Section 6.6

To solve a proportion

To solve application problems

To solve problems involving similar triangles

Section 6.7

To solve a literal equation for one of the variables

Section 6.8

To solve work problems

To solve uniform motion problems

Measurement of the Circumference of the Earth

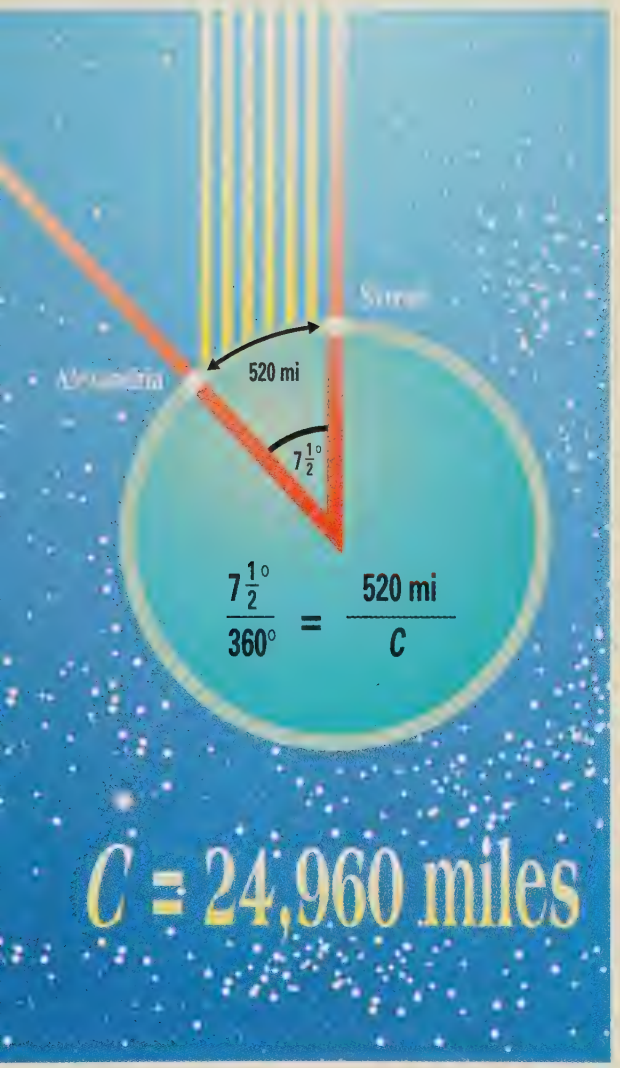
Distances on the earth, the circumference of the earth, and the distance to the moon and stars are known to great precision. Eratosthenes, the fifth librarian of Alexandria (230 B.C.), laid the foundation of scientific geography with his determination of the circumference of the earth.

Eratosthenes was familiar with certain astronomical data that enabled him to calculate the circumference of the earth by using a proportion statement.

Eratosthenes knew that on a midsummer day, the sun was directly overhead at Syrene, as shown in the diagram. At the same time, at Alexandria the sun was at a $7\frac{1}{2}°$ angle from the zenith. The distance from Syrene to Alexandria was 5000 stadia (about 520 mi).

Knowing that the ratio of the $7\frac{1}{2}°$ angle to one revolution (360°) is equal to the ratio of the arc length (520 mi) to the circumference, Eratosthenes was able to write and solve a proportion.

This result, calculated over 2000 years ago is very close to the accepted value of 24,800 miles.



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6.1 Multiplication and Division of Rational Expressions

Objective A To simplify a rational expression

A fraction in which the numerator or denominator is a polynomial is called a **rational expression.** Examples of rational expressions are shown at the right.

$$\frac{5}{z}, \quad \frac{x^2+1}{2x-1}, \quad \frac{y^2+y-1}{4y^2+1}$$

Care must be exercised with a rational expression to ensure that when the variables are replaced with numbers, the resulting denominator is not zero.

Consider the rational expression at the right. The value of x cannot be 3 because the denominator would then be zero.

$$\frac{4x^2-9}{2x-6}$$

$$\frac{4(3)^2-9}{2(3)-6}=\frac{27}{0}$$ Not a real number

A rational expression is in **simplest form** when the numerator and denominator have no common factors. The Multiplication Property of One is used to write a rational expression in simplest form.

➡ Simplify: $\frac{x^2-4}{x^2-2x-8}$

$$\frac{x^2-4}{x^2-2x-8}=\frac{(x-2)(x+2)}{(x-4)(x+2)}$$

- Factor the numerator and denominator.

$$=\frac{x-2}{x-4}\cdot\frac{x+2}{x+2}=\frac{x-2}{x-4}\cdot 1$$

$$=\frac{x-2}{x-4}, x\neq -2, 4$$

- The restrictions, $x \neq -2$ or 4, are necessary to prevent division by zero.

This simplification is usually shown with slashes through the common factors. The last simplification would be shown as

$$\frac{x^2-4}{x^2-2x-8}=\frac{(x-2)\cancel{(x+2)}^{1}}{(x-4)\cancel{(x+2)}_{1}}$$

- Factor the numerator and denominator.

$$=\frac{x-2}{x-4}, x\neq -2, 4$$

- Divide by the common factors. The restrictions, $x \neq -2$ or 4, are necessary to prevent division by zero.

➡ Simplify: $\frac{10+3x-x^2}{x^2-4x-5}$

$$\frac{10+3x-x^2}{x^2-4x-5}=\frac{(5-x)(2+x)}{(x-5)(x+1)}$$

- Factor the numerator and denominator.

$$=\frac{\cancel{(5-x)}^{-1}(2+x)}{\cancel{(x-5)}_{1}(x+1)}$$

- Recall that $5-x=-(x-5)$. Therefore, $\frac{5-x}{x-5}=\frac{-(x-5)}{x-5}=\frac{-1}{1}=-1$.

$$=-\frac{x+2}{x+1}, x\neq -1, 5$$

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For the remaining examples, we will omit the restrictions on the variables that prevent division by zero and assume the values of the variables are such that division by zero is not possible.

Example 1

Simplify: $\frac{4x^3y^4}{6x^4y}$

Solution

$\frac{4x^3y^4}{6x^4y} = \frac{2y^3}{3x}$ • Use rules of exponents.

You Try It 1

Simplify: $\frac{6x^5y}{12x^2y^3}$

Your solution

Example 2

Simplify: $\frac{9 - x^2}{x^2 + x - 12}$

Solution

$$\frac{9 - x^2}{x^2 + x - 12} = \frac{(3 - x)(3 + x)}{(x - 3)(x + 4)} = \frac{x + 3}{x + 4}$$

You Try It 2

Simplify: $\frac{x^2 + 2x - 24}{16 - x^2}$

Your solution

Example 3

Simplify: $\frac{x^2 + 2x - 15}{x^2 - 7x + 12}$

Solution

$$\frac{x^2 + 2x - 15}{x^2 - 7x + 12} = \frac{(x + 5)(x - 3)}{(x - 3)(x - 4)} = \frac{x + 5}{x - 4}$$

You Try It 3

Simplify: $\frac{x^2 + 4x - 12}{x^2 - 3x + 2}$

Your solution

Solutions on p. S12

Objective B To multiply rational expressions 19 CT

The product of two fractions is a fraction whose numerator is the product of the numerators of the two fractions and whose denominator is the product of the denominators of the two fractions.

Rule for Multiplying Fractions

If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions and $b \neq 0$, $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

$\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$ $\quad \frac{3x}{y} \cdot \frac{2}{z} = \frac{6x}{yz}$ $\quad \frac{x + 2}{x} \cdot \frac{3}{x - 2} = \frac{3x + 6}{x^2 - 2x}$

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Simplify: $\dfrac{x^2+3x}{x^2-3x-4}\cdot\dfrac{x^2-5x+4}{x^2+2x-3}$

$$\frac{x^2+3x}{x^2-3x-4}\cdot\frac{x^2-5x+4}{x^2+2x-3}$$

$$=\frac{x(x+3)}{(x-4)(x+1)}\cdot\frac{(x-4)(x-1)}{(x+3)(x-1)}$$

Factor the numerator and denominator of each fraction.

$$=\frac{\overset{1}{\cancel{x}}(\overset{1}{\cancel{x+3}})(\overset{1}{\cancel{x-4}})(\overset{1}{\cancel{x-1}})}{(\underset{1}{\cancel{x-4}})(x+1)(\underset{1}{\cancel{x+3}})(\underset{1}{\cancel{x-1}})}$$

Multiply.

$$=\frac{x}{x+1}$$

Write the answer in simplest form.

Example 4

Simplify: $\dfrac{10x^2-15x}{12x-8}\cdot\dfrac{3x-2}{20x-25}$

Solution

$$\frac{10x^2-15x}{12x-8}\cdot\frac{3x-2}{20x-25}$$

$$=\frac{5x(2x-3)}{4(3x-2)}\cdot\frac{(3x-2)}{5(4x-5)}$$

$$=\frac{\overset{1}{\cancel{5}}x(2x-3)(\overset{1}{\cancel{3x-2}})}{4(\underset{1}{\cancel{3x-2}})\underset{1}{\cancel{5}}(4x-5)}=\frac{x(2x-3)}{4(4x-5)}$$

You Try It 4

Simplify: $\dfrac{12x^2+3x}{10x-15}\cdot\dfrac{8x-12}{9x+18}$

Your solution

Example 5

Simplify: $\dfrac{x^2+x-6}{x^2+7x+12}\cdot\dfrac{x^2+3x-4}{4-x^2}$

Solution

$$\frac{x^2+x-6}{x^2+7x+12}\cdot\frac{x^2+3x-4}{4-x^2}$$

$$=\frac{(x+3)(x-2)}{(x+3)(x+4)}\cdot\frac{(x+4)(x-1)}{(2-x)(2+x)}$$

$$=\frac{\overset{1}{\cancel{(x+3)}}(\overset{-1}{\cancel{x-2}})(\overset{1}{\cancel{x+4}})(x-1)}{(\underset{1}{\cancel{x+3}})(\underset{1}{\cancel{x+4}})(\underset{1}{\cancel{2-x}})(2+x)}=-\frac{x-1}{x+2}$$

You Try It 5

Simplify: $\dfrac{x^2+2x-15}{9-x^2}\cdot\dfrac{x^2-3x-18}{x^2-7x+6}$

Your solution

Solutions on p. S12

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Objective C To divide rational expressions

19 CT

The reciprocal of a fraction is a fraction with the numerator and denominator interchanged.

Fraction	Reciprocal
$\frac{a}{b}$	$\frac{b}{a}$
$x^2 = \frac{x^2}{1}$	$\frac{1}{x^2}$
$\frac{x+2}{x}$	$\frac{x}{x+2}$

Rule for Dividing Fractions

Divide fractions by multiplying the dividend by the reciprocal of the divisor. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

$\frac{4}{x} \div \frac{y}{5} = \frac{4}{x} \cdot \frac{5}{y} = \frac{20}{xy}$

$\frac{x+4}{x} \div \frac{x-2}{4} = \frac{x+4}{x} \cdot \frac{4}{x-2} = \frac{4(x+4)}{x(x-2)}$

The basis for the division rule is shown at the right.

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b}}{\frac{c}{d}} \cdot \frac{\frac{d}{c}}{\frac{d}{c}} = \frac{\frac{a}{b} \cdot \frac{d}{c}}{1} = \frac{a}{b} \cdot \frac{d}{c}$$

Example 6

Simplify: $\frac{xy^2 - 3x^2y}{z^2} \div \frac{6x^2 - 2xy}{z^3}$

Solution

$$\frac{xy^2 - 3x^2y}{z^2} \div \frac{6x^2 - 2xy}{z^3}$$

$$= \frac{xy^2 - 3x^2y}{z^2} \cdot \frac{z^3}{6x^2 - 2xy}$$

$$= \frac{xy(y - 3x) \cdot z^3}{z^2 \cdot 2x(3x - y)} = \frac{yz}{2}$$

You Try It 6

Simplify: $\frac{a^2}{4bc^2 - 2b^2c} \div \frac{a}{6bc - 3b^2}$

Your solution

Example 7

Simplify: $\frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \div \frac{3x^2 + 13x + 4}{2x^2 + 7x - 4}$

Solution

$$\frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \div \frac{3x^2 + 13x + 4}{2x^2 + 7x - 4}$$

$$= \frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \cdot \frac{2x^2 + 7x - 4}{3x^2 + 13x + 4}$$

$$= \frac{(2x + 1)(x + 2) \cdot (2x - 1)(x + 4)}{(2x - 1)(x + 2) \cdot (3x + 1)(x + 4)} = \frac{2x + 1}{3x + 1}$$

You Try It 7

Simplify: $\frac{3x^2 + 26x + 16}{3x^2 - 7x - 6} \div \frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$

Your solution

Solutions on p. S13

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6.1 Exercises

Objective A

Simplify.

1. $\frac{9x^3}{12x^4}$

2. $\frac{16x^2y}{24xy^3}$

3. $\frac{(x+3)^2}{(x+3)^3}$

4. $\frac{(2x-1)^5}{(2x-1)^4}$

5. $\frac{3n-4}{4-3n}$

6. $\frac{5-2x}{2x-5}$

7. $\frac{6y(y+2)}{9y^2(y+2)}$

8. $\frac{12x^2(3-x)}{18x(3-x)}$

9. $\frac{6x(x-5)}{8x^2(5-x)}$

10. $\frac{14x^3(7-3x)}{21x(3x-7)}$

11. $\frac{a^2+4a}{ab+4b}$

12. $\frac{x^2-3x}{2x-6}$

13. $\frac{4-6x}{3x^2-2x}$

14. $\frac{5xy-3y}{9-15x}$

15. $\frac{y^2-3y+2}{y^2-4y+3}$

16. $\frac{x^2+5x+6}{x^2+8x+15}$

17. $\frac{x^2+3x-10}{x^2+2x-8}$

18. $\frac{a^2+7a-8}{a^2+6a-7}$

19. $\frac{x^2+x-12}{x^2-6x+9}$

20. $\frac{x^2+8x+16}{x^2-2x-24}$

21. $\frac{x^2-3x-10}{25-x^2}$

22. $\frac{4-y^2}{y^2-3y-10}$

23. $\frac{2x^3+2x^2-4x}{x^3+2x^2-3x}$

24. $\frac{3x^3-12x}{6x^3-24x^2+24x}$

25. $\frac{6x^2-7x+2}{6x^2+5x-6}$

26. $\frac{2n^2-9n+4}{2n^2-5n-12}$

27. $\frac{x^2+3x-28}{24-2x-x^2}$

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Objective B

Simplify.

28. $\frac{8x^2}{9y^3} \cdot \frac{3y^2}{4x^3}$

29. $\frac{14a^2b^3}{15x^5y^2} \cdot \frac{25x^3y}{16ab}$

30. $\frac{12x^3y^4}{7a^2b^3} \cdot \frac{14a^3b^4}{9x^2y^2}$

31. $\frac{18a^4b^2}{25x^2y^3} \cdot \frac{50x^5y^6}{27a^6b^2}$

32. $\frac{3x - 6}{5x - 20} \cdot \frac{10x - 40}{27x - 54}$

33. $\frac{8x - 12}{14x + 7} \cdot \frac{42x + 21}{32x - 48}$

34. $\frac{3x^2 + 2x}{2xy - 3y} \cdot \frac{2xy^3 - 3y^3}{3x^3 + 2x^2}$

35. $\frac{4a^2x - 3a^2}{2by + 5b} \cdot \frac{2b^3y + 5b^3}{4ax - 3a}$

36. $\frac{x^2 + 5x + 4}{x^3y^2} \cdot \frac{x^2y^3}{x^2 + 2x + 1}$

37. $\frac{x^2 + x - 2}{xy^2} \cdot \frac{x^3y}{x^2 + 5x + 6}$

38. $\frac{x^4y^2}{x^2 + 3x - 28} \cdot \frac{x^2 - 49}{xy^4}$

39. $\frac{x^5y^3}{x^2 + 13x + 30} \cdot \frac{x^2 + 2x - 3}{x^7y^2}$

40. $\frac{2x^2 - 5x}{2xy + y} \cdot \frac{2xy^2 + y^2}{5x^2 - 2x^3}$

41. $\frac{3a^3 + 4a^2}{5ab - 3b} \cdot \frac{3b^3 - 5ab^3}{3a^2 + 4a}$

42. $\frac{x^2 - 2x - 24}{x^2 - 5x - 6} \cdot \frac{x^2 + 5x + 6}{x^2 + 6x + 8}$

43. $\frac{x^2 - 8x + 7}{x^2 + 3x - 4} \cdot \frac{x^2 + 3x - 10}{x^2 - 9x + 14}$

44. $\frac{x^2 + 2x - 35}{x^2 + 4x - 21} \cdot \frac{x^2 + 3x - 18}{x^2 + 9x + 18}$

45. $\frac{y^2 + y - 20}{y^2 + 2y - 15} \cdot \frac{y^2 + 4y - 21}{y^2 + 3y - 28}$

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46. $\dfrac{x^2 - 3x - 4}{x^2 + 6x + 5} \cdot \dfrac{x^2 + 5x + 6}{8 + 2x - x^2}$

47. $\dfrac{25 - n^2}{n^2 - 2n - 35} \cdot \dfrac{n^2 - 8n - 20}{n^2 - 3n - 10}$

48. $\dfrac{12x^2 - 6x}{x^2 + 6x + 5} \cdot \dfrac{2x^4 + 10x^3}{4x^2 - 1}$

49. $\dfrac{8x^3 + 4x^2}{x^2 - 3x + 2} \cdot \dfrac{x^2 - 4}{16x^2 + 8x}$

50. $\dfrac{16 + 6x - x^2}{x^2 - 10x - 24} \cdot \dfrac{x^2 - 6x - 27}{x^2 - 17x + 72}$

51. $\dfrac{x^2 - 11x + 28}{x^2 - 13x + 42} \cdot \dfrac{x^2 + 7x + 10}{20 - x - x^2}$

52. $\dfrac{2x^2 + 5x + 2}{2x^2 + 7x + 3} \cdot \dfrac{x^2 - 7x - 30}{x^2 - 6x - 40}$

53. $\dfrac{x^2 - 4x - 32}{x^2 - 8x - 48} \cdot \dfrac{3x^2 + 17x + 10}{3x^2 - 22x - 16}$

Objective C

Simplify.

54. $\dfrac{4x^2y^3}{15a^2b^3} \div \dfrac{6xy}{5a^3b^5}$

55. $\dfrac{9x^3y^4}{16a^4b^2} \div \dfrac{45x^4y^2}{14a^7b}$

56. $\dfrac{6x - 12}{8x + 32} \div \dfrac{18x - 36}{10x + 40}$

57. $\dfrac{28x + 14}{45x - 30} \div \dfrac{14x + 7}{30x - 20}$

58. $\dfrac{6x^3 + 7x^2}{12x - 3} \div \dfrac{6x^2 + 7x}{36x - 9}$

59. $\dfrac{5a^2y + 3a^2}{2x^3 + 5x^2} \div \dfrac{10ay + 6a}{6x^3 + 15x^2}$

60. $\dfrac{x^2 + 4x + 3}{x^2y} \div \dfrac{x^2 + 2x + 1}{xy^2}$

61. $\dfrac{x^3y^2}{x^2 - 3x - 10} \div \dfrac{xy^4}{x^2 - x - 20}$

62. $\dfrac{x^2 - 49}{x^4y^3} \div \dfrac{x^2 - 14x + 49}{x^4y^3}$

63. $\dfrac{x^2y^5}{x^2 - 11x + 30} \div \dfrac{xy^6}{x^2 - 7x + 10}$

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64. $\dfrac{4ax - 8a}{c^2} \div \dfrac{2y - xy}{c^3}$

65. $\dfrac{3x^2y - 9xy}{a^2b} \div \dfrac{3x^2 - x^3}{ab^2}$

66. $\dfrac{x^2 - 5x + 6}{x^2 - 9x + 18} \div \dfrac{x^2 - 6x + 8}{x^2 - 9x + 20}$

67. $\dfrac{x^2 + 3x - 40}{x^2 + 2x - 35} \div \dfrac{x^2 + 2x - 48}{x^2 + 3x - 18}$

68. $\dfrac{x^2 + 2x - 15}{x^2 - 4x - 45} \div \dfrac{x^2 + x - 12}{x^2 - 5x - 36}$

69. $\dfrac{y^2 - y - 56}{y^2 + 8y + 7} \div \dfrac{y^2 - 13y + 40}{y^2 - 4y - 5}$

70. $\dfrac{8 + 2x - x^2}{x^2 + 7x + 10} \div \dfrac{x^2 - 11x + 28}{x^2 - x - 42}$

71. $\dfrac{x^2 - x - 2}{x^2 - 7x + 10} \div \dfrac{x^2 - 3x - 4}{40 - 3x - x^2}$

72. $\dfrac{2x^2 - 3x - 20}{2x^2 - 7x - 30} \div \dfrac{2x^2 - 5x - 12}{4x^2 + 12x + 9}$

73. $\dfrac{6n^2 + 13n + 6}{4n^2 - 9} \div \dfrac{6n^2 + n - 2}{4n^2 - 1}$

APPLYING THE CONCEPTS

74. Given the expression $\dfrac{9}{x^2 + 1}$, choose some values of x and evaluate the expression for those values. Is it possible to choose a value of x for which the value of the expression is greater than 10? If so, what is that value of x? If not, explain why it is not possible.

75. Given the expression $\dfrac{1}{y - 3}$, choose some values of y and evaluate the expression for those values. Is it possible to choose a value of y for which the value of the expression is greater than 10,000,000? If so, what is that value of y? If not, explain why it is not possible.

For what values of x is the algebraic fraction undefined?

76. $\dfrac{x}{(x - 2)(x + 5)}$

77. $\dfrac{7}{x^2 - 25}$

78. $\dfrac{3x - 8}{3x^2 - 10x - 8}$

Simplify.

79. $\dfrac{xy}{3} \cdot \dfrac{x}{y^2} \div \dfrac{x}{4}$

80. $\left(\dfrac{y}{3}\right) \div \left(\dfrac{y}{2} \cdot \dfrac{y}{4}\right)$

81. $\left(\dfrac{x - 4}{y^2}\right)^3 \cdot \left(\dfrac{y}{4 - x}\right)^3$

82. $\dfrac{x - 2}{x + 5} \div \dfrac{x - 3}{x + 5} \cdot \dfrac{x - 3}{x - 2}$

83. $\dfrac{b + 4}{b - 1} \div \dfrac{b + 4}{b + 2} \cdot \dfrac{b - 1}{b - 5}$

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6.2 Expressing Fractions in Terms of the Least Common Multiple (LCM)

Objective A To find the least common multiple (LCM) of two or more polynomials

The **least common multiple (LCM)** of two or more numbers is the smallest number that contains the prime factorization of each number.

The LCM of 12 and 18 is 36 because 36 contains the prime factors of 12 and the prime factors of 18.

$$12 = 2 \cdot 2 \cdot 3$$
$$18 = 2 \cdot 3 \cdot 3$$
$$\text{LCM} = 36 = \overbrace{2 \cdot \underbrace{2 \cdot 3}}^{\text{Factors of 12}} \cdot \underbrace{3}_{\text{Factors of 18}}$$

The least common multiple of two or more polynomials is the polynomial of least degree that contains the factors of each polynomial.

To find the LCM of two or more polynomials, first factor each polynomial completely. The LCM is the product of each factor the greatest number of times it occurs in any one factorization.

➡ Find the LCM of $4x^2 + 4x$ and $x^2 + 2x + 1$.

The LCM of the polynomials is the product of the LCM of the numerical coefficients and each variable factor the greatest number of times it occurs in any one factorization.

$$4x^2 + 4x = 4x(x + 1) = 2 \cdot 2 \cdot x(x + 1)$$
$$x^2 + 2x + 1 = (x + 1)(x + 1)$$
$$\text{LCM} = \overbrace{2 \cdot 2 \cdot \underbrace{x(x + 1)}}^{\text{Factors of } 4x^2 + 4x}\underbrace{(x + 1)}_{\text{Factors of } x^2 + 2x + 1} = 4x(x + 1)(x + 1)$$

TAKE NOTE

The LCM must contain the factors of each polynomial. As shown with the braces at the right, the LCM contains the factors of $4x^2 + 4x$ and the factors of $x^2 + 2x + 1$.

Example 1

Find the LCM of $4x^2y$ and $6xy^2$.

Solution

$4x^2y = 2 \cdot 2 \cdot x \cdot x \cdot y$
$6xy^2 = 2 \cdot 3 \cdot x \cdot y \cdot y$
$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y = 12x^2y^2$

You Try It 1

Find the LCM of $8uv^2$ and $12uw$.

Your solution

Example 2

Find the LCM of $x^2 - x - 6$ and $9 - x^2$.

Solution

$x^2 - x - 6 = (x - 3)(x + 2)$
$9 - x^2 = -(x^2 - 9) = -(x + 3)(x - 3)$
$\text{LCM} = (x - 3)(x + 2)(x + 3)$

You Try It 2

Find the LCM of $m^2 - 6m + 9$ and $m^2 - 2m - 3$.

Your solution

Solutions on p. S13

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Objective B To express two fractions in terms of the LCM of their denominators

20 CT

When adding and subtracting fractions, it is frequently necessary to express two or more fractions in terms of a common denominator. This common denominator is the LCM of the denominators of the fractions.

Write the fractions $\frac{x+1}{4x^2}$ and $\frac{x-3}{6x^2-12x}$ in terms of the LCM of the denominators.

Find the LCM of the denominators.

The LCM is $12x^2(x-2)$.

For each fraction, multiply the numerator and denominator by the factors whose product with the denominator is the LCM.

$$\frac{x+1}{4x^2} = \frac{x+1}{4x^2} \cdot \frac{3(x-2)}{3(x-2)} = \frac{3x^2-3x-6}{12x^2(x-2)}$$

$$\frac{x-3}{6x^2-12x} = \frac{x-3}{6x(x-2)} \cdot \frac{2x}{2x} = \frac{2x^2-6x}{12x(x-2)}$$

LCM

Example 3

Write the fractions $\frac{x+2}{3x^2}$ and $\frac{x-1}{8xy}$ in terms of the LCM of the denominators.

Solution

The LCM is $24x^2y$.

$$\frac{x+2}{3x^2} = \frac{x+2}{3x^2} \cdot \frac{8y}{8y} = \frac{8xy+16y}{24x^2y}$$

$$\frac{x-1}{8xy} = \frac{x-1}{8xy} \cdot \frac{3x}{3x} = \frac{3x^2-3x}{24x^2y}$$

You Try It 3

Write the fractions $\frac{x-3}{4xy^2}$ and $\frac{2x+1}{9y^2z}$ in terms of the LCM of the denominators.

Your solution

Example 4

Write the fractions $\frac{2x-1}{2x-x^2}$ and $\frac{x}{x^2+x-6}$ in terms of the LCM of the denominators.

Solution

$$\frac{2x-1}{2x-x^2} = \frac{2x-1}{-(x^2-2x)} = -\frac{2x-1}{x^2-2x}$$

The LCM is $x(x-2)(x+3)$.

$$\frac{2x-1}{2x-x^2} = -\frac{2x-1}{x(x-2)} \cdot \frac{x+3}{x+3} = -\frac{2x^2+5x-3}{x(x-2)(x+3)}$$

$$\frac{x}{x^2+x-6} = \frac{x}{(x-2)(x+3)} \cdot \frac{x}{x} = \frac{x^2}{x(x-2)(x+3)}$$

You Try It 4

Write the fractions $\frac{x+4}{x^2-3x-10}$ and $\frac{2x}{25-x^2}$ in terms of the LCM of the denominators.

Your solution

Solutions on p. S13

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6.2 Exercises

Objective A

Find the LCM of the expressions.

1. $8x^3y$, $12xy^2$

2. $6ab^2$, $18ab^3$

3. $10x^4y^2$, $15x^3y$

4. $12a^2b$, $18ab^3$

5. $8x^2$, $4x^2 + 8x$

6. $6y^2$, $4y + 12$

7. $2x^2y$, $3x^2 + 12x$

8. $4xy^2$, $6xy^2 + 12y^2$

9. $9x(x + 2)$, $12(x + 2)^2$

10. $8x^2(x - 1)^2$, $10x^3(x - 1)$

11. $3x + 3$, $2x^2 + 4x + 2$

12. $4x - 12$, $2x^2 - 12x + 18$

13. $(x - 1)(x + 2)$, $(x - 1)(x + 3)$

14. $(2x - 1)(x + 4)$, $(2x + 1)(x + 4)$

15. $(2x + 3)^2$, $(2x + 3)(x - 5)$

16. $(x - 7)(x + 2)$, $(x - 7)^2$

17. $x - 1$, $x - 2$, $(x - 1)(x - 2)$

18. $(x + 4)(x - 3)$, $x + 4$, $x - 3$

19. $x^2 - x - 6$, $x^2 + x - 12$

20. $x^2 + 3x - 10$, $x^2 + 5x - 14$

21. $x^2 + 5x + 4$, $x^2 - 3x - 28$

22. $x^2 - 10x + 21$, $x^2 - 8x + 15$

23. $x^2 - 2x - 24$, $x^2 - 36$

24. $x^2 + 7x + 10$, $x^2 - 25$

25. $x^2 - 7x - 30$, $x^2 - 5x - 24$

26. $2x^2 - 7x + 3$, $2x^2 + x - 1$

27. $3x^2 - 11x + 6$, $3x^2 + 4x - 4$

28. $2x^2 - 9x + 10$, $2x^2 + x - 15$

29. $6 + x - x^2$, $x + 2$, $x - 3$

30. $15 + 2x - x^2$, $x - 5$, $x + 3$

31. $5 + 4x - x^2$, $x - 5$, $x + 1$

32. $x^2 + 3x - 18$, $3 - x$, $x + 6$

33. $x^2 - 5x + 6$, $1 - x$, $x - 6$

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Objective B

Write each fraction in terms of the LCM of the denominators.

34. $\frac{4}{x}, \frac{3}{x^2}$

35. $\frac{5}{ab^2}, \frac{6}{ab}$

36. $\frac{x}{3y^2}, \frac{z}{4y}$

37. $\frac{5y}{6x^2}, \frac{7}{9xy}$

38. $\frac{y}{x(x-3)}, \frac{6}{x^2}$

39. $\frac{a}{y^2}, \frac{6}{y(y+5)}$

40. $\frac{9}{(x-1)^2}, \frac{6}{x(x-1)}$

41. $\frac{a^2}{y(y+7)}, \frac{a}{(y+7)^2}$

42. $\frac{3}{x-3}, \frac{5}{x(3-x)}$

43. $\frac{b}{y(y-4)}, \frac{b^2}{4-y}$

44. $\frac{3}{(x-5)^2}, \frac{2}{5-x}$

45. $\frac{3}{7-y}, \frac{2}{(y-7)^2}$

46. $\frac{3}{x^2+2x}, \frac{4}{x^2}$

47. $\frac{2}{y-3}, \frac{3}{y^3-3y^2}$

48. $\frac{x-2}{x+3}, \frac{x}{x-4}$

49. $\frac{x^2}{2x-1}, \frac{x+1}{x+4}$

50. $\frac{3}{x^2+x-2}, \frac{x}{x+2}$

51. $\frac{3x}{x-5}, \frac{4}{x^2-25}$

52. $\frac{x}{x^2+x-6}, \frac{2x}{x^2-9}$

53. $\frac{x-1}{x^2+2x-15}, \frac{x}{x^2+6x+5}$

APPLYING THE CONCEPTS

54. When is the LCM of two expressions equal to their product?

Write each expression in terms of the LCM of the denominators.

55. $\frac{8}{10^3}, \frac{9}{10^5}$

56. $3, \frac{2}{n}$

57. $x, \frac{x}{x^2-1}$

58. $\frac{x^2+1}{(x-1)^3}, \frac{x+1}{(x-1)^2}, \frac{1}{x-1}$

59. $\frac{c}{6c^2+7cd+d^2}, \frac{d}{3c^2-3d^2}$

60. $\frac{1}{ab+3a-3b-b^2}, \frac{1}{ab+3a+3b+b^2}$

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6.3 Addition and Subtraction of Rational Expressions

Objective A To add or subtract rational expressions with the same denominators

When adding rational expressions in which the denominators are the same, add the numerators. The denominator of the sum is the common denominator.

Rule for Adding Fractions

If $\frac{a}{b}$ and $\frac{c}{b}$ are fractions and $b \neq 0$, then $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$.

$$\frac{5x}{18} + \frac{7x}{18} = \frac{5x + 7x}{18} = \frac{12x}{18} = \frac{2x}{3}$$

$$\frac{x}{x^2 - 1} + \frac{1}{x^2 - 1} = \frac{x + 1}{x^2 - 1} = \frac{\cancel{(x + 1)}^{1}}{(x - 1)\cancel{(x + 1)}_{1}} = \frac{1}{x - 1}$$

Note that the sum is written in simplest form.

When subtracting rational expressions with like denominators, subtract the numerators. The denominator of the difference is the common denominator. Write the answer in simplest form.

$$\frac{2x}{x - 2} - \frac{4}{x - 2} = \frac{2x - 4}{x - 2} = \frac{2\cancel{(x - 2)}^{1}}{\cancel{x - 2}_{1}} = 2$$

$$\frac{3x - 1}{x^2 - 5x + 4} - \frac{2x + 3}{x^2 - 5x + 4} = \frac{(3x - 1) - (2x + 3)}{x^2 - 5x + 4} = \frac{3x - 1 - 2x - 3}{x^2 - 5x + 4}$$

$$= \frac{x - 4}{x^2 - 5x + 4} = \frac{\cancel{(x - 4)}^{1}}{\cancel{(x - 4)}_{1}(x - 1)} = \frac{1}{x - 1}$$

Example 1

Simplify: $\frac{3x^2}{x^2 - 1} - \frac{x + 4}{x^2 - 1}$

Solution

$$\frac{3x^2}{x^2 - 1} - \frac{x + 4}{x^2 - 1} = \frac{3x^2 - (x + 4)}{x^2 - 1}$$

$$= \frac{3x^2 - x - 4}{x^2 - 1}$$

$$= \frac{(3x - 4)\cancel{(x + 1)}^{1}}{(x - 1)\cancel{(x + 1)}_{1}} = \frac{3x - 4}{x - 1}$$

You Try It 1

Simplify: $\frac{2x^2}{x^2 - x - 12} - \frac{7x + 4}{x^2 - x - 12}$

Your solution

Solution on p. S13

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Example 2

Simplify:

$$\frac{2x^2 + 5}{x^2 + 2x - 3} - \frac{x^2 - 3x}{x^2 + 2x - 3} + \frac{x - 2}{x^2 + 2x - 3}$$

Solution

$$\frac{2x^2 + 5}{x^2 + 2x - 3} - \frac{x^2 - 3x}{x^2 + 2x - 3} + \frac{x - 2}{x^2 + 2x - 3}$$

$$= \frac{(2x^2 + 5) - (x^2 - 3x) + (x - 2)}{x^2 + 2x - 3}$$

$$= \frac{2x^2 + 5 - x^2 + 3x + x - 2}{x^2 + 2x - 3}$$

$$= \frac{x^2 + 4x + 3}{x^2 + 2x - 3} = \frac{\cancel{(x + 3)}^{1}(x + 1)}{\cancel{(x + 3)}_{1}(x - 1)} = \frac{x + 1}{x - 1}$$

You Try It 2

Simplify:

$$\frac{x^2 - 1}{x^2 - 8x + 12} - \frac{2x + 1}{x^2 - 8x + 12} + \frac{x}{x^2 - 8x + 12}$$

Your solution

Solution on p. S13

Objective B To add or subtract rational expressions with different denominators

20 CT

Before two fractions with unlike denominators can be added or subtracted, each fraction must be expressed in terms of a common denominator. This common denominator is the LCM of the denominators of the fractions.

➡ Simplify: $\frac{x - 3}{x^2 - 2x} + \frac{6}{x^2 - 4}$

The LCM is $x(x - 2)(x + 2)$.

• Find the LCM of the denominators.

$$\frac{x - 3}{x^2 - 2x} + \frac{6}{x^2 - 4} = \frac{x - 3}{x(x - 2)} \cdot \frac{x + 2}{x + 2} + \frac{6}{(x - 2)(x + 2)} \cdot \frac{x}{x}$$

• Write each fraction in terms of the LCM.

$$= \frac{x^2 - x - 6}{x(x - 2)(x + 2)} + \frac{6x}{x(x - 2)(x + 2)}$$

$$= \frac{(x^2 - x - 6) + 6x}{x(x - 2)(x + 2)}$$

$$= \frac{x^2 + 5x - 6}{x(x - 2)(x + 2)}$$

$$= \frac{(x + 6)(x - 1)}{x(x - 2)(x + 2)}$$

• Multiply the factors in the numerator and then add the fractions.

The last step is to factor the numerator to determine whether there are common factors in the numerator and denominator. For this example, there are no common factors, so the answer is in simplest form.

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Example 3

Simplify: $\frac{y}{x} - \frac{4y}{3x} + \frac{3y}{4x}$

Solution

The LCM of the denominators is $12x$.

$$\frac{y}{x} - \frac{4y}{3x} + \frac{3y}{4x} = \frac{y}{x} \cdot \frac{12}{12} - \frac{4y}{3x} \cdot \frac{4}{4} + \frac{3y}{4x} \cdot \frac{3}{3}$$

$$= \frac{12y}{12x} - \frac{16y}{12x} + \frac{9y}{12x}$$

$$= \frac{12y - 16y + 9y}{12x} = \frac{5y}{12x}$$

You Try It 3

Simplify: $\frac{z}{8y} - \frac{4z}{3y} + \frac{5z}{4y}$

Your solution

Example 4

Simplify: $\frac{2x}{x-3} - \frac{5}{3-x}$

Solution

Remember $3 - x = -(x - 3)$.

Therefore, $\frac{5}{3-x} = \frac{5}{-(x-3)} = \frac{-5}{x-3}$.

$$\frac{2x}{x-3} - \frac{5}{3-x} = \frac{2x}{x-3} - \frac{-5}{x-3}$$

$$= \frac{2x - (-5)}{x-3} = \frac{2x+5}{x-3}$$

You Try It 4

Simplify: $\frac{5x}{x-2} - \frac{3}{2-x}$

Your solution

Example 5

Simplify: $\frac{2x}{2x-3} - \frac{1}{x+1}$

Solution

The LCM is $(2x - 3)(x + 1)$.

$$\frac{2x}{2x-3} - \frac{1}{x+1}$$

$$= \frac{2x}{2x-3} \cdot \frac{x+1}{x+1} - \frac{1}{x+1} \cdot \frac{2x-3}{2x-3}$$

$$= \frac{2x^2 + 2x}{(2x-3)(x+1)} - \frac{2x-3}{(2x-3)(x+1)}$$

$$= \frac{(2x^2 + 2x) - (2x - 3)}{(2x-3)(x+1)} = \frac{2x^2+3}{(2x-3)(x+1)}$$

You Try It 5

Simplify: $\frac{4x}{3x-1} - \frac{9}{x+4}$

Your solution

Solutions on p. S13

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Example 6

Simplify: $1 + \frac{3}{x^2}$

Solution

The LCM is x^2.

$$1 + \frac{3}{x^2} = 1 \cdot \frac{x^2}{x^2} + \frac{3}{x^2}$$

$$= \frac{x^2}{x^2} + \frac{3}{x^2}$$

$$= \frac{x^2 + 3}{x^2}$$

You Try It 6

Simplify: $2 - \frac{1}{x - 3}$

Your solution

Example 7

Simplify: $\frac{x + 3}{x^2 - 2x - 8} + \frac{3}{4 - x}$

Solution

The LCM is $(x - 4)(x + 2)$.

Recall: $\frac{3}{4 - x} = \frac{-3}{x - 4}$

$$\frac{x + 3}{x^2 - 2x - 8} + \frac{3}{4 - x}$$

$$= \frac{x + 3}{(x - 4)(x + 2)} + \frac{(-3)}{x - 4}$$

$$= \frac{x + 3}{(x - 4)(x + 2)} + \frac{(-3)}{x - 4} \cdot \frac{x + 2}{x + 2}$$

$$= \frac{x + 3}{(x - 4)(x + 2)} + \frac{(-3)(x + 2)}{(x - 4)(x + 2)}$$

$$= \frac{(x + 3) + (-3)(x + 2)}{(x - 4)(x + 2)}$$

$$= \frac{x + 3 - 3x - 6}{(x - 4)(x + 2)} = \frac{-2x - 3}{(x - 4)(x + 2)}$$

You Try It 7

Simplify: $\frac{2x - 1}{x^2 - 25} + \frac{2}{5 - x}$

Your solution

Example 8

Simplify: $\frac{3x + 2}{2x^2 - x - 1} - \frac{3}{2x + 1} + \frac{4}{x - 1}$

Solution

The LCM is $(2x + 1)(x - 1)$.

$$\frac{3x + 2}{2x^2 - x - 1} - \frac{3}{2x + 1} + \frac{4}{x - 1}$$

$$= \frac{3x + 2}{(2x + 1)(x - 1)} - \frac{3}{2x + 1} \cdot \frac{x - 1}{x - 1} + \frac{4}{x - 1} \cdot \frac{2x + 1}{2x + 1}$$

$$= \frac{3x + 2}{(2x + 1)(x - 1)} - \frac{3x - 3}{(2x + 1)(x - 1)} + \frac{8x + 4}{(2x + 1)(x - 1)}$$

$$= \frac{(3x + 2) - (3x - 3) + (8x + 4)}{(2x + 1)(x - 1)}$$

$$= \frac{3x + 2 - 3x + 3 + 8x + 4}{(2x + 1)(x - 1)} = \frac{8x + 9}{(2x + 1)(x - 1)}$$

You Try It 8

Simplify: $\frac{2x - 3}{3x^2 - x - 2} + \frac{5}{3x + 2} - \frac{1}{x - 1}$

Your solution

Solutions on p. S14

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6.3 Exercises

Objective A

Simplify.

1. $\frac{3}{y^2} + \frac{8}{y^2}$

2. $\frac{6}{ab} - \frac{2}{ab}$

3. $\frac{3}{x+4} - \frac{10}{x+4}$

4. $\frac{x}{x+6} - \frac{2}{x+6}$

5. $\frac{3x}{2x+3} + \frac{5x}{2x+3}$

6. $\frac{6y}{4y+1} - \frac{11y}{4y+1}$

7. $\frac{2x+1}{x-3} + \frac{3x+6}{x-3}$

8. $\frac{4x+3}{2x-7} + \frac{3x-8}{2x-7}$

9. $\frac{5x-1}{x+9} - \frac{3x+4}{x+9}$

10. $\frac{6x-5}{x-10} - \frac{3x-4}{x-10}$

11. $\frac{x-7}{2x+7} - \frac{4x-3}{2x+7}$

12. $\frac{2n}{3n+4} - \frac{5n-3}{3n+4}$

13. $\frac{x}{x^2+2x-15} - \frac{3}{x^2+2x-15}$

14. $\frac{3x}{x^2+3x-10} - \frac{6}{x^2+3x-10}$

15. $\frac{2x+3}{x^2-x-30} - \frac{x-2}{x^2-x-30}$

16. $\frac{3x-1}{x^2+5x-6} - \frac{2x-7}{x^2+5x-6}$

17. $\frac{4y+7}{2y^2+7y-4} - \frac{y-5}{2y^2+7y-4}$

18. $\frac{x+1}{2x^2-5x-12} + \frac{x+2}{2x^2-5x-12}$

19. $\frac{2x^2+3x}{x^2-9x+20} + \frac{2x^2-3}{x^2-9x+20} - \frac{4x^2+2x+1}{x^2-9x+20}$

20. $\frac{2x^2+3x}{x^2-2x-63} - \frac{x^2-3x+21}{x^2-2x-63} - \frac{x-7}{x^2-2x-63}$

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Objective B

Simplify.

21. $\frac{4}{x} + \frac{5}{y}$

22. $\frac{7}{a} + \frac{5}{b}$

23. $\frac{12}{x} - \frac{5}{2x}$

24. $\frac{5}{3a} - \frac{3}{4a}$

25. $\frac{1}{2x} - \frac{5}{4x} + \frac{7}{6x}$

26. $\frac{7}{4y} + \frac{11}{6y} - \frac{8}{3y}$

27. $\frac{5}{3x} - \frac{2}{x^2} + \frac{3}{2x}$

28. $\frac{6}{y^2} + \frac{3}{4y} - \frac{2}{5y}$

29. $\frac{2}{x} - \frac{3}{2y} + \frac{3}{5x} - \frac{1}{4y}$

30. $\frac{5}{2a} + \frac{7}{3b} - \frac{2}{b} - \frac{3}{4a}$

31. $\frac{2x+1}{3x} + \frac{x-1}{5x}$

32. $\frac{4x-3}{6x} + \frac{2x+3}{4x}$

33. $\frac{x-3}{6x} + \frac{x+4}{8x}$

34. $\frac{2x-3}{2x} + \frac{x+3}{3x}$

35. $\frac{2x+9}{9x} - \frac{x-5}{5x}$

36. $\frac{3y-2}{12y} - \frac{y-3}{18y}$

37. $\frac{x+4}{2x} - \frac{x-1}{x^2}$

38. $\frac{x-2}{3x^2} - \frac{x+4}{x}$

39. $\frac{x-10}{4x^2} + \frac{x+1}{2x}$

40. $\frac{x+5}{3x^2} + \frac{2x+1}{2x}$

41. $\frac{4}{x+4} - x$

42. $2x + \frac{1}{x}$

43. $5 - \frac{x-2}{x+1}$

44. $3 + \frac{x-1}{x+1}$

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45. $\frac{x+3}{6x} - \frac{x-3}{8x^2}$

46. $\frac{x+2}{xy} - \frac{3x-2}{x^2y}$

47. $\frac{3x-1}{xy^2} - \frac{2x+3}{xy}$

48. $\frac{4x-3}{3x^2y} + \frac{2x+1}{4xy^2}$

49. $\frac{5x+7}{6xy^2} - \frac{4x-3}{8x^2y}$

50. $\frac{x-2}{8x^2} - \frac{x+7}{12xy}$

51. $\frac{3x-1}{6y^2} - \frac{x+5}{9xy}$

52. $\frac{4}{x-2} + \frac{5}{x+3}$

53. $\frac{2}{x-3} + \frac{5}{x-4}$

54. $\frac{6}{x-7} - \frac{4}{x+3}$

55. $\frac{3}{y+6} - \frac{4}{y-3}$

56. $\frac{2x}{x+1} + \frac{1}{x-3}$

57. $\frac{3x}{x-4} + \frac{2}{x+6}$

58. $\frac{4x}{2x-1} - \frac{5}{x-6}$

59. $\frac{6x}{x+5} - \frac{3}{2x+3}$

60. $\frac{2a}{a-7} + \frac{5}{7-a}$

61. $\frac{4x}{6-x} + \frac{5}{x-6}$

62. $\frac{x}{x^2-9} + \frac{3}{x-3}$

63. $\frac{y}{y^2-16} + \frac{1}{y-4}$

64. $\frac{2x}{x^2-x-6} - \frac{3}{x+2}$

65. $\frac{(x-1)^2}{(x+1)^2} - 1$

66. $1 - \frac{(y-2)^2}{(y+2)^2}$

67. $\frac{x}{1-x^2} - 1 + \frac{x}{1+x}$

68. $\frac{y}{x-y} + 2 - \frac{x}{y-x}$

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69. $\dfrac{3x-1}{x^2-10x+25}-\dfrac{3}{x-5}$

70. $\dfrac{2a+3}{a^2-7a+12}-\dfrac{2}{a-3}$

71. $\dfrac{x+4}{x^2-x-42}+\dfrac{3}{7-x}$

72. $\dfrac{x+3}{x^2-3x-10}+\dfrac{2}{5-x}$

73. $\dfrac{1}{x+1}+\dfrac{x}{x-6}-\dfrac{5x-2}{x^2-5x-6}$

74. $\dfrac{x}{x-4}+\dfrac{5}{x+5}-\dfrac{11x-8}{x^2+x-20}$

75. $\dfrac{3x+1}{x-1}-\dfrac{x-1}{x-3}+\dfrac{x+1}{x^2-4x+3}$

76. $\dfrac{4x+1}{x-8}-\dfrac{3x+2}{x+4}-\dfrac{49x+4}{x^2-4x-32}$

77. $\dfrac{2x+9}{3-x}+\dfrac{x+5}{x+7}-\dfrac{2x^2+3x-3}{x^2+4x-21}$

78. $\dfrac{3x+5}{x+5}-\dfrac{x+1}{2-x}-\dfrac{4x^2-3x-1}{x^2+3x-10}$

APPLYING THE CONCEPTS

79. Find the sum of the following:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}$$

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}$$

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}$$

Note the pattern in these sums, and find the sum of 50 terms, of 100 terms, and of 1000 terms.

80. In your own words, explain the procedure for adding rational expressions with different denominators.

Simplify.

81. $\dfrac{x^2+x-6}{x^2+2x-8}\cdot\dfrac{x^2+5x+4}{x^2+2x-3}-\dfrac{2}{x-1}$

82. $\dfrac{x^2+9x+20}{x^2+4x-5}-\dfrac{x^2-49}{x^2+6x-7}-\dfrac{x}{x-7}$

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6.4 Complex Fractions

Objective A To simplify a complex fraction

21 CT

POINT OF INTEREST

There are many instances of complex fractions in application problems. The fraction $\dfrac{1}{\frac{1}{r_1}+\frac{1}{r_2}}$ is used to determine the total resistance in certain electric circuits.

A **complex fraction** is a fraction whose numerator or denominator contains one or more fractions. Examples of complex fractions are shown at the right.

$$\frac{3}{2-\frac{1}{2}}, \quad \frac{4+\frac{1}{x}}{3+\frac{2}{x}}, \quad \frac{\frac{1}{x-1}+x+3}{x-3+\frac{1}{x+4}}$$

➡ Simplify: $\dfrac{1-\frac{4}{x^2}}{1+\frac{2}{x}}$

The LCM of x and x^2 is x^2.

- Find the LCM of the denominators of the fractions in the numerator and denominator.

$$\frac{1-\frac{4}{x^2}}{1+\frac{2}{x}} = \frac{1-\frac{4}{x^2}}{1+\frac{2}{x}} \cdot \frac{x^2}{x^2}$$

- Multiply the numerator and denominator by the LCM.

$$= \frac{1 \cdot x^2 - \frac{4}{x^2} \cdot x^2}{1 \cdot x^2 + \frac{2}{x} \cdot x^2}$$

- Simplify.

$$= \frac{x^2-4}{x^2+2x} = \frac{(x-2)\cancel{(x+2)}^{1}}{x\cancel{(x+2)}_{1}}$$

$$= \frac{x-2}{x}$$

Example 1

Simplify: $\dfrac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}}$

Solution

The LCM of x, 2, x^2, and 4 is $4x^2$.

$$\frac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}} = \frac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}} \cdot \frac{4x^2}{4x^2} = \frac{\frac{1}{x} \cdot 4x^2 + \frac{1}{2} \cdot 4x^2}{\frac{1}{x^2} \cdot 4x^2 - \frac{1}{4} \cdot 4x^2}$$

$$= \frac{4x+2x^2}{4-x^2} = \frac{2x\cancel{(2+x)}^{1}}{(2-x)\cancel{(2+x)}_{1}} = \frac{2x}{2-x}$$

You Try It 1

Simplify: $\dfrac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}}$

Your solution

Solution on p. S14

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Example 2

Simplify: $\dfrac{1-\frac{2}{x}-\frac{15}{x^2}}{1-\frac{11}{x}+\frac{30}{x^2}}$

Solution

The LCM of x and x^2 is x^2.

$$\frac{1-\frac{2}{x}-\frac{15}{x^2}}{1-\frac{11}{x}+\frac{30}{x^2}} = \frac{1-\frac{2}{x}-\frac{15}{x^2}}{1-\frac{11}{x}+\frac{30}{x^2}} \cdot \frac{x^2}{x^2}$$

$$= \frac{1 \cdot x^2 - \frac{2}{x} \cdot x^2 - \frac{15}{x^2} \cdot x^2}{1 \cdot x^2 - \frac{11}{x} \cdot x^2 + \frac{30}{x^2} \cdot x^2}$$

$$= \frac{x^2 - 2x - 15}{x^2 - 11x + 30}$$

$$= \frac{\cancel{(x-5)}^{1}(x+3)}{\cancel{(x-5)}_{1}(x-6)} = \frac{x+3}{x-6}$$

You Try It 2

Simplify: $\dfrac{1+\frac{4}{x}+\frac{3}{x^2}}{1+\frac{10}{x}+\frac{21}{x^2}}$

Your solution

Example 3

Simplify: $\dfrac{x-8+\frac{20}{x+4}}{x-10+\frac{24}{x+4}}$

Solution

The LCM is $x+4$.

$$\frac{x-8+\frac{20}{x+4}}{x-10+\frac{24}{x+4}}$$

$$= \frac{x-8+\frac{20}{x+4}}{x-10+\frac{24}{x+4}} \cdot \frac{x+4}{x+4}$$

$$= \frac{(x-8)(x+4) + \frac{20}{x+4} \cdot (x+4)}{(x-10)(x+4) + \frac{24}{x+4} \cdot (x+4)}$$

$$= \frac{x^2 - 4x - 32 + 20}{x^2 - 6x - 40 + 24} = \frac{x^2 - 4x - 12}{x^2 - 6x - 16}$$

$$= \frac{(x-6)\cancel{(x+2)}^{1}}{(x-8)\cancel{(x+2)}_{1}} = \frac{x-6}{x-8}$$

You Try It 3

Simplify: $\dfrac{x+3-\frac{20}{x-5}}{x+8+\frac{30}{x-5}}$

Your solution

Solutions on p. S14

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6.4 Exercises

Objective A

Simplify.

1. $\dfrac{1+\frac{3}{x}}{1-\frac{9}{x^2}}$

2. $\dfrac{1+\frac{4}{x}}{1-\frac{16}{x^2}}$

3. $\dfrac{2-\frac{8}{x+4}}{3-\frac{12}{x+4}}$

4. $\dfrac{5-\frac{25}{x+5}}{1-\frac{3}{x+5}}$

5. $\dfrac{1+\frac{5}{y-2}}{1-\frac{2}{y-2}}$

6. $\dfrac{2-\frac{11}{2x-1}}{3-\frac{17}{2x-1}}$

7. $\dfrac{4-\frac{2}{x+7}}{5+\frac{1}{x+7}}$

8. $\dfrac{5+\frac{3}{x-8}}{2-\frac{1}{x-8}}$

9. $\dfrac{1-\frac{1}{x}-\frac{6}{x^2}}{1-\frac{9}{x^2}}$

10. $\dfrac{1+\frac{4}{x}+\frac{4}{x^2}}{1-\frac{2}{x}-\frac{8}{x^2}}$

11. $\dfrac{1-\frac{5}{x}-\frac{6}{x^2}}{1+\frac{6}{x}+\frac{5}{x^2}}$

12. $\dfrac{1-\frac{7}{a}+\frac{12}{a^2}}{1+\frac{1}{a}-\frac{20}{a^2}}$

13. $\dfrac{1-\frac{6}{x}+\frac{8}{x^2}}{\frac{4}{x^2}+\frac{3}{x}-1}$

14. $\dfrac{1+\frac{3}{x}-\frac{18}{x^2}}{\frac{21}{x^2}-\frac{4}{x}-1}$

15. $\dfrac{x-\frac{4}{x+3}}{1+\frac{1}{x+3}}$

16. $\dfrac{y+\frac{1}{y-2}}{1+\frac{1}{y-2}}$

17. $\dfrac{1-\frac{x}{2x+1}}{x-\frac{1}{2x+1}}$

18. $\dfrac{1-\frac{2x-2}{3x-1}}{x-\frac{4}{3x-1}}$

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19. $\dfrac{x-5+\dfrac{14}{x+4}}{x+3-\dfrac{2}{x+4}}$

20. $\dfrac{a+4+\dfrac{5}{a-2}}{a+6+\dfrac{15}{a-2}}$

21. $\dfrac{x+3-\dfrac{10}{x-6}}{x+2-\dfrac{20}{x-6}}$

22. $\dfrac{x-7+\dfrac{5}{x-1}}{x-3+\dfrac{1}{x-1}}$

23. $\dfrac{y-6+\dfrac{22}{2y+3}}{y-5+\dfrac{11}{2y+3}}$

24. $\dfrac{x+2-\dfrac{12}{2x-1}}{x+1-\dfrac{9}{2x-1}}$

25. $\dfrac{x-\dfrac{2}{2x-3}}{2x-1-\dfrac{8}{2x-3}}$

26. $\dfrac{x+3-\dfrac{18}{2x+1}}{x-\dfrac{6}{2x+1}}$

27. $\dfrac{\dfrac{1}{x}-\dfrac{2}{x-1}}{\dfrac{3}{x}+\dfrac{1}{x-1}}$

28. $\dfrac{\dfrac{3}{n+1}+\dfrac{1}{n}}{\dfrac{2}{n+1}+\dfrac{3}{n}}$

29. $\dfrac{\dfrac{3}{2x-1}-\dfrac{1}{x}}{\dfrac{4}{x}+\dfrac{2}{2x-1}}$

30. $\dfrac{\dfrac{4}{3x+1}+\dfrac{3}{x}}{\dfrac{6}{x}-\dfrac{2}{3x+1}}$

APPLYING THE CONCEPTS

Simplify.

31. $1+\dfrac{1}{1+\dfrac{1}{2}}$

32. $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{2}}}$

33. $1-\dfrac{1}{1-\dfrac{1}{x}}$

34. $\dfrac{a^{-1}-b^{-1}}{a^{-2}-b^{-2}}$

35. $\left(\dfrac{y}{4}-\dfrac{4}{y}\right)\div\left(\dfrac{4}{y}-3+\dfrac{y}{2}\right)$

36. $\left(\dfrac{b}{8}-\dfrac{8}{b}\right)\div\left(\dfrac{8}{b}-5+\dfrac{b}{2}\right)$

37. $\dfrac{1+x^{-1}}{1-x^{-1}}$

38. $\dfrac{x+x^{-1}}{x-x^{-1}}$

39. $\dfrac{x^{-1}}{y^{-1}}+\dfrac{x}{y}$

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6.5 Solving Equations Containing Fractions

Objective A To solve an equation containing fractions



To solve an equation containing fractions, **clear denominators** by multiplying each side of the equation by the LCM of the denominators. Then solve for the variable.

➡ Solve: $\frac{3x-1}{4}+\frac{2}{3}=\frac{7}{6}$

$$\frac{3x-1}{4}+\frac{2}{3}=\frac{7}{6}$$

$$12\left(\frac{3x-1}{4}+\frac{2}{3}\right)=12\cdot\frac{7}{6}$$

• The LCM is 12. Multiply each side of the equation by the LCM.

$$12\left(\frac{3x-1}{4}\right)+12\cdot\frac{2}{3}=12\cdot\frac{7}{6}$$

• Simplify using the Distributive Property and the Properties of Fractions.

$$\frac{\overset{3}{\cancel{12}}}{1}\left(\frac{3x-1}{\underset{1}{\cancel{4}}}\right)+\frac{\overset{4}{\cancel{12}}}{1}\cdot\frac{2}{\underset{1}{\cancel{3}}}=\frac{\overset{2}{\cancel{12}}}{1}\cdot\frac{7}{\underset{1}{\cancel{6}}}$$

$$9x-3+8=14$$

• Solve for x.

$$9x+5=14$$

$$9x=9$$

$$x=1$$

1 checks as a solution. The solution is 1.

Occasionally, a value of the variable that appears to be a solution of an equation will make one of the denominators zero. In this case, that value is not a solution of the equation.

➡ Solve: $\frac{2x}{x-2}=1+\frac{4}{x-2}$

$$\frac{2x}{x-2}=1+\frac{4}{x-2}$$

$$(x-2)\frac{2x}{x-2}=(x-2)\left(1+\frac{4}{x-2}\right)$$

• The LCM is $x-2$. Multiply each side of the equation by the LCM.

$$(x-2)\frac{2x}{x-2}=(x-2)\cdot 1+(x-2)\frac{4}{x-2}$$

• Simplify using the Distributive Property and the Properties of Fractions.

$$\frac{\overset{1}{\cancel{(x-2)}}}{1}\cdot\frac{2x}{\underset{1}{\cancel{x-2}}}=(x-2)\cdot 1+\frac{\overset{1}{\cancel{(x-2)}}}{1}\cdot\frac{4}{\underset{1}{\cancel{x-2}}}$$

$$2x=x-2+4$$

• Solve for x.

$$2x=x+2$$

$$x=2$$

When x is replaced by 2, the denominators of $\frac{2x}{x-2}$ and $\frac{4}{x-2}$ are zero. Therefore, the equation has no solution.

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Example 1

Solve: $\frac{x}{x+4} = \frac{2}{x}$

Solution

The LCM is $x(x + 4)$.

$$\frac{x}{x+4} = \frac{2}{x}$$

$$x(x+4)\left(\frac{x}{x+4}\right) = x(x+4)\left(\frac{2}{x}\right)$$

$$\frac{\overset{1}{\cancel{x(x+4)}}}{1} \cdot \frac{x}{\underset{1}{\cancel{x+4}}} = \frac{\overset{1}{\cancel{x}}(x+4)}{1} \cdot \frac{2}{\underset{1}{\cancel{x}}}$$

$$x^2 = (x+4)2$$
$$x^2 = 2x + 8$$

Solve the quadratic equation by factoring.

$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x - 4 = 0 \qquad x + 2 = 0$$
$$x = 4 \qquad x = -2$$

Both 4 and -2 check as solutions.
The solutions are 4 and -2.

You Try It 1

Solve: $\frac{x}{x+6} = \frac{3}{x}$

Your solution

Example 2

Solve: $\frac{3x}{x-4} = 5 + \frac{12}{x-4}$

Solution

The LCM is $x - 4$.

$$\frac{3x}{x-4} = 5 + \frac{12}{x-4}$$

$$(x-4)\left(\frac{3x}{x-4}\right) = (x-4)\left(5 + \frac{12}{x-4}\right)$$

$$\frac{\overset{1}{\cancel{(x-4)}}}{1} \cdot \frac{3x}{\underset{1}{\cancel{x-4}}} = (x-4)5 + \frac{\overset{1}{\cancel{(x-4)}}}{1} \cdot \frac{12}{\underset{1}{\cancel{x-4}}}$$

$$3x = (x-4)5 + 12$$
$$3x = 5x - 20 + 12$$
$$3x = 5x - 8$$
$$-2x = -8$$
$$x = 4$$

4 does not check as a solution.
The equation has no solution.

You Try It 2

Solve: $\frac{5x}{x+2} = 3 - \frac{10}{x+2}$

Your solution

Solutions on p. S15

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6.5 Exercises

Objective A

Solve.

1. $\frac{2x}{3} - \frac{5}{2} = -\frac{1}{2}$

2. $\frac{x}{3} - \frac{1}{4} = \frac{1}{12}$

3. $\frac{x}{3} - \frac{1}{4} = \frac{x}{4} - \frac{1}{6}$

4. $\frac{2y}{9} - \frac{1}{6} = \frac{y}{9} + \frac{1}{6}$

5. $\frac{2x - 5}{8} + \frac{1}{4} = \frac{x}{8} + \frac{3}{4}$

6. $\frac{3x + 4}{12} - \frac{1}{3} = \frac{5x + 2}{12} - \frac{1}{2}$

7. $\frac{6}{2a + 1} = 2$

8. $\frac{12}{3x - 2} = 3$

9. $\frac{9}{2x - 5} = -2$

10. $\frac{6}{4 - 3x} = 3$

11. $2 + \frac{5}{x} = 7$

12. $3 + \frac{8}{n} = 5$

13. $1 - \frac{9}{x} = 4$

14. $3 - \frac{12}{x} = 7$

15. $\frac{2}{y} + 5 = 9$

16. $\frac{6}{x} + 3 = 11$

17. $\frac{3}{x - 2} = \frac{4}{x}$

18. $\frac{5}{x + 3} = \frac{3}{x - 1}$

19. $\frac{2}{3x - 1} = \frac{3}{4x + 1}$

20. $\frac{5}{3x - 4} = \frac{-3}{1 - 2x}$

21. $\frac{-3}{2x + 5} = \frac{2}{x - 1}$

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22. $\frac{4}{5y-1} = \frac{2}{2y-1}$

23. $\frac{4x}{x-4} + 5 = \frac{5x}{x-4}$

24. $\frac{2x}{x+2} - 5 = \frac{7x}{x+2}$

25. $2 + \frac{3}{a-3} = \frac{a}{a-3}$

26. $\frac{x}{x+4} = 3 - \frac{4}{x+4}$

27. $\frac{x}{x-1} = \frac{8}{x+2}$

28. $\frac{x}{x+12} = \frac{1}{x+5}$

29. $\frac{2x}{x+4} = \frac{3}{x-1}$

30. $\frac{5}{3n-8} = \frac{n}{n+2}$

31. $x + \frac{6}{x-2} = \frac{3x}{x-2}$

32. $x - \frac{6}{x-3} = \frac{2x}{x-3}$

33. $\frac{8}{y} = \frac{2}{y-2} + 1$

APPLYING THE CONCEPTS

34. Explain the procedure for solving an equation containing fractions. Include in your discussion how the LCM is used to eliminate fractions in the equation.

Solve.

35. $\frac{3}{5}y - \frac{1}{3}(1-y) = \frac{2y-5}{15}$

36. $\frac{3}{4}a = \frac{1}{2}(3-a) + \frac{a-2}{4}$

37. $\frac{b+2}{5} = \frac{1}{4}b - \frac{3}{10}(b-1)$

38. $\frac{x}{2x^2-x-1} = \frac{3}{x^2-1} + \frac{3}{2x+1}$

39. $\frac{x+1}{x^2+x-2} = \frac{x+2}{x^2-1} + \frac{3}{x+2}$

40. $\frac{y+2}{y^2-y-2} + \frac{y+1}{y^2-4} = \frac{1}{y+1}$

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6.6 Ratio and Proportion

Objective A To solve a proportion

Quantities such as 4 meters, 15 seconds, and 8 gallons are number quantities written with units. In these examples the units are meters, seconds, and gallons.

A **ratio** is the quotient of two quantities that have the same unit.

The length of a living room is 16 ft and the width is 12 ft. The ratio of the length to the width is written

$$\frac{16 \text{ ft}}{12 \text{ ft}} = \frac{16}{12} = \frac{4}{3}$$

A ratio is in simplest form when the two numbers do not have a common factor. Note that the units are not written.

A **rate** is the quotient of two quantities that have different units.

There are 2 lb of salt in 8 gal of water. The salt-to-water rate is

$$\frac{2 \text{ lb}}{8 \text{ gal}} = \frac{1 \text{ lb}}{4 \text{ gal}}$$

A rate is in simplest form when the two numbers do not have a common factor. The units are written as part of the rate.

A **proportion** is an equation that states the equality of two ratios or rates. Examples of proportions are shown at the right.

$$\frac{30 \text{ mi}}{4 \text{ h}} = \frac{15 \text{ mi}}{2 \text{ h}}$$

$$\frac{4}{6} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{x}{8}$$

Solve the proportion $\frac{4}{x} = \frac{2}{3}$.

$$\frac{4}{x} = \frac{2}{3}$$

$$3x\left(\frac{4}{x}\right) = 3x\left(\frac{2}{3}\right)$$

The LCM of the denominators is $3x$. Multiply each side of the proportion by the LCM.

$$12 = 2x$$

Solve the equation.

$$6 = x$$

The solution is 6.

Example 1

Solve the proportion $\frac{8}{x+3} = \frac{4}{x}$.

Solution

$$\frac{8}{x+3} = \frac{4}{x}$$

$$x(x+3)\frac{8}{x+3} = x(x+3)\frac{4}{x}$$

$$8x = 4(x+3)$$

$$8x = 4x + 12$$

$$4x = 12$$

$$x = 3$$

The solution is 3.

You Try It 1

Solve the proportion $\frac{2}{x+3} = \frac{6}{5x+5}$.

Your solution

Solution on p. S15

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Objective B To solve application problems

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Example 2

The monthly loan payment for a car is \$28.35 for each \$1000 borrowed. At this rate, find the monthly payment for a \$6000 car loan.

Strategy

To find the monthly payment, write and solve a proportion, using P to represent the monthly car payment.

Solution

$$\frac{28.35}{1000} = \frac{P}{6000}$$

$$6000\left(\frac{28.35}{1000}\right) = 6000\left(\frac{P}{6000}\right)$$

$$170.10 = P$$

The monthly payment is \$170.10.

You Try It 2

Sixteen ceramic tiles are needed to tile a 9-square-foot area. At this rate how many square feet can be tiled using 256 ceramic tiles?

Your strategy

Your solution

Solution on p. S15

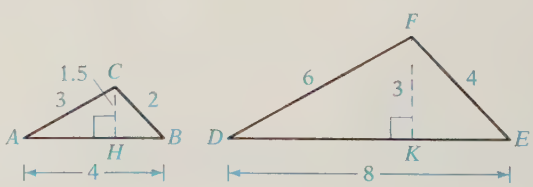
Objective C To solve problems involving similar triangles

CT

Similar objects have the same shape but not necessarily the same size. A tennis ball is similar to a basketball. A model ship is similar to an actual ship.

Similar objects have corresponding parts; for example, the rudder on the model ship corresponds to the rudder on the actual ship. The relationship between the sizes of each of the corresponding parts can be written as a ratio, and each ratio will be the same. If the rudder on the model ship is $\frac{1}{100}$ the size of the rudder on the actual ship, then the model wheelhouse is $\frac{1}{100}$ the size of the actual wheelhouse, the width of the model is $\frac{1}{100}$ the width of the actual ship, and so on.

The two triangles ABC and DEF shown at the right are similar. Side AB corresponds to DE, side BC corresponds to EF, and side AC corresponds to DF. The height CH corresponds to the height FK. The ratios of corresponding parts are equal.



$$\frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}, \quad \frac{AC}{DF} = \frac{3}{6} = \frac{1}{2}, \quad \frac{BC}{EF} = \frac{2}{4} = \frac{1}{2}, \quad \text{and} \quad \frac{CH}{FK} = \frac{1.5}{3} = \frac{1}{2}$$

Since the ratios of corresponding parts are equal, three proportions can be formed using the sides of the triangles.

$$\frac{AB}{DE} = \frac{AC}{DF}, \quad \frac{AB}{DE} = \frac{BC}{EF}, \quad \text{and} \quad \frac{AC}{DF} = \frac{BC}{EF}$$

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Three proportions can also be formed by using the sides and height of the triangles.

$$\frac{AB}{DE} = \frac{CH}{FK}, \qquad \frac{AC}{DF} = \frac{CH}{FK}, \qquad \text{and} \qquad \frac{BC}{EF} = \frac{CH}{FK}$$

The corresponding angles in similar triangles are equal. Therefore,

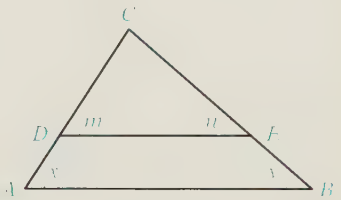
$$\angle A = \angle D, \qquad \angle B = \angle E, \qquad \text{and} \qquad \angle C = \angle F$$

It is also true that if the three angles of one triangle are equal respectively to the three angles of another triangle, then the two triangles are similar.

TAKE NOTE

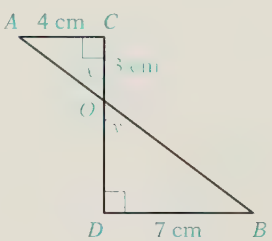
Vertical angles of intersecting lines, parallel lines, and angles of a triangle are discussed in the section "Geometry Problems" in the Solving Equations chapter.

A line DE is drawn parallel to the base AB in the triangle at the right. $\angle x = \angle m$ and $\angle y = \angle n$ because corresponding angles are equal. Angle C = angle C, thus the three angles of triangle DEC are equal respectively to the three angles of triangle ABC. The triangle DEC is similar to the triangle ABC.



The sum of the three angles of a triangle is 180°. If two angles of one triangle are equal to two angles of another triangle, then the third angles must be equal. Thus we can say that if two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar.

➡ The lines AB and CD intersect at point O in the figure at the right. Angles C and D are right angles. Find the length of DO.



First we must determine if triangle AOC is similar to triangle BOD.

Angle C = angle D because they are right angles.

Angle x = angle y because they are vertical angles.

Therefore triangle AOC is similar to triangle BOD because two angles of one triangle are equal to two angles of the other triangle.

$$\frac{AC}{DB} = \frac{CO}{DO}$$

• Use a proportion to find the length of the unknown side.

$$\frac{4}{7} = \frac{3}{DO}$$

$$7(DO)\frac{4}{7} = 7(DO)\frac{3}{DO}$$

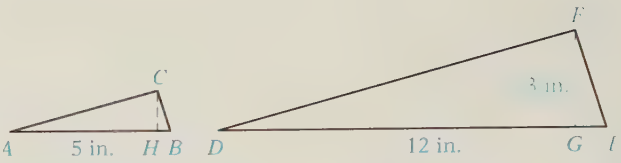
$$4(DO) = 7(3)$$

$$4(DO) = 21$$

$$DO = 5.25$$

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Triangles ABC and DEF at the right are similar. Find the area of triangle ABC.



$$\frac{AB}{DE} = \frac{CH}{FG}$$

Solve a proportion to find the height of triangle *ABC*.

$$\frac{5}{12} = \frac{CH}{3}$$

$$12 \cdot \frac{5}{12} = 12 \cdot \frac{CH}{3}$$

$$5 = 4(CH)$$

$$1.25 = CH$$

The height is 1.25 in. The base is 5 in.

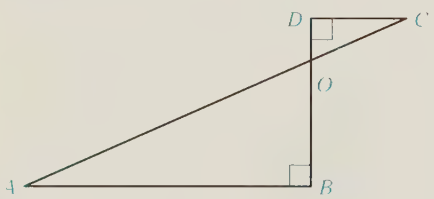
$$A = \frac{1}{2}bh = \frac{1}{2}(5)(1.25) = 3.125$$

Use the formula for the area of a triangle.

The area of triangle ABC is 3.125 in^2.

Example 3

In the figure below, AB is parallel to DC and angles B and D are right angles. $AB = 12$ m, $DC = 4$ m, and $AC = 18$ m. Find the length of CO.



Strategy

Triangle AOB is similar to triangle DOC. Solve a proportion to find the length of CO. Let x represent the length of CO and $18 - x$ represent the length of AO.

Solution

$$\frac{DC}{AB} = \frac{CO}{AO}$$

$$\frac{4}{12} = \frac{x}{18 - x}$$

$$12(18 - x) \cdot \frac{4}{12} = 12(18 - x) \cdot \frac{x}{18 - x}$$

$$4(18 - x) = 12x$$

$$72 - 4x = 12x$$

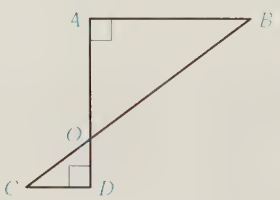
$$72 = 16x$$

$$4.5 = x$$

The length of CO is 4.5 m.

You Try It 3

In the figure below, AB is parallel to DC and angles A and D are right angles. $AB = 10$ cm, $CD = 4$ cm, and $DO = 3$ cm. Find the area of triangle AOB.



Your strategy

Your solution

Solution on p. S15

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6.6 Exercises

Objective A

Solve.

1. $\frac{x}{12} = \frac{3}{4}$

2. $\frac{6}{x} = \frac{2}{3}$

3. $\frac{4}{9} = \frac{x}{27}$

4. $\frac{16}{9} = \frac{64}{x}$

5. $\frac{x+3}{12} = \frac{5}{6}$

6. $\frac{3}{5} = \frac{x-4}{10}$

7. $\frac{18}{x+4} = \frac{9}{5}$

8. $\frac{2}{11} = \frac{20}{x-3}$

9. $\frac{2}{x} = \frac{4}{x+1}$

10. $\frac{16}{x-2} = \frac{8}{x}$

11. $\frac{x+3}{4} = \frac{x}{8}$

12. $\frac{x-6}{3} = \frac{x}{5}$

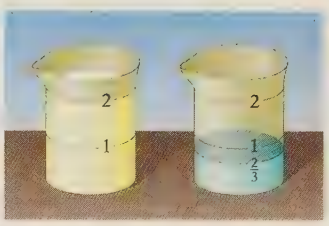
13. $\frac{2}{x-1} = \frac{6}{2x+1}$

14. $\frac{9}{x+2} = \frac{3}{x-2}$

15. $\frac{2x}{7} = \frac{x-2}{14}$

Objective B Application Problems

16. Simple syrup used in making some desserts requires 2 cups (c) of sugar for every $\frac{2}{3}$ c of boiling water. At this rate, how many cups of sugar are required for 2 c boiling water?



17. An exit poll survey showed that 4 out of every 7 voters cast a ballot in favor of an amendment to a city charter. At this rate, how many voters voted in favor of the amendment if 35,000 people voted?

18. In a city of 25,000 homes, a survey was taken to determine the number with cable television. Of the 300 homes surveyed, 210 had cable television. Estimate the number of homes in the city that have cable television.



19. On a map, two cities are $2\frac{5}{8}$ in. apart. If $\frac{3}{8}$ in. on the map represents 25 mi, find the number of miles between the two cities.

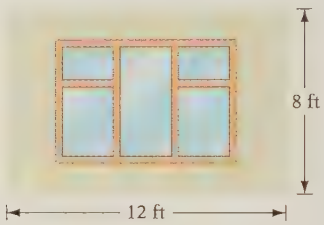
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20. A company decides to accept a large shipment of 10,000 computer chips if there are 2 or fewer defects in a sample of 100 randomly chosen chips. Assuming that there are 300 defective chips in the shipment and that the rate of defective chips in the sample is the same as the rate in the shipment, will the shipment be accepted?

21. The sales tax on a car that sold for \$12,000 is \$780. At this rate, how much higher is the sales tax on a car that sells for \$13,500?

22. The lighting for some billboards is provided by using solar energy. If 3 small solar energy panels can generate 10 W of power, how many panels are necessary to provide 600 W of power?

23. To conserve energy and still allow for as much natural lighting as possible, an architect suggests that the ratio of the area of a window to the area of the total wall surface be 5 to 12. Using this ratio, determine the recommended area of a window to be installed in a wall that measures 8 ft by 12 ft.



24. As part of a conservation effort for a lake, 40 fish are caught, tagged, and then released. Later 80 fish are caught. Four of the 80 fish are found to have tags. Estimate the number of fish in the lake.

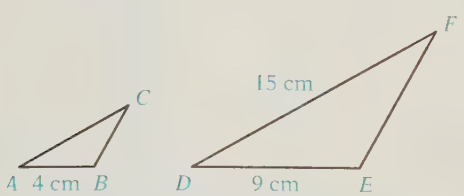
25. In a wildlife preserve, 10 elk are captured, tagged, and then released. Later 15 elk are captured and 2 are found to have tags. Estimate the number of elk in the preserve.

26. The engine of a small rocket burns 170,000 lb of fuel in 1 min. At this rate, how many pounds of fuel does the rocket burn in 45 s?

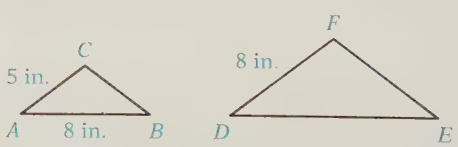
Objective C

Triangles *ABC* and *DEF* in Exercises 27 to 34 are similar. Round answers to the nearest tenth.

27. Find side *AC*.

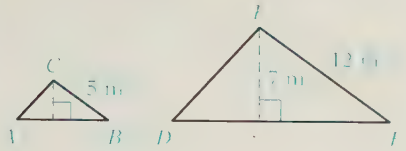


28. Find side *DE*.



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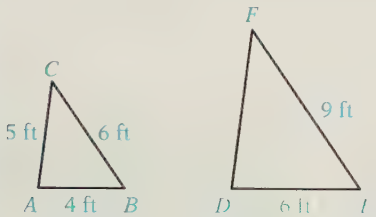
29. Find the height of triangle *ABC*.



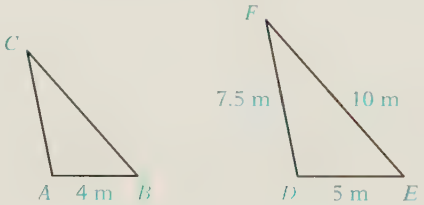
30. Find the height of triangle *DEF*.



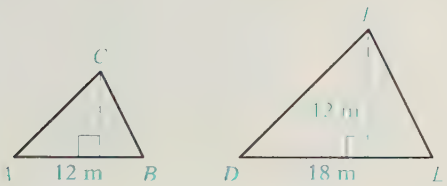
31. Find the perimeter of triangle *DEF*.



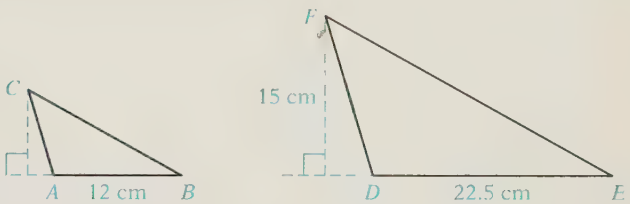
32. Find the perimeter of triangle *ABC*.



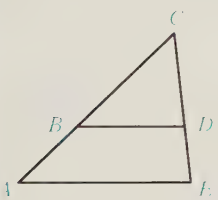
33. Find the area of triangle *ABC*.



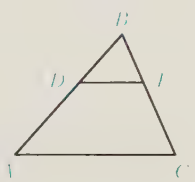
34. Find the area of triangle *ABC*.



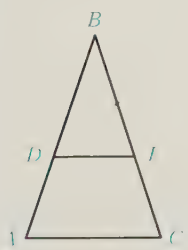
35. Given $BD \parallel AE$, BD measures 5 cm, AE measures 8 cm, and AC measures 10 cm, find the length of BC.



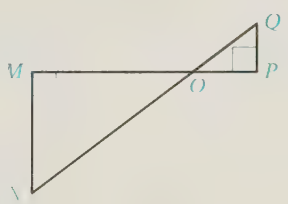
36. Given $AC \parallel DE$, BD measures 8 m, AD measures 12 m, and BE measures 6 m, find the length of BC.



37. Given $DE \parallel AC$, DE measures 6 in., AC measures 10 in., and AB measures 15 in., find the length of DA.

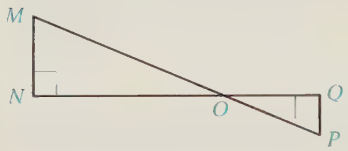


38. Given MP and NQ intersect at O, NO measures 25 ft, MO measures 20 ft, and PO measures 8 ft, find the length of QO.

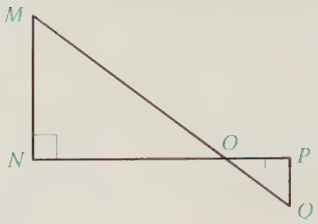


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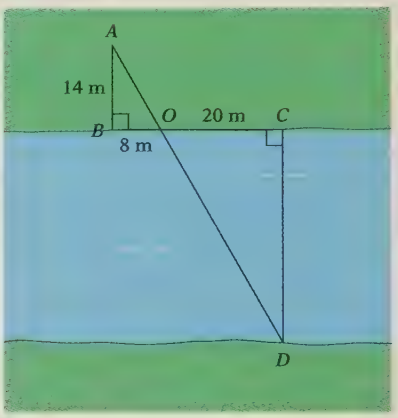
39. Given MP and NQ intersect at O, NO measures 24 cm, MN measures 10 cm, MP measures 39 cm, and QO measures 12 cm, find the length of OP.



40. Given MQ and NP intersect at O, NO measures 12 m, MN measures 9 m, PQ measures 3 m, and MQ measures 20 m, find the perimeter of triangle OPQ.



41. Similar triangles can be used as an indirect way to measure inaccessible distances. The diagram at the right represents a river of width DC. The triangles AOB and DOC are similar. The distances AB, BO, and OC can be measured. Find the width of the river.



42. The sun's rays cast a shadow as shown in the diagram at the right. Find the height of the flagpole. Write the answer in terms of feet.



APPLYING THE CONCEPTS

43. Three people put their money together to buy lottery tickets. The first person put in \$25, the second person put in \$30, and the third person put in \$35. One of their tickets was a winning ticket. If they won \$4.5 million, what was the first person's share of the winnings?

44. No one belongs to both the Math Club and the Photography Club, but the two clubs join to hold a car wash. Ten members of the Math Club and 6 members of the Photography Club participate. The profits from the car wash are \$120. If each club's profits are proportional to the number of members participating, what share of the profits does the Math Club receive?

45. A basketball player has made 5 out of every 6 foul shots attempted in one year of play. If 42 foul shots were missed that year, how many shots did the basketball player make?

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6.7 Literal Equations

Objective A *To solve a literal equation for one of the variables* ..... 9 CT

A **literal equation** is an equation that contains more than one variable. Examples of literal equations are shown at the right.

$2x + 3y = 6$
$4w - 2x + z = 0$

Formulas are used to express a relationship among physical quantities. A **formula** is a literal equation that states rules about measurements. Examples of formulas are shown at the right.

$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R}$ (Physics)
$s = a + (n - 1)d$ (Mathematics)
$A = P + Prt$ (Business)

The Addition and Multiplication Properties can be used to solve a literal equation for one of the variables. The goal is to rewrite the equation so that the variable being solved for is alone on one side of the equation and all the other numbers and variables are on the other side.

➡ Solve $A = P(1 + i)$ for i.

The goal is to rewrite the equation so that i is on one side of the equation and all other variables are on the other side.

$A = P(1 + i)$
$A = P + Pi$ • Use the Distributive Property to remove parentheses.
$A - P = P - P + Pi$ • Subtract P from each side of the equation.
$A - P = Pi$
$\frac{A - P}{P} = \frac{Pi}{P}$ • Divide each side of the equation by P.
$\frac{A - P}{P} = i$

Example 1
Solve $3x - 4y = 12$ for y.

Solution

$3x - 4y = 12$
$3x - 3x - 4y = -3x + 12$
$-4y = -3x + 12$
$\frac{-4y}{-4} = \frac{-3x + 12}{-4}$
$y = \frac{3}{4}x - 3$

You Try It 1
Solve $5x - 2y = 10$ for y.

Your solution

Solution on p. S15

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Example 2

Solve $I = \frac{E}{R + r}$ for R.

Solution

$$I = \frac{E}{R + r}$$

$$(R + r)I = (R + r)\frac{E}{R + r}$$

$$RI + rI = E$$

$$RI + rI - rI = E - rI$$

$$RI = E - rI$$

$$\frac{RI}{I} = \frac{E - rI}{I}$$

$$R = \frac{E - rI}{I}$$

You Try It 2

Solve $s = \frac{A + L}{2}$ for L.

Your solution

Example 3

Solve $L = a(1 + ct)$ for c.

Solution

$$L = a(1 + ct)$$

$$L = a + act$$

$$L - a = a - a + act$$

$$L - a = act$$

$$\frac{L - a}{at} = \frac{act}{at}$$

$$\frac{L - a}{at} = c$$

You Try It 3

Solve $S = a + (n - 1)d$ for n.

Your solution

Example 4

Solve $S = C - rC$ for C.

Solution

$$S = C - rC$$

$$S = (1 - r)C$$

$$\frac{S}{1 - r} = \frac{(1 - r)C}{1 - r}$$

$$\frac{S}{1 - r} = C$$

You Try It 4

Solve $S = C + rC$ for C.

Your solution

Solutions on p. S16

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6.7 Exercises

Objective A

Solve for y.

1. $3x + y = 10$ **2.** $2x + y = 5$ **3.** $4x - y = 3$ **4.** $5x - y = 7$

5. $3x + 2y = 6$ **6.** $2x + 3y = 9$ **7.** $2x - 5y = 10$ **8.** $5x - 2y = 4$

9. $2x + 7y = 14$ **10.** $6x - 5y = 10$ **11.** $x + 3y = 6$ **12.** $x + 2y = 8$

13. $2x - 9y - 18 = 0$ **14.** $3x - y + 7 = 0$ **15.** $2x - y + 5 = 0$

Solve for x.

16. $x + 3y = 6$ **17.** $x + 6y = 10$ **18.** $3x - y = 3$

19. $2x - y = 6$ **20.** $2x + 5y = 10$ **21.** $4x + 3y = 12$

22. $x - 2y + 1 = 0$ **23.** $x - 4y - 3 = 0$ **24.** $5x + 4y + 20 = 0$

Solve the formula for the given variable.

25. $d = rt; t$ (Physics) **26.** $E = IR; R$ (Physics)

27. $PV = nRT; T$ (Chemistry) **28.** $A = bh; h$ (Geometry)

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29. $P = 2l + 2w; l$ (Geometry)

30. $F = \frac{9}{5}C + 32; C$ (Temperature conversion)

31. $A = \frac{1}{2}h(b_1 + b_2); b_1$ (Geometry)

32. $C = \frac{5}{9}(F - 32); F$ (Temperature conversion)

33. $V = \frac{1}{3}Ah; h$ (Geometry)

34. $P = R - C; C$ (Business)

35. $R = \frac{C - S}{t}; S$ (Business)

36. $P = \frac{R - C}{n}; R$ (Business)

37. $A = P + Prt; P$ (Business)

38. $T = fm - gm; m$ (Engineering)

39. $A = Sw + w; w$ (Physics)

40. $a = S - Sr; S$ (Mathematics)

APPLYING THE CONCEPTS

Breakeven analysis is a method used to determine the sales volume required for a company to break even, or experience neither a profit nor a loss on the sale of a product. The breakeven point represents the number of units that must be made and sold for income from sales to equal the cost of the product. The breakeven point can be calculated using the formula $B = \frac{F}{S - V}$, where F is the fixed costs, S is the selling price per unit, and V is the variable costs per unit.

41. **a.** Solve the formula $B = \frac{F}{S - V}$ for S.

b. Use your answer to part **a** to find the required selling price per unit for a company to break even. The fixed costs are \$20,000, the variable costs per unit are \$80, and the company plans to make and sell 200 desks.

c. Use your answer to part **a** to find the required selling price per unit for a company to break even. The fixed costs are \$15,000, the variable costs per unit are \$50, and the company plans to make and sell 600 cameras.

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6.8 Application Problems

Objective A To solve work problems

If a painter can paint a room in 4 h, then in 1 h the painter can paint $\frac{1}{4}$ of the room. The painter's rate of work is $\frac{1}{4}$ of the room each hour. The **rate of work** is the part of a task that is completed in one unit of time.

A pipe can fill a tank in 30 min. This pipe can fill $\frac{1}{30}$ of the tank in 1 min. The rate of work is $\frac{1}{30}$ of the tank each minute. If a second pipe can fill the tank in x min, the rate of work for the second pipe is $\frac{1}{x}$ of the tank each minute.

In solving a work problem, the goal is to determine the time it takes to complete a task. The basic equation that is used to solve work problems is

Rate of work × time worked = part of task completed

For example, if a faucet can fill a sink in 6 min, then in 5 min the faucet will fill $\frac{1}{6} \times 5 = \frac{5}{6}$ of the sink. In 5 min the faucet completes $\frac{5}{6}$ of the task.

➡ A painter can paint a wall in 20 min. The painter's apprentice can paint the same wall in 30 min. How long will it take to paint the wall when they work together?

Strategy for Solving a Work Problem

1. For each person or machine, write a numerical or variable expression for the rate of work, the time worked, and the part of the task completed. The results can be recorded in a table.

Unknown time to paint the wall working together: t

TAKE NOTE

Use the information given in the problem to fill in the "Rate" and "Time" columns of the table. Fill in the "Part Completed" column by multiplying the two expressions you wrote in each row.

	Rate of Work	·	*Time Worked*	=	*Part of Task Completed*
Painter	$\frac{1}{20}$	·	t	=	$\frac{t}{20}$
Apprentice	$\frac{1}{30}$	·	t	=	$\frac{t}{30}$

2. Determine how the parts of the task completed are related. Use the fact that the sum of the parts of the task completed must equal 1, the complete task.

The sum of the part of the task completed by the painter and the part of the task completed by the apprentice is 1.

$$\frac{t}{20} + \frac{t}{30} = 1$$

$$60\left(\frac{t}{20} + \frac{t}{30}\right) = 60 \cdot 1 \quad \text{Multiply by the LCM of 20 and 30.}$$

$$3t + 2t = 60$$

$$5t = 60$$

$$t = 12$$

Working together, they will paint the wall in 12 min.

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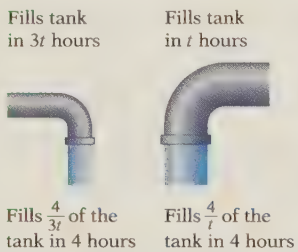
Example 1

A small water pipe takes three times longer to fill a tank than does a large water pipe. With both pipes open it takes 4 h to fill the tank. Find the time it would take the small pipe, working alone, to fill the tank.

Strategy

Time for large pipe to fill the tank: t

Time for small pipe to fill the tank: $3t$



	Rate	*Time*	*Part*
Small pipe	$\frac{1}{3t}$	4	$\frac{4}{3t}$
Large pipe	$\frac{1}{t}$	4	$\frac{4}{t}$

The sum of the parts of the task completed by each pipe must equal 1.

Solution

$$\frac{4}{3t} + \frac{4}{t} = 1$$

$$3t\left(\frac{4}{3t} + \frac{4}{t}\right) = 3t \cdot 1$$

$$4 + 12 = 3t$$

$$16 = 3t$$

$$\frac{16}{3} = t$$

$$3t = 3\left(\frac{16}{3}\right) = 16$$

The small pipe working alone takes 16 h to fill the tank.

You Try It 1

Two computer printers that work at the same rate are working together to print the payroll checks for a large corporation. After they work together for 2 h, one of the printers quits. The second requires 3 h more to complete the payroll checks. Find the time it would take one printer, working alone, to print the payroll.

Your strategy

Your solution

Solution on p. S16

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Objective B To solve uniform motion problems

A car that travels constantly in a straight line at 30 mph is in uniform motion. **Uniform motion** means that the speed or direction of an object does not change.

The basic equation used to solve uniform motion problems is

$$\textbf{Distance} = \textbf{rate} \times \textbf{time}$$

An alternative form of this equation can be written by solving the equation for time.

$$\frac{\textbf{Distance}}{\textbf{Rate}} = \textbf{time}$$

This form of the equation is useful when the total time of travel for two objects or the time of travel between two points is known.

➡ The speed of a boat in still water is 20 mph. The boat traveled 75 mi down a river in the same amount of time it took to travel 45 mi up the river. Find the rate of the river's current.

Strategy for Solving a Uniform Motion Problem

1. For each object, write a numerical or variable expression for the distance, rate, and time. The results can be recorded in a table.

The unknown rate of the river's current: r

TAKE NOTE

Use the information given in the problem to fill in the "Distance" and "Rate" columns of the table. Fill in the "Time" column by dividing the two expressions you wrote in each row.

	Distance	÷	*Rate*	=	*Time*
Down river	75	÷	$20 + r$	=	$\frac{75}{20 + r}$
Up river	45	÷	$20 - r$	=	$\frac{45}{20 - r}$

2. Determine how the times traveled by each object are related. For example, it may be known that the times are equal, or the total time may be known.

$$\frac{75}{20 + r} = \frac{45}{20 - r}$$

• The time down the river is equal to the time up the river.

$$(20 + r)(20 - r)\frac{75}{20 + r} = (20 + r)(20 - r)\frac{45}{20 - r}$$

• Multiply by the LCM.

$$\begin{aligned}(20 - r)75 &= (20 + r)45\\ 1500 - 75r &= 900 + 45r\\ -120r &= -600\\ r &= 5\end{aligned}$$

The rate of the river's current is 5 mph.

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Example 2

A cyclist rode the first 20 mi of a trip at a constant rate. For the next 16 mi, the cyclist reduced the speed by 2 mph. The total time for the 36 mi was 4 h. Find the rate of the cyclist for each leg of the trip.

Strategy

Rate for the first 20 mi: r

Rate for the next 16 mi: $r - 2$



	Distance	*Rate*	*Time*
First 20 mi	20	r	$\frac{20}{r}$
Next 16 mi	16	$r - 2$	$\frac{16}{r-2}$

The total time for the trip was 4 h.

Solution

$$\frac{20}{r} + \frac{16}{r-2} = 4$$

$$r(r-2)\left[\frac{20}{r} + \frac{16}{r-2}\right] = r(r-2) \cdot 4$$

$$(r-2)20 + 16r = 4r^2 - 8r$$

$$20r - 40 + 16r = 4r^2 - 8r$$

$$36r - 40 = 4r^2 - 8r$$

Solve the quadratic equation by factoring.

$$0 = 4r^2 - 44r + 40$$

$$0 = 4(r^2 - 11r + 10)$$

$$0 = 4(r-10)(r-1)$$

$r - 10 = 0$ $\qquad$ $r - 1 = 0$

$r = 10$ $\qquad$ $r = 1$

The solution $r = 1$ mph is not possible, because the rate on the last 16 mi would then be -1 mph.

10 mph was the rate for the first 20 mi.
8 mph was the rate for the next 16 mi.

You Try It 2

The total time it took for a sailboat to sail back and forth across a lake 6 km wide was 2 h. The rate sailing back was three times the rate sailing across. Find the rate sailing out across the lake.

Your strategy

Your solution

Solution on p. S16

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6.8 Exercises

Objective A Application Problems

1. A park has two sprinklers that are used to fill a fountain. One sprinkler can fill the fountain in 3 h, whereas the second sprinkler can fill the fountain in 6 h. How long will it take to fill the fountain with both sprinklers operating?

2. One grocery clerk can stock a shelf in 20 min, whereas a second clerk requires 30 min to stock the same shelf. How long would it take to stock the shelf if the two clerks worked together?

3. One person with a skiploader requires 12 h to remove a large quantity of earth. A second, larger skiploader can remove the same amount of earth in 4 h. How long would it take to remove the earth with both skiploaders working together?

4. An experienced painter can paint a fence twice as fast as an inexperienced painter. Working together, the painters require 4 h to paint the fence. How long would it take the experienced painter working alone to paint the fence?

5. One computer can solve a complex prime factorization problem in 75 h. A second computer can solve the same problem in 50 h. How long would it take both computers, working together, to solve the problem?

6. A new machine can make 10,000 aluminum cans three times faster than an older machine. With both machines working, 10,000 cans can be made in 9 h. How long would it take the new machine, working alone, to make the 10,000 cans?

7. A small air conditioner can cool a room 5° in 75 min. A larger air conditioner can cool the room 5° in 50 min. How long would it take to cool the room 5° with both air conditioners working?

8. One printing press can print the first edition of a book in 55 min, whereas a second printing press requires 66 min to print the same number of copies. How long would it take to print the first edition with both presses operating?

9. Two oil pipelines can fill a small tank in 30 min. Using one of the pipelines would require 45 min to fill the tank. How long would it take the second pipeline alone to fill the tank?

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10. Working together, two dock workers can load a crate in 6 min. One dock worker, working alone, can load the crate in 15 min. How long would it take the second dock worker, working alone, to load the crate?

11. A mason can construct a retaining wall in 10 h. With the mason's apprentice assisting, the task would take 6 h. How long would it take the apprentice working alone to construct the wall?

12. A mechanic requires 2 h to repair a transmission, whereas an apprentice requires 6 h to make the same repairs. The mechanic worked alone for 1 h and then stopped. How long will it take the apprentice, working alone, to complete the repairs?

13. One computer technician can wire a modem in 4 h, whereas it takes 6 h for a second technician to do the same job. After working alone for 2 h, the first technician quit. How long will it take the second technician to complete the wiring?

14. A wallpaper hanger requires 2 h to hang the wallpaper on one wall of a room. A second wallpaper hanger requires 4 h to hang the same amount of paper. The first wallpaper hanger worked alone for 1 h and then quit. How long will it take the second wallpaper hanger, working alone, to complete the wall?

15. Two welders who work at the same rate are welding the girders of a building. After they work together for 10 h, one of the welders quits. The second welder requires 20 more hours to complete the welds. Find the time it would have taken one of the welders, working alone, to complete the welds.

16. A large and a small heating unit are being used to heat the water of a pool. The larger unit, working alone, requires 8 h to heat the pool. After both units have been operating for 2 h, the larger unit is turned off. The small unit requires 9 h more to heat the pool. How long would it take the small unit, working alone, to heat the pool?

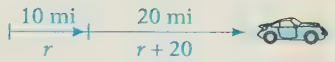
17. Two machines that fill cereal boxes work at the same rate. After they work together for 7 h, one machine breaks down. The second machine requires 14 h more to finish filling the boxes. How long would it have taken one of the machines, working alone, to fill the boxes?

18. A large and a small drain are opened to drain a pool. The large drain can empty the pool in 6 h. After both drains have been open for 1 h, the large drain becomes clogged and is closed. The smaller drain remains open and requires 9 h more to empty the pool. How long would it have taken the small drain, working alone, to empty the pool?

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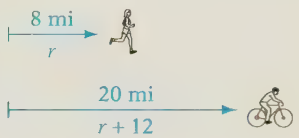
Objective B Application Problems

19. Commuting from work to home, a lab technician traveled 10 mi at a constant rate through congested traffic. On reaching the expressway, the technician increased the speed by 20 mph. An additional 20 mi was traveled at the increased speed. The total time for the trip was 1 h. Find the rate of travel through the congested traffic.



20. The president of a company traveled 1800 mi by jet and 300 mi on a prop plane. The rate of the jet was four times the rate of the prop plane. The entire trip took a total of 5 h. Find the rate of the jet plane.

21. As part of a conditioning program, a jogger ran 8 mi in the same time a cyclist rode 20 mi. The rate of the cyclist was 12 mph faster than the rate of the jogger. Find the rate of the jogger and that of the cyclist.



22. An express train travels 600 mi in the same amount of time it takes a freight train to travel 360 mi. The rate of the express train is 20 mph faster than that of the freight train. Find the rate of each train.

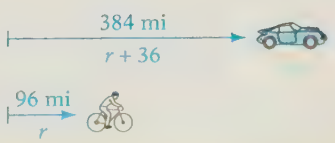
23. To assess the damage done by a fire, a forest ranger traveled 1080 mi by jet and then an additional 180 mi by helicopter. The rate of the jet was 4 times the rate of the helicopter. The entire trip took a total of 5 h. Find the rate of the jet.

24. A twin-engine plane can fly 800 mi in the same time that it takes a single-engine plane to fly 600 mi. The rate of the twin-engine plane is 50 mph faster than that of the single-engine plane. Find the rate of the twin-engine plane.

25. The rate of a motorcycle is 36 mph faster than the rate of a bicycle. The motorcycle travels 192 mi in the same amount of time as the bicycle travels 48 mi. Find the rate of the motorcycle.

26. A car and a bus leave a town at 1 P.M. and head for a town 300 mi away. The rate of the car is twice the rate of the bus. The car arrives 5 h ahead of the bus. Find the rate of the car.

27. A car is traveling at a rate that is 36 mph faster than the rate of a cyclist. The car travels 384 mi in the same time it takes the cyclist to travel 96 mi. Find the rate of the car.

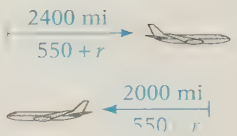


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28. A backpacker hiking into a wilderness area walked 9 mi at a constant rate and then reduced this rate by 1 mph. Another 4 mi was hiked at this reduced rate. The time required to hike the 4 mi was 1 h less than the time required to walk the 9 mi. Find the rate at which the hiker walked the first 9 mi.

29. A plane can fly 180 mph in calm air. Flying with the wind, the plane can fly 600 mi in the same amount of time it takes to fly 480 mi against the wind. Find the rate of the wind.

30. A commercial jet can fly 550 mph in calm air. Traveling with the jet stream, the plane flew 2400 mi in the same amount of time it takes to fly 2000 mi against the jet stream. Find the rate of the jet stream.



31. A cruise ship can sail at 28 mph in calm water. Sailing with the gulf current, the ship can sail 170 mi in the same amount of time that it can sail 110 mi against the gulf current. Find the rate of the gulf current.

32. Rowing with the current of a river, a rowing team can row 25 mi in the same amount of time it takes to row 15 mi against the current. The rate of the rowing team in calm water is 20 mph. Find the rate of the current.

33. On a recent trip, a trucker traveled 330 mi at a constant rate. Because of road construction, the trucker then had to reduce speed by 25 mph. An additional 30 mi was traveled at the reduced rate. The total time for the entire trip was 7 h. Find the rate of the trucker for the first 330 mi.

APPLYING THE CONCEPTS

34. One pipe can fill a tank in 2 h, a second pipe can fill the tank in 4 h, and a third pipe can fill the tank in 5 h. How long will it take to fill the tank with all three pipes working?

35. A mason can construct a retaining wall in 10 h. The mason's more-experienced apprentice can do the same job in 15 h. How long would it take the mason's less-experienced apprentice to do the job if, working together, all three can complete the job in 5 h?

36. A surveyor traveled 32 mi by canoe and then hiked 4 mi. The rate of speed by boat was four times the rate on foot. If the time spent walking was 1 h less than the time spent canoeing, find the amount of time spent traveling by canoe.

37. Because of bad weather, a bus driver reduced the usual speed along a 150-mile bus route by 10 mph. The bus arrived only 30 min later than its usual arrival time. How fast does the bus usually travel?

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Focus on Problem Solving

Negations and If ... then Sentences

The sentence "George Washington was the first president of the United States" is a true sentence. The **negation** of that sentence is "George Washington was **not** the first president of the United States." That sentence is false. In general, the negation of a true sentence is a false statement.

The negation of a false sentence is a true sentence. For instance, the sentence "The moon is made of green cheese" is a false statement. The negation of that sentence, "The moon is **not** made of green cheese," is true.

The words *all*, *no* (or *none*), and *some* are called **quantifiers.** Writing the negation of a sentence that contains these words requires special attention. Consider the sentence "All pets are dogs." This sentence is not true because there are pets that are not dogs; cats, for example, are pets. Because the sentence is false, its negation must be true. You might be tempted to write "All pets are not dogs," but that sentence is not true because some pets are dogs. The correct negation of "All pets are dogs" is "Some pets are not dogs." Note the use of the word *some* in the negation.

Now consider the sentence "Some computers are portable." Because that sentence is true, its negation must be false. Writing "Some computers are not portable" as the negation is not correct, because that sentence is true. The negation of "Some computers are portable" is "No computers are portable."

The sentence "No flowers have red blooms" is false, because there is at least one flower (some roses, for example) that has red blooms. Because the sentence is false, its negation must be true. The negation is "Some flowers have red blooms."

Statement	*Negation*
All *A* are *B*.	Some *A* are not *B*.
No *A* are *B*.	Some *A* are *B*.
Some *A* are *B*.	No *A* are *B*.
Some *A* are not *B*.	All *A* are *B*.

Write the negation of the sentence.

1. All cats like milk.

2. All computers need people.

3. Some trees are tall.

4. No politicians are honest.

5. No houses have kitchens.

6. All police officers are tall.

7. All lakes are not polluted.

8. Some drivers are unsafe.

9. Some speeches are interesting.

10. All laws are good.

11. All businesses are not profitable.

12. All motorcycles are not large.

13. Some vegetables are good for you to eat.

14. Some banks are not open on Sunday.

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A **premise** is a known or assumed fact. A premise can be stated using one of the quantifiers (*all, no, none,* or *some*) or can be stated using an *If ... then* sentence. For instance, the sentence "All triangles have three sides" can be written "*If* the figure is a triangle, *then* it has three sides."

We can write the sentence "No whole numbers are negative numbers" as an *If ... then* sentence: If a number is a whole number, then it is not a negative number.

Write the sentence as an *If ... then* sentence.

15. All students at Barlock College must take a life science course.

16. All baseballs are round.

17. All computers need people.

18. All cats like milk.

19. No odd number is evenly divisible by 2.

20. No prime number greater than 2 is an even number.

21. No rectangles have five sides.

22. All roads lead to Rome.

23. All dogs have fleas.

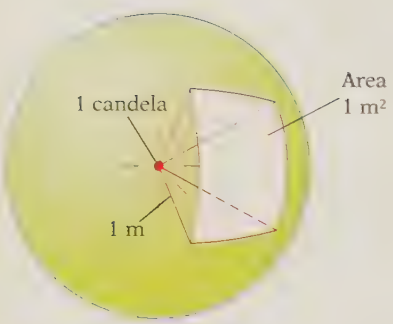
24. No triangle has four angles.

Projects and Group Activities

Intensity of Illumination

You are already aware that the standard unit of length in the metric system is the meter (m) and that the standard unit of mass in the metric system is the gram (g). You may not know that the standard unit of light intensity is the candela (cd).

The rate at which light falls on a 1-square-unit area of surface is called the **intensity of illumination.** Intensity of illumination is measured in **lumens** (lm). A lumen is defined in the following illustration.



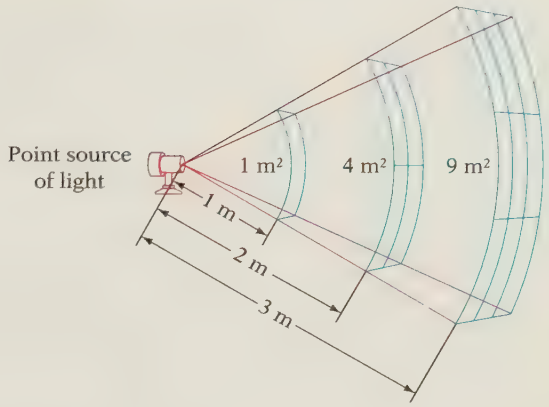
Picture a source of light equal to 1 cd positioned at the center of a hollow sphere that has a radius of 1 m. The rate at which light falls on 1 m^2 of the inner surface of the sphere is equal to 1 lm. If a light source equal to 4 cd is positioned at the center of the sphere, each square meter of the inner surface receives four times as much illumination, or 4 lm.

Light rays diverge as they leave a light source. The light that falls on an area of 1 m^2 at a distance of 1 m from the source of light spreads out over an area of 4 m^2 when it is 2 m from the source. The same light spreads out over an area of 9 m^2 when it is 3 m from the light source and over an area of 16 m^2 when it is 4 m from the light source. Therefore, as a surface moves farther away from the source of light, the intensity of illumination on the surface decreases from its value at 1 m to $\left(\frac{1}{2}\right)^2$, or $\frac{1}{4}$, that value at 2 m; to $\left(\frac{1}{3}\right)^2$, or $\frac{1}{9}$, that value at 3 m; and to $\left(\frac{1}{4}\right)^2$, or $\frac{1}{16}$, that value at 4 m.

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The formula for the intensity of illumination is

$$I = \frac{s}{r^2}$$



where I is the intensity of illumination in lumens, s is the strength of the light source in candelas, and r is the distance in meters between the light source and the illuminated surface.

A 30-candela lamp is 0.5 m above a desk. Find the illumination on the desk.

$$I = \frac{s}{r^2}$$

$$I = \frac{30}{(0.5)^2} = 120$$

The illumination on the desk is 120 lm.

1. A 100-candela light is hanging 5 m above a floor. What is the intensity of the illumination on the floor beneath it?

2. A 25-candela source of light is 2 m above a desk. Find the intensity of illumination on the desk.

3. How strong a light source is needed to cast 20 lm of light on a surface 4 m from the source?

4. How strong a light source is needed to cast 80 lm of light on a surface 5 m from the source?

5. How far from the desk surface must a 40-candela light source be positioned if the desired intensity of illumination is 10 lm?

6. Find the distance between a 36-candela light source and a surface if the intensity of illumination on the surface is 0.01 lm.

7. Two lights cast the same intensity of illumination on a wall. One light is 6 m from the wall and has a rating of 36 candelas. The second light is 8 m from the wall. Find the candela rating of the second light.

8. A 40-candela light source and a 10-candela light source both throw the same intensity of illumination on a wall. The 10-candela light is 6 m from the wall. Find the distance from the 40-candela light to the wall.

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Chapter Summary

Key Words

A *rational expression* is a fraction in which the numerator or denominator is a polynomial.

A *rational expression* is in *simplest form* when the numerator and denominator have no common factors.

The *reciprocal* of a fraction is a fraction with the numerator and denominator interchanged.

The *least common multiple* (LCM) of two or more numbers is the smallest number that contains the prime factorization of each number.

A *complex fraction* is a fraction whose numerator or denominator contains one or more fractions.

A *ratio* is the quotient of two quantities that have the same unit.

A *rate* is the quotient of two quantities that have different units.

A *proportion* is an equation that states the equality of two ratios or rates.

Similar triangles have the same shape but not necessarily the same size.

A *literal equation* is an equation that contains more than one variable.

A *formula* is a literal equation that states rules about measurement.

Essential Rules

To multiply rational numbers, multiply the numerators and multiply the denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To divide two fractions, multiply the dividend by the reciprocal of the divisor.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

To add fractions, write each fraction in terms of a common denominator, then add the numerators.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To determine whether two triangles are similar, show that two angles of one triangle are equal to two angles of the other triangle.

To solve an equation containing fractions, clear denominators by multiplying each side of the equation by the LCM of the denominators.

Equation for Work Problems

$$\text{Rate of Work} \times \text{time worked} = \text{part of task completed}$$

Uniform Motion Equation

$$\frac{\text{Distance}}{\text{Rate}} = \text{time}$$

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Chapter Review

1. Simplify: $\frac{6a^2b^7}{25x^3y} \div \frac{12a^3b^4}{5x^2y^2}$

2. Simplify: $\frac{x+7}{15x} + \frac{x-2}{20x}$

3. Simplify: $\dfrac{x - \frac{16}{5x-2}}{3x - 4 - \frac{88}{5x-2}}$

4. Simplify: $\frac{x^2 + x - 30}{15 + 2x - x^2}$

5. Simplify: $\frac{16x^5y^3}{24xy^{10}}$

6. Solve: $\frac{20}{x+2} = \frac{5}{16}$

7. Simplify: $\frac{10 - 23y + 12y^2}{6y^2 - y - 5} \div \frac{4y^2 - 13y + 10}{18y^2 + 3y - 10}$

8. Simplify: $\frac{8ab^2}{15x^3y} \cdot \frac{5xy^4}{16a^2b}$

9. Simplify: $\dfrac{1 - \frac{1}{x}}{1 - \frac{8x-7}{x^2}}$

10. Write each fraction in terms of the LCM of the denominators.

$$\frac{x}{12x^2 + 16x - 3}, \frac{4x^2}{6x^2 + 7x - 3}$$

11. Solve $T = 2(ab + bc + ca)$ for a.

12. Solve: $\frac{5}{7} + \frac{x}{2} = 2 - \frac{x}{7}$

13. Solve $i = \frac{100m}{c}$ for c.

14. Solve: $\frac{x+8}{x+4} = 1 + \frac{5}{x+4}$

15. Simplify: $\frac{20x^2 - 45x}{6x^3 + 4x^2} \div \frac{40x^3 - 90x^2}{12x^2 + 8x}$

16. Simplify: $\frac{2y}{5y - 7} + \frac{3}{7 - 5y}$

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17. Simplify: $\frac{5x+3}{2x^2+5x-3} - \frac{3x+4}{2x^2+5x-3}$

18. Find the LCM of $10x^2 - 11x + 3$ and $20x^2 - 17x + 3$.

19. Solve $4x + 9y = 18$ for y.

20. Simplify: $\frac{24x^2-94x+15}{12x^2-49x+15} \cdot \frac{24x^2+7x-5}{4-27x+18x^2}$

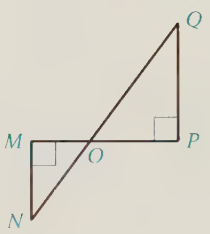
21. Solve: $\frac{20}{2x+3} = \frac{17x}{2x+3} - 5$

22. Simplify: $\frac{x-1}{x+2} + \frac{3x-2}{5-x} + \frac{5x^2+15x-11}{x^2-3x-10}$

23. Solve: $\frac{6}{x-7} = \frac{8}{x-6}$

24. Solve: $\frac{3}{20} = \frac{x}{80}$

25. Given MP and NQ intersect at O, NQ measures 25 cm, MO measures 6 cm, and PO measures 9 cm, find the length of QO.



26. One hose can fill a pool in 15 h. The second hose can fill the pool in 10 h. How long would it take to fill the pool using both hoses?

27. A car travels 315 mi in the same amount of time that a bus travels 245 mi. The rate of the car is 10 mph faster than that of the bus. Find the rate of the car.

28. The rate of a jet is 400 mph in calm air. Traveling with the wind, the jet can fly 2100 mi in the same amount of time it takes to fly 1900 mi against the wind. Find the rate of the wind.

29. A pitcher's earned run average (ERA) is the average number of runs allowed in 9 innings of pitching. If a pitcher allows 15 runs in 100 innings, find the pitcher's ERA.

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Chapter Test

1. Simplify: $\frac{x}{x+3} - \frac{2x-5}{x^2+x-6}$

2. Solve the proportion: $\frac{3}{x+4} = \frac{5}{x+6}$

3. Simplify: $\frac{x^2+2x-3}{x^2+6x+9} \cdot \frac{2x^2-11x+5}{2x^2+3x-5}$

4. Simplify: $\frac{16x^5y}{24x^2y^4}$

5. Solve $d = s + rt$ for t.

6. Solve: $\frac{6}{x} - 2 = 1$

7. Simplify: $\frac{x^2+4x-5}{1-x^2}$

8. Find the LCM of $6x - 3$ and $2x^2 + x - 1$.

9. Simplify: $\frac{2}{2x-1} - \frac{3}{3x+1}$

10. Simplify: $\frac{x^2+3x+2}{x^2+5x+4} \div \frac{x^2-x-6}{x^2+2x-15}$

11. Simplify: $\dfrac{1+\frac{1}{x}-\frac{12}{x^2}}{1+\frac{2}{x}-\frac{8}{x^2}}$

12. Write each fraction in terms of the LCM of the denominators.

$$\frac{3}{x^2-2x}, \frac{x}{x^2-4}$$

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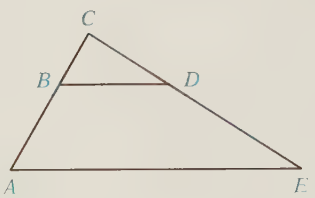
13. Simplify: $\frac{2x}{x^2 + 3x - 10} - \frac{4}{x^2 + 3x - 10}$

14. Solve $3x - 8y = 16$ for y.

15. Solve: $\frac{2x}{x + 1} - 3 = \frac{-2}{x + 1}$

16. Simplify: $\frac{x^3y^4}{x^2 - 4x + 4} \cdot \frac{x^2 \cdot x \quad 2}{x^6y^4}$

17. Given $AE \parallel BD$, AB measures 5 ft, ED measures 8 ft, and BC measures 3 ft, find the length of CE.



18. A saltwater solution is formed by mixing 4 lb of salt with 10 gal of water. At this rate, how many additional pounds of salt are required for 15 gal of water?

19. A pool can be filled with one pipe in 6 h, whereas a second pipe requires 12 h to fill the pool. How long would it take to fill the pool with both pipes turned on?

20. A small plane can fly at 110 mph in calm air. Flying with the wind, the plane can fly 260 mi in the same amount of time it takes to fly 180 mi against the wind. Find the rate of the wind.

21. A landscape architect uses three sprinklers for each 200 ft^2 of lawn. At this rate, how many sprinklers are needed for a 3600–square-foot lawn?

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Cumulative Review

1. Simplify: $\left(\frac{2}{3}\right)^2 \div \left(\frac{3}{2} - \frac{2}{3}\right) + \frac{1}{2}$

2. Evaluate $-a^2 + (a - b)^2$ when $a = -2$ and $b = 3$.

3. Simplify: $-2x - (-3y) + 7x - 5y$

4. Simplify: $2[3x - 7(x - 3) - 8]$

5. Solve: $4 - \frac{2}{3}x = 7$

6. Solve: $3[x - 2(x - 3)] = 2(3 - 2x)$

7. Find $16\frac{2}{3}\%$ of 60.

8. Simplify: $(a^2b^5)(ab^2)$

9. Simplify: $(a - 3b)(a + 4b)$

10. Simplify: $\frac{15b^4 - 5b^2 + 10b}{5b}$

11. Simplify: $(x^3 - 8) \div (x - 2)$

12. Factor: $12x^2 - x - 1$

13. Factor: $y^2 - 7y + 6$

14. Factor: $2a^3 + 7a^2 - 15a$

15. Factor: $4b^2 - 100$

16. Solve: $(x + 3)(2x - 5) = 0$

17. Simplify: $\frac{12x^4y^2}{18xy^7}$

18. Simplify: $\frac{x^2 - 7x + 10}{25 - x^2}$

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19. Simplify: $\dfrac{x^2 - x - 56}{x^2 + 8x + 7} \div \dfrac{x^2 - 13x + 40}{x^2 - 4x - 5}$

20. Simplify: $\dfrac{2}{2x - 1} - \dfrac{1}{x + 1}$

21. Simplify: $\dfrac{1 - \dfrac{2}{x} - \dfrac{15}{x^2}}{1 - \dfrac{25}{x^2}}$

22. Solve: $\dfrac{3x}{x - 3} - 2 = \dfrac{10}{x - 3}$

23. Solve the proportion: $\dfrac{2}{x - 2} = \dfrac{12}{x + 3}$

24. Solve $f = v + at$ for t.

25. Translate "the difference between five times a number and thirteen is the opposite of eight" into an equation and solve.

26. A silversmith mixes 60 g of an alloy that is 40% silver with 120 g of another silver alloy. The resulting alloy is 60% silver. Find the percent of silver in the 120-gram alloy.

27. The length of the base of a triangle is 2 in. less than twice the height. The area of the triangle is 30 in^2. Find the base and height of the triangle.

28. A life insurance policy costs \$16 for every \$1000 of coverage. At this rate, how much money would a policy of \$5000 cost?

29. One water pipe can fill a tank in 9 min, whereas a second pipe requires 18 min to fill the tank. How long would it take both pipes, working together, to fill the tank?

30. The rower of a boat can row at a rate of 5 mph in calm water. Rowing with the current, the boat travels 14 mi in the same amount of time it takes to travel 6 mi against the current. Find the rate of the current.

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CHAPTER

7 Linear Equations in Two Variables



Broadcast technicians operate the electronic equipment used to record and transmit radio and television programs, and they help prepare movie soundtracks for production studios. They work with television cameras, microphones, tape recorders, light and sound effects, transmitters, antennas, and other equipment. These technicians must be able to read and interpret graphs such as those discussed in this chapter.

Objectives

Section 7.1

To graph points in a rectangular coordinate system

To determine ordered-pair solutions of an equation in two variables

To determine whether a set of ordered pairs is a function

To evaluate a function written in functional notation

Section 7.2

To graph an equation of the form $y = mx + b$

To graph an equation of the form $Ax + By = C$

To solve application problems

Section 7.3

To find the x- and y-intercepts of a straight line

To find the slope of a straight line

To graph a line using the slope and the y-intercept

Section 7.4

To find the equation of a line given a point and the slope

To find the equation of a line given two points

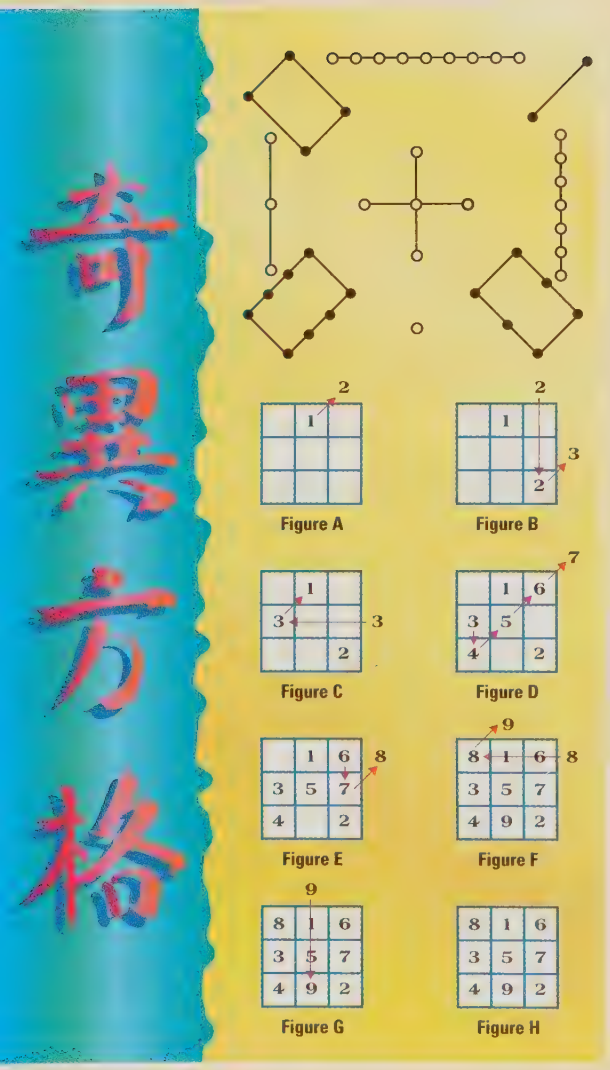
To solve application problems

Magic Squares

A magic square is a square array of distinct integers so arranged that the numbers along any row, column, or main diagonal have the same sum. An example of a magic square is shown at the right.

8	3	4
1	5	9
6	7	2

The oldest known example of a magic square comes from China. Estimates are that this magic square is over 4000 years old. It is shown at the left.



There is a simple way to produce a magic square with an odd number of cells. Start by writing a 1 in the top middle cell. The rule then is to proceed diagonally upward to the right with the successive integers.

When the rule takes you outside the square, write the number by shifting either across the square from right to left or down the square from top to bottom, as the case may be. For example, in Figure B the second number (2) is outside the square above a column. Because the 2 is above a column, it should be shifted down to the bottom cell in that column. In Figure C, the 3 is outside the square to the right of a column and should therefore be shifted all the way to the left.

If the rule takes you to a square that is already filled (as shown in Figure D), then write the number in the cell directly below the last number written. Continue until the entire square is filled.

It is possible to begin a magic square with any integer and proceed by using the above rule and consecutive integers.

For an odd magic square beginning with 1, the sum of a row, column, or diagonal is $\frac{n(n^2 + 1)}{2}$, where n is the number of rows.

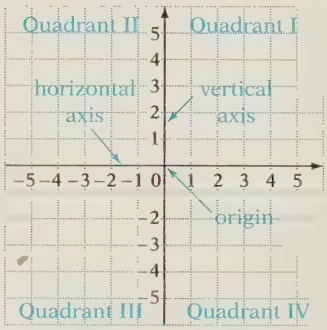
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7.1 The Rectangular Coordinate System

Objective A To graph points in a rectangular coordinate system

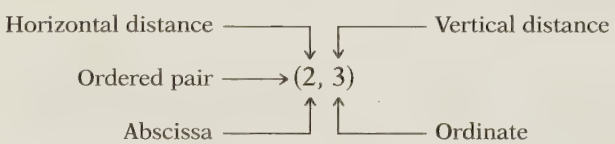
Before the 15th century, geometry and algebra were considered separate branches of mathematics. That all changed when René Descartes, a French mathematician who lived from 1596 to 1650, founded **analytic geometry.** In this geometry, a *coordinate system* is used to study relationships between variables.

A **rectangular coordinate system** is formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. The point of intersection is called the **origin.** The two lines are called **coordinate axes,** or simply **axes.**

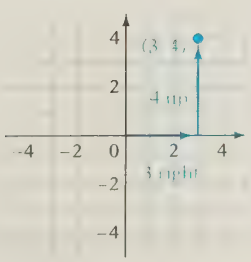


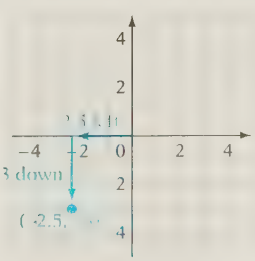
The axes determine a **plane,** which can be thought of as a large, flat sheet of paper. The two axes divide the plane into four regions called **quadrants,** which are numbered counterclockwise from I to IV.

Each point in the plane can be identified by a pair of numbers called an **ordered pair.** The first number of the pair measures a horizontal distance and is called the **abscissa.** The second number of the pair measures a vertical distance and is called the **ordinate.** The **coordinates** of the point are the numbers in the ordered pair associated with the point. The abscissa is also called the **first coordinate** of the ordered pair, and the ordinate is also called the **second coordinate** of the ordered pair.



To **graph** or **plot** a point in the plane, place a dot at the location given by the ordered pair. The **graph of an ordered pair** is the dot drawn at the coordinates of the point in the plane. The points whose coordinates are (3, 4) and (−2.5, −3) are graphed in the figures below.



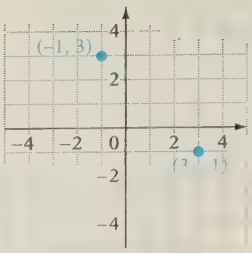


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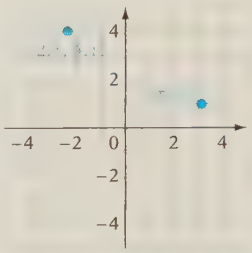
TAKE NOTE

This is very important. An **ordered pair** is a *pair* of coordinates, and the *order* in which the coordinates appear is important.

The points whose coordinates are $(3, -1)$ and $(-1, 3)$ are graphed at the right. Note that the graphed points are in different locations. *The order of the coordinates of an ordered pair is important.*

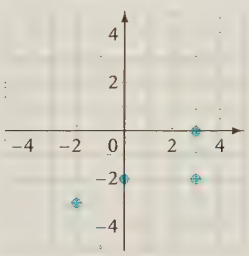


Each point in the plane is associated with an ordered pair, and each ordered pair is associated with a point in the plane. Although only the labels for integers are given on a coordinate grid, the graph of any ordered pair can be approximated. For example, the points whose coordinates are $(-2.3, 4.1)$ and $(\pi, 1)$ are shown on the graph at the right.



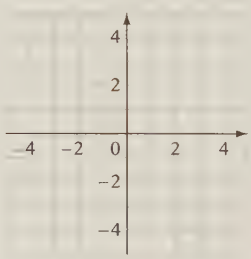
Example 1 Graph the ordered pairs $(-2, -3)$, $(3, -2)$, $(0, -2)$, and $(3, 0)$.

Solution

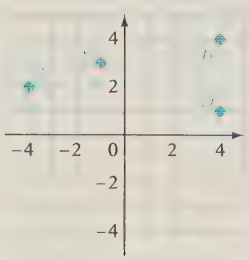


You Try It 1 Graph the ordered pairs $(-4, 1)$, $(3, -3)$, $(0, 4)$, and $(-3, 0)$.

Your solution



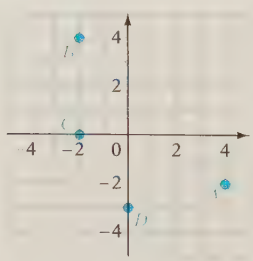
Example 2 Give the coordinates of the points labeled A and B. Give the abscissa of point C and the ordinate of point D.



Solution

The coordinates of A are $(-4, 2)$.
The coordinates of B are $(4, 4)$.
The abscissa of C is -1.
The ordinate of D is 1.

You Try It 2 Give the coordinates of the points labeled A and B. Give the abscissa of point D and the ordinate of point C.

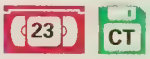


Your solution

Solutions on p. S16

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Objective B To determine ordered-pair solutions of an equation in two variables



When drawing a rectangular coordinate system, we often label the horizontal axis x and the vertical axis y. In this case, the coordinate system is called an ***xy*-coordinate system.** The coordinates of the points are given by ordered pairs (x, y), where the abscissa is called the ***x*-coordinate** and the ordinate is called the ***y*-coordinate.**

A coordinate system is used to study the relationship between two variables. Frequently this relationship is given by an equation. Examples of equations in two variables include

$$y = 2x - 3 \qquad 3x + 2y = 6 \qquad x^2 - y = 0$$

A **solution of an equation in two variables** is an ordered pair (x, y) whose coordinates make the equation a true statement.

➡ Is $(-3, 7)$ a solution of $y = -2x + 1$?

$$\begin{array}{r|l} y = -2x + 1 & \\ \hline 7 & -2(-3) + 1 \\ & 6 + 1 \end{array} \qquad \text{• Replace } x \text{ by } -3\text{; replace } y \text{ by } 7.$$

$7 = 7$ • The results are equal.

$(-3, 7)$ is a solution of the equation $y = -2x + 1$.

Besides $(-3, 7)$, there are many other ordered-pair solutions of $y = -2x + 1$. For example, $(0, 1)$, $\left(-\frac{3}{2}, 4\right)$, and $(4, -7)$ are also solutions. In general, an equation in two variables has an infinite number of solutions. By choosing any value of x and substituting that value into the equation, we can calculate a corresponding value of y.

➡ Find the ordered-pair solution of $y = \frac{2}{3}x - 3$ that corresponds to $x = 6$.

$$\begin{aligned} y &= \frac{2}{3}x - 3 \\ &= \frac{2}{3}(6) - 3 && \text{• Replace } x \text{ by } 6. \\ &= 4 - 3 = 1 && \text{• Simplify.} \end{aligned}$$

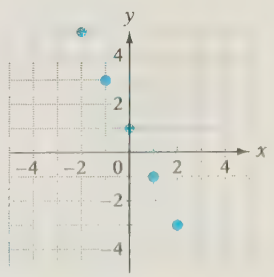
The ordered-pair solution is $(6, 1)$.

The solutions of an equation in two variables can be graphed in an xy-coordinate system.

➡ Graph the ordered-pair solutions of $y = -2x + 1$ when $x = -2, -1, 0, 1$, and 2.

Use the values of x to determine ordered-pair solutions of the equation. It is convenient to record these in a table.

x	$y = -2x + 1$	y	(x, y)
-2	$-2(-2) + 1$	5	$(-2, 5)$
-1	$-2(-1) + 1$	3	$(-1, 3)$
0	$-2(0) + 1$	1	$(0, 1)$
1	$-2(1) + 1$	-1	$(1, -1)$
2	$-2(2) + 1$	-3	$(2, -3)$



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Example 3

Is $(3, -2)$ a solution of $3x - 4y = 15$?

Solution

$$3x - 4y = 15$$

$3(3) - 4(-2)$	15
$9 + 8$	
$17 \neq 15$	

Replace x by 3 and y by -2.

No. $(3, -2)$ is not a solution of $3x - 4y = 15$.

You Try It 3

Is $(-2, 4)$ a solution of $x - 3y = -14$?

Your solution

Example 4

Graph the ordered-pair solutions of $2x - 3y = 6$ when $x = -3$, 0, 3, and 6.

Solution

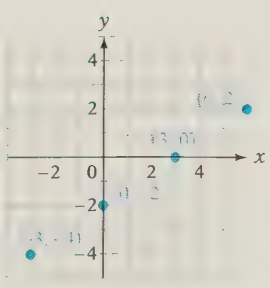
$2x - 3y = 6$ Solve $2x - 3y = 6$ for y.

$-3y = -2x + 6$

$y = \frac{2}{3}x - 2$

Replace x in $y = \frac{2}{3}x - 2$ by -3, 0, 3, and 6. For each value of x, determine the value of y.

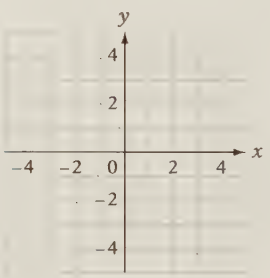
x	$y = \frac{2}{3}x - 2$	y	(x, y)
-3	$\frac{2}{3}(-3) - 2$	-4	$(-3, -4)$
0	$\frac{2}{3}(0) - 2$	-2	$(0, -2)$
3	$\frac{2}{3}(3) - 2$	0	$(3, 0)$
6	$\frac{2}{3}(6) - 2$	2	$(6, 2)$



You Try It 4

Graph the ordered-pair solutions of $x + 2y = 4$ when $x = -4$, -2, 0, and 2.

Your solution



Solutions on pp. S16–S17

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Objective C To determine whether a set of ordered pairs is a function

Discovering a relationship between two variables is an important task in the application of mathematics. Here are some examples.

- Botanists study the relationship between the number of bushels of wheat yielded per acre and the amount of watering per acre.
- Environmental scientists study the relationship between the incidents of skin cancer and the amount of ozone in the atmosphere.
- Business analysts study the relationship between the price of a product and the number of products that are sold at that price.

Each of these relationships can be described by a set of ordered pairs.

Definition of a Relation

A **relation** is any set of ordered pairs.

The following table shows the number of hours that each of 9 students spent studying for a midterm exam and the grade that each of these 9 students received.

Hours	3	3.5	2.75	2	4	4.5	3	2.5	5
Grade	78	75	70	65	85	85	80	75	90

This information can be written as the relation

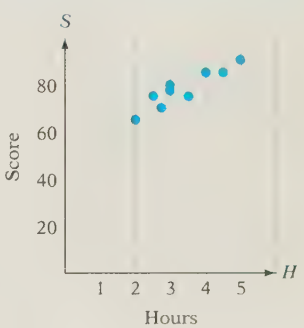
$$\{(3, 78), (3.5, 75), (2.75, 70), (2, 65), (4, 85), (4.5, 85), (3, 80), (2.5, 75), (5, 90)\}$$

where the first coordinate of the ordered pair is the hours spent studying and the second coordinate is the score on the midterm.

The **domain** of a relation is the set of first coordinates of the ordered pairs; the **range** is the set of second coordinates. For the relation above,

$$\text{Domain} = \{2, 2.5, 2.75, 3, 3.5, 4, 4.5, 5\} \qquad \text{Range} = \{65, 70, 75, 78, 80, 85, 90\}$$

The **graph of a relation** is the graph of the ordered pairs that belong to the relation. The graph of the relation given above is shown at the right. The horizontal axis represents the hours spent studying (the domain); the vertical axis represents the test score (the range). The axes could be labeled H for hours studied and S for test score.



A *function* is a special type of relation for which no two ordered pairs have the same first coordinate.

Definition of a Function

A **function** is a relation in which no two ordered pairs that have the same first coordinate have different second coordinates.

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The table at the right is the grading scale for a 100-point test. This table defines a relationship between the *score* on the test and a *letter grade*. Some of the ordered pairs of this function are (78, C), (97, A), (84, B), and (82, B).

Score	*Grade*
90–100	A
80–89	B
70–79	C
60–69	D
0–59	F

The grading-scale table defines a function, because no two ordered pairs can have the *same* first coordinate and *different* second coordinates. For instance, it is not possible to have the ordered pairs (72, C), and (72, B)—same first coordinate (test score) but different second coordinate (test grade). The domain of this function is $\{0, 1, 2, \ldots, 99, 100\}$. The range is $\{A, B, C, D, F\}$.

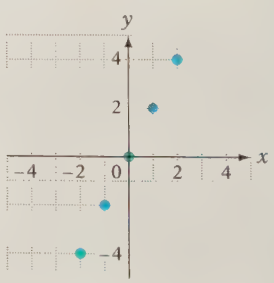
The example of hours spent studying and test score given earlier is *not* a function, because (3, 78) and (3, 80) are ordered pairs of the relation that have the *same* first coordinate but *different* second coordinates.

Consider, again, the grading-scale example. Note that (84, B) and (82, B) are ordered pairs of the function. Ordered pairs of a function may have the same *second* coordinates but not the same first coordinates.

Although relations and functions are given by tables, they are frequently given by an equation in two variables.

The equation $y = 2x$ expresses the relationship between a number, x, and twice the number, y. For instance, if $x = 3$, then $y = 6$, which is twice 3. To indicate exactly which ordered pairs are determined by the equation, the domain (values of x) is specified. If $x \in \{-2, -1, 0, 1, 2\}$, then the ordered pairs determined by the equation are $\{(-2, -4), (-1, -2), (0, 0), (1, 2), (2, 4)\}$. This relation is a function because no two ordered pairs have the same first coordinate.

The graph of the function $y = 2x$ with domain $\{-2, -1, 0, 1, 2\}$ is shown at the right. The horizontal axis (domain) is labeled x; the vertical axis (range) is labeled y.



The domain $\{-2, -1, 0, 1, 2\}$ was chosen arbitrarily. Other domains could have been selected. The type of application usually influences the choice of the domain.

For the equation $y = 2x$, we say that "y is a function of x" because the set of ordered pairs is a function.

Not all equations, however, define a function. For instance, the equation $|y| = x + 2$ does not define y as a function of x. The ordered pairs (2, 4) and (2, −4) both satisfy the equation. Thus there are two ordered pairs with the same first coordinate but different second coordinates.

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Example 5

The number of gold medals earned by several countries during the 1992 and 1996 Olympic Summer Games are recorded in the following table.

Country	1992	1996
United States	37	44
Canada	6	3
China	16	16
Italy	6	13
France	8	15
Germany	33	20

Write a relation where the first coordinate is the number of gold medals earned in 1992 and the second coordinate is the number earned in 1996. Is the relation a function?

Solution

{(37, 44), (6, 3), (16, 16), (6, 13), (8, 15), (33, 20)}

No. The relation is not a function. The two ordered pairs (6, 3) and (6, 13) have the same first coordinate but different second coordinates.

You Try It 5

Five students decided to go on a diet and fitness program over the summer. Their weights (in pounds) at the beginning and end of the program are given in the table below.

Beginning	*End*
145	140
140	125
150	130
165	150
140	130
165	160

Write a relation where the first coordinate is the weight at the beginning of the summer and the second coordinate is the weight at the end of the summer. Is the relation a function?

Your solution

Example 6

Does $y = x^2 + 3$, where $x \in \{-2, -1, 1, 3\}$ define y as a function of x?

Solution

Determine the ordered pairs defined by the equation. Replace x in $y = x^2 + 3$ by the given values and solve for y.

$\{(-2, 7), (-1, 4), (1, 4), (3, 12)\}$

No two ordered pairs have the same first coordinate. Therefore, the relation is a function and the equation $y = x^2 + 3$ defines y as a function of x.

Note that $(-1, 4)$ and $(1, 4)$ are ordered pairs that belong to this function. Ordered pairs of a function may have the same *second* coordinates but not the same *first* coordinates.

You Try It 6

Does $y = \frac{1}{2}x + 1$, where $x \in \{-4, 0, 2\}$, define y as a function of x?

Your solution

Solutions on p. S17

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Objective D To evaluate a function written in functional notation

When an equation defines y as a function of x, **functional notation** is frequently used to emphasize that the relation is a function. In this case, it is common to use the notation $f(x)$, where

$f(x)$ is read "f of x" or "the value of f at x."

For instance, the equation $y = x^2 + 3$ from Example 6 defined y as a function of x. The equation can also be written in functional notation as

$$f(x) = x^2 + 3$$

where y has been replaced by $f(x)$.

The symbol $f(x)$ is called the **value of the function** at x because it is the result of evaluating a variable expression. For instance, $f(4)$ means to replace x by 4 and then simplify the resulting numerical expression.

$$\begin{aligned} f(x) &= x^2 + 3 \\ f(4) &= 4^2 + 3 \qquad \text{• Replace } x \text{ by 4.} \\ &= 16 + 3 = 19 \end{aligned}$$

This process is called **evaluating the function.**

Given $f(x) = x^2 + x - 3$, find $f(-2)$.

$$\begin{aligned} f(x) &= x^2 + x - 3 \\ f(-2) &= (-2)^2 + (-2) - 3 \qquad \text{• Replace } x \text{ by } -2. \\ &= 4 - 2 - 3 = -1 \end{aligned}$$

$f(-2) = -1$

In this example, $f(-2)$ is the second coordinate of an ordered pair of the function; the first coordinate is -2. Therefore, an ordered pair of this function is $(-2, f(-2))$, which simplifies to $(-2, -1)$.

For the function given by $y = f(x) = x^2 + x - 3$, y is called the **dependent variable** because its value depends on the value of x. The **independent variable** is x.

Functions can be written using other letters or even combinations of letters. For instance, some calculators use $ABS(x)$ for the absolute-value function. Thus the equation $y = |x|$ would be written $ABS(x) = |x|$, where $ABS(x)$ replaces y.

Example 7

Given $G(t) = \dfrac{3t}{t + 4}$, find $G(1)$.

Solution

$$G(t) = \frac{3t}{t + 4}$$

$$G(1) = \frac{3(1)}{1 + 4} \qquad \text{• Replace } t \text{ by 1. Then simplify.}$$

$$G(1) = \frac{3}{5}$$

You Try It 7

Given $H(x) = \dfrac{x}{x - 4}$, find $H(8)$.

Your solution

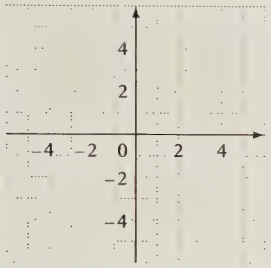
Solution on p. S17

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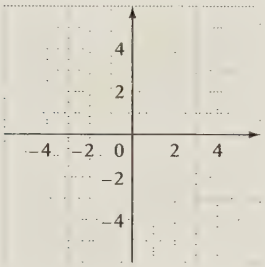
7.1 Exercises

Objective A

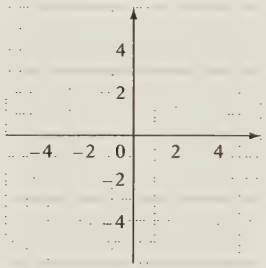
1. Graph $(-2, 1)$, $(3, -5)$, $(-2, 4)$, and $(0, 3)$.



2. Graph $(5, -1)$, $(-3, -3)$, $(-1, 0)$, and $(1, -1)$.



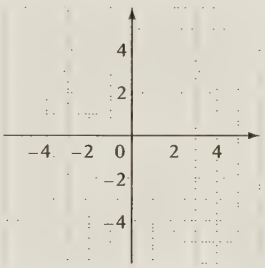
3. Graph $(0, 0)$, $(0, -5)$, $(-3, 0)$, and $(0, 2)$.



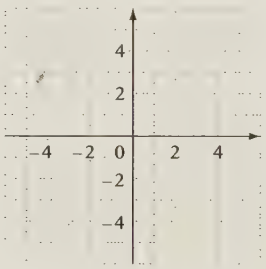
4. Graph $(-4, 5)$, $(-3, 1)$, $(3, -4)$, and $(5, 0)$.



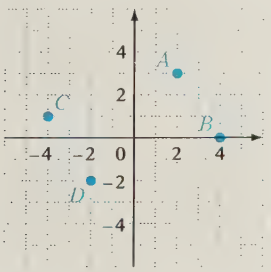
5. Graph $(-1, 4)$, $(-2, -3)$, $(0, 2)$, and $(4, 0)$.



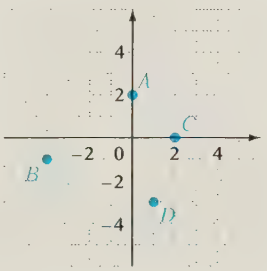
6. Graph $(5, 2)$, $(-4, -1)$, $(0, 0)$, and $(0, 3)$.



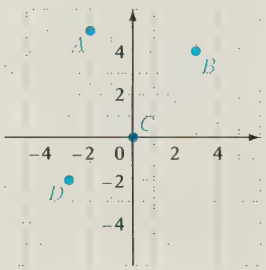
7. Find the coordinates of each of the points.



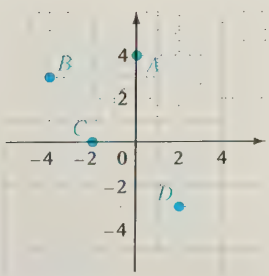
8. Find the coordinates of each of the points.



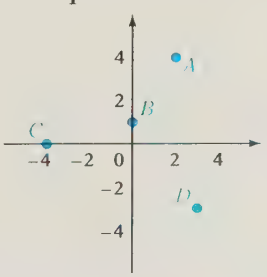
9. Find the coordinates of each of the points.



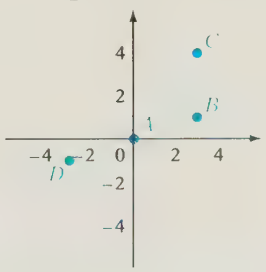
10. Find the coordinates of each of the points.



11. **a.** Name the abscissas of points A and C.
b. Name the ordinates of points B and D.



12. **a.** Name the abscissas of points A and C.
b. Name the ordinates of points B and D.



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Objective B

13. Is (3, 4) a solution of $y = -x + 7$?

14. Is (2, −3) a solution of $y = x + 5$?

15. Is (−1, 2) a solution of $y = \frac{1}{2}x - 1$?

16. Is (1, −3) a solution of $y = -2x - 1$?

17. Is (4, 1) a solution of $2x - 5y = 4$?

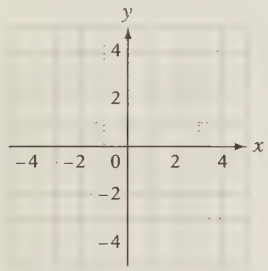
18. Is (−5, 3) a solution of $3x - 2y = 9$?

19. Is (0, 4) a solution of $3x - 4y = -4$?

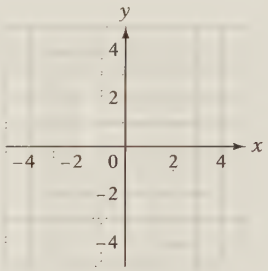
20. Is (−2, 0) a solution of $x + 2y = -1$?

Graph the ordered-pair solutions of each equation for the given values of x.

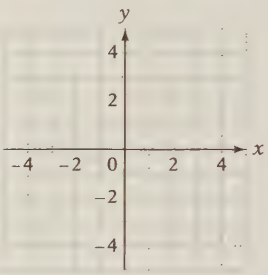
21. $y = 2x; x = -2, -1, 0, 2$



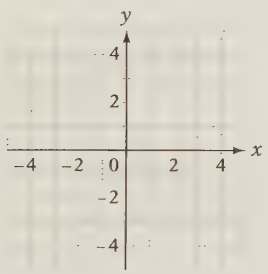
22. $y = -2x; x = -2, -1, 0, 2$



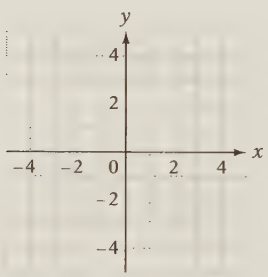
23. $y = x + 2; x = -4, -2, 0, 3$



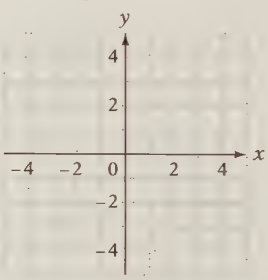
24. $y = \frac{1}{2}x - 1; x = -2, 0, 2, 4$



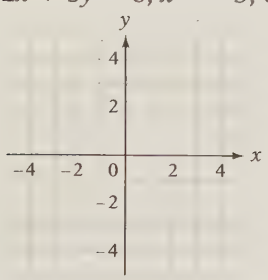
25. $y = \frac{2}{3}x + 1; x = -3, 0, 3$



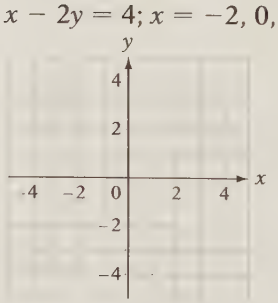
26. $y = -\frac{1}{3}x - 2; x = -3, 0, 3$



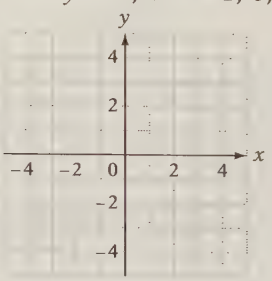
27. $2x + 3y = 6; x = -3, 0, 3$



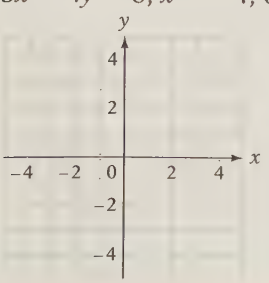
28. $x - 2y = 4; x = -2, 0, 2$



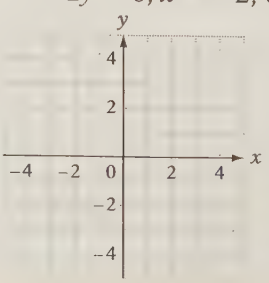
29. $2x + y = 3; x = -1, 0, 1, 2$



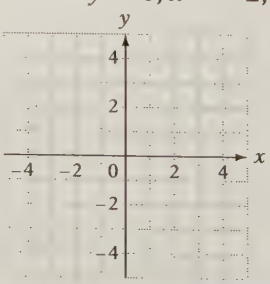
30. $3x - 4y = 8; x = -4, 0, 4$



31. $5x + 2y = 0; x = -2, 0, 2$



32. $-x - 2y = 0; x = -2, 0, 2$



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Objective C

33. The table below records the runs scored by the New York Yankees for the six games of the 1996 World Series and whether the Yankees won, W, or lost, L, the game. Write a relation where the first coordinate is the runs scored and the second coordinate is a win or a loss. Is the relation a function?

Runs scored	1	0	5	8	1	3
Won or lost	L	L	W	W	W	W

34. The table below records the goals scored by the Mighty Ducks hockey team for six games of the 1996–97 season and whether the Mighty Ducks won, W, tied, T, or lost, L, the game. Write a relation where the first coordinate is the goals scored and the second coordinate is a win, tie, or loss. Is the relation a function?

Goals scored	3	4	1	3	5	1
Won, lost, or tied	L	W	W	T	T	W

35. The number of sunny days in a year in five different cities and the average cost of a four bedroom house (in thousands of dollars) in those cities is given in the table to the right. Write a relation where the first coordinate is the number of sunny days annually, and the second coordinate is the cost of a house. Is the relation a function?

City	*Sunny Days Annually*	*4-Bedroom House Cost*
Nashua, NH	197	125
Portsmouth, NH	205	122
San Jose, CA	257	498
Jacksonville, FL	226	108
Manchester, NH	205	150

Source: *Money Magazine*, July, 1997

36. The budgets (in millions of dollars) for several movies and the gross ticket sales (in millions of dollars) are recorded in the table to the right. Write a relation where the first coordinate is the budget and the second coordinate is the gross sales. Is the relation a function?

Movie	*Budget*	*Gross Sales*
Seven	30	93
Apollo 13	62	172
Nixon	30	15
Toy Story	50	200
Batman Forever	100	184

37. Does $y = -2x - 3$, where $x \in \{-2, -1, 0, 3\}$, define y as a function of x?

38. Does $y = 2x + 3$, where $x \in \{-2, -1, 1, 4\}$, define y as a function of x?

39. Does $|y| = x - 1$, where $x \in \{1, 2, 3, 4\}$, define y as a function of x?

40. Does $|y| = x + 2$, where $x \in \{-2, -1, 0, 3\}$, define y as a function of x?

41. Does $y = x^2$, where $x \in \{-2, -1, 0, 1, 2\}$, define y as a function of x?

42. Does $y = x^2 - 1$, where $x \in \{-2, -1, 0, 1, 2\}$, define y as a function of x?

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Objective D

43. Given $f(x) = 3x - 4$, find $f(4)$.

44. Given $f(x) = 5x + 1$, find $f(2)$.

45. Given $f(x) = x^2$, find $f(3)$.

46. Given $f(x) = x^2 - 1$, find $f(1)$.

47. Given $G(x) = x^2 + x$, find $G(-2)$.

48. Given $H(x) = x^2 - x$, find $H(-2)$.

49. Given $s(t) = \frac{3}{t - 1}$, find $s(-2)$.

50. Given $P(x) = \frac{4}{2x + 1}$, find $P(-2)$.

51. Given $h(x) = 3x^2 - 2x + 1$, find $h(3)$.

52. Given $Q(r) = 4r^2 - r - 3$, find $Q(2)$.

53. Given $f(x) = \frac{x}{x + 5}$, find $f(-3)$.

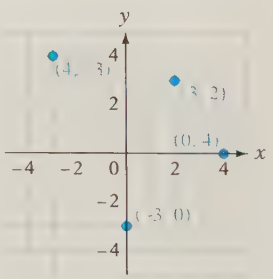
54. Given $v(t) = \frac{2t}{2t + 1}$, find $v(3)$.

55. Given $g(x) = x^3 - x^2 + 2x - 7$, find $g(0)$.

56. Given $F(z) = \frac{z}{z^2 + 1}$, find $F(0)$.

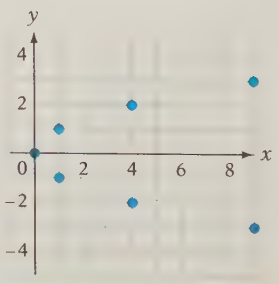
APPLYING THE CONCEPTS

57. Suppose you are helping a student who is having trouble graphing ordered pairs. The work of the student is at the right. What can you say to this student to correct the error that is being made?



58. Write a few sentences that describe the similarities and differences between relations and functions.

59. The graph of $y^2 = x$, where $x \in \{0, 1, 4, 9\}$, is shown at the right. Is this the graph of a function? Explain your answer.



60. Is it possible to evaluate $f(x) = \frac{5}{x - 1}$ when $x = 1$? If so, what is $f(1)$? If not, explain why not.

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7.2 Linear Equations in Two Variables

Objective A *To graph an equation of the form $y = mx + b$* ..... 23 CT

The **graph of an equation in two variables** is a graph of the ordered-pair solutions of the equation.

Consider $y = 2x + 1$. Choosing $x = -2, -1, 0, 1$, and 2 and determining the corresponding values of y produces some of the ordered pairs of the equation. These are recorded in the table at the right. See the graph of the ordered pairs in Figure 1.

x	$y = 2x + 1$	y	(x, y)
-2	$2(-2) + 1$	-3	$(-2, -3)$
-1	$2(-1) + 1$	-1	$(-1, -1)$
0	$2(0) + 1$	1	$(0, 1)$
1	$2(1) + 1$	3	$(1, 3)$
2	$2(2) + 1$	5	$(2, 5)$

Choosing values of x that are not integers produces more ordered pairs to graph, such as $\left(-\frac{5}{2}, -4\right)$ and $\left(\frac{3}{2}, 4\right)$, as shown in Figure 2. Choosing still other values of x would result in more and more ordered pairs being graphed. The result would be so many dots that the graph would appear as the straight line shown in Figure 3, which is the graph of $y = 2x + 1$.

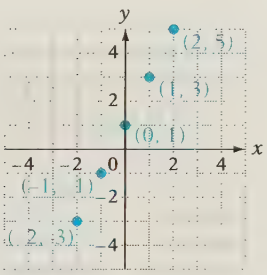


Figure 1

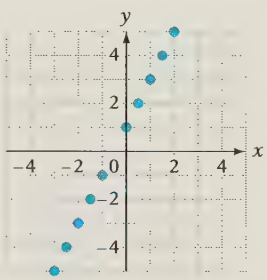


Figure 2

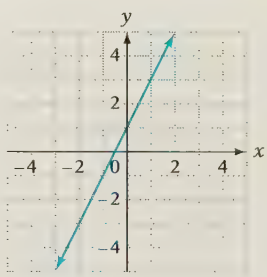


Figure 3

Equations in two variables have characteristic graphs. The equation $y = 2x + 1$ is an example of a *linear equation*, or *linear function*, because its graph is a straight line. It is also called a *first-degree equation* in two variables because the exponent on each variable is the first power.

Linear Equation in Two Variables

Any equation of the form $y = mx + b$, where m and b are constants, is a **linear equation in two variables or a first-degree equation in two variables.** The graph of a linear equation in two variables is a straight line.

Examples of linear equations are shown at the right. These equations represent linear functions because there is only one possible y for each x. Note that for $y = 3 - 2x$, m is the coefficient of x and b is the constant.

$y = 2x + 1$	$(m = 2, b = 1)$
$y = x - 4$	$(m = 1, b = -4)$
$y = -\frac{3}{4}x$	$\left(m = -\frac{3}{4}, b = 0\right)$
$y = 3 - 2x$	$(m = -2, b = 3)$

The equation $y = x^2 + 4x + 3$ is not a linear equation in two variables because there is a term with a variable squared. The equation $y = \frac{3}{x - 4}$ is not a linear equation because a variable occurs in the denominator of a fraction.

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POINT OF INTEREST

The "Projects and Group Activities" at the end of this chapter contains information on using calculators to graph an equation.

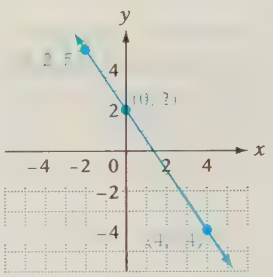
To graph a linear equation, choose some values of x and then find the corresponding values of y. Because a straight line is determined by two points, it is sufficient to find only two ordered-pair solutions. However, it is recommended that at least three ordered-pair solutions be used to ensure accuracy.

➡ Graph $y = -\frac{3}{2}x + 2$.

This is a linear equation with $m = -\frac{3}{2}$ and $b = 2$. Find at least three solutions.

Because m is a fraction, choose values of x that will simplify the calculations. We have chosen -2, 0, and 4 for x. (Any values of x could have been selected.)

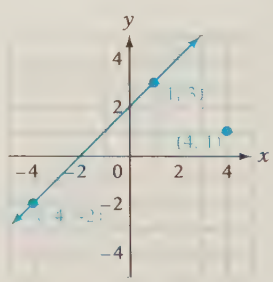
x	$y = -\frac{3}{2}x + 2$	y	(x, y)
-2	$-\frac{3}{2}(-2) + 2$	5	$(-2, 5)$
0	$-\frac{3}{2}(0) + 2$	2	$(0, 2)$
4	$-\frac{3}{2}(4) + 2$	-4	$(4, -4)$



The graph of $y = -\frac{3}{2}x + 2$ is shown at the right.

Remember that a graph is a drawing of the ordered-pair solutions of the equation. Therefore, every point on the graph is a solution of the equation, and every solution of the equation is a point on the graph.

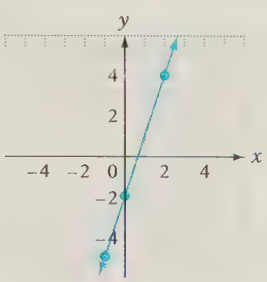
The graph at the right is the graph of $y = x + 2$. Note that $(-4, -2)$ and $(1, 3)$ are points on the graph and that these points are solutions of $y = x + 2$. The point whose coordinates are $(4, 1)$ is not a point on the graph and is not a solution of the equation.



Example 1 Graph $y = 3x - 2$.

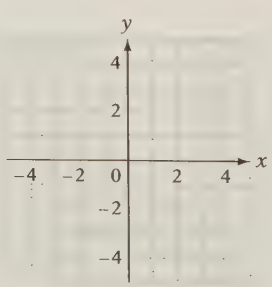
Solution

x	y
0	-2
-1	-5
2	4



You Try It 1 Graph $y = 3x + 1$.

Your solution



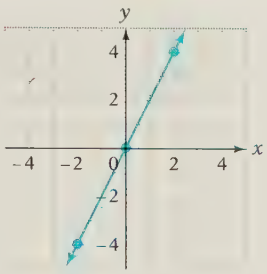
Solution on p. S17

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Example 2 Graph $y = 2x$.

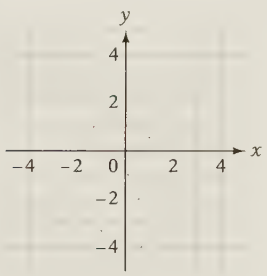
Solution

x	y
0	0
2	4
−2	−4



You Try It 2 Graph $y = -2x$.

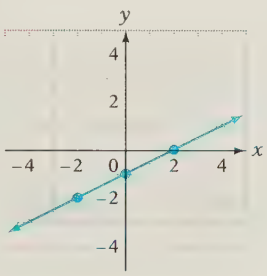
Your solution



Example 3 Graph $y = \frac{1}{2}x - 1$.

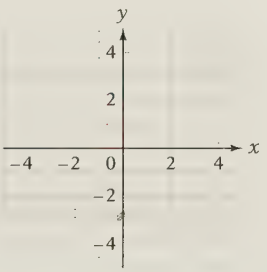
Solution

x	y
0	−1
2	0
−2	−2



You Try It 3 Graph $y = \frac{1}{3}x - 3$.

Your solution



Solutions on p. S17

Objective B To graph an equation of the form Ax + By = C

The equation $Ax + By = C$, where A, B, and C are constants, is also a linear equation. Examples of these equations are shown at the right.

$2x + 3y = 6$	$(A = 2, B = 3, C = 6)$
$x - 2y = -4$	$(A = 1, B = -2, C = -4)$
$2x + y = 0$	$(A = 2, B = 1, C = 0)$
$4x - 5y = 2$	$(A = 4, B = -5, C = 2)$

To graph an equation of the form $Ax + By = C$, first solve the equation for y. Then follow the same procedure used for graphing $y = mx + b$.

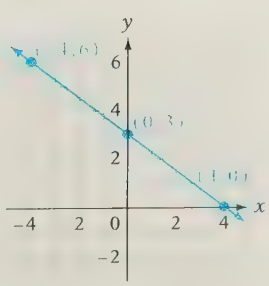
➡ Graph $3x + 4y = 12$.

$3x + 4y = 12$
$4y = -3x + 12$ • Solve for y.
• Subtract $3x$ from each side of the equation.

$y = -\frac{3}{4}x + 3$ • Divide each side of the equation by 4.

x	y
0	3
4	0
−4	6

• Find 3 ordered-pair solutions of the equation.



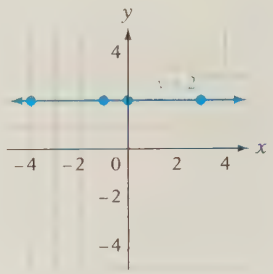
• Graph the ordered pairs and then draw a line through the points.

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The graph of a linear equation with one of the variables missing is either a horizontal or a vertical line.

The equation $y = 2$ could be written $0 \cdot x + y = 2$. Because $0 \cdot x = 0$ for any value of x, the value of y is always 2 no matter what value of x is chosen. For instance, replace x by -4, -1, 0, or 3. In each case, $y = 2$.

$$\begin{aligned} 0x + y &= 2 \\ 0(-4) + y &= 2 \quad (-4, 2) \text{ is a solution.} \\ 0(-1) + y &= 2 \quad (-1, 2) \text{ is a solution.} \\ 0(0) + y &= 2 \quad (0, 2) \text{ is a solution.} \\ 0(3) + y &= 2 \quad (3, 2) \text{ is a solution.} \end{aligned}$$



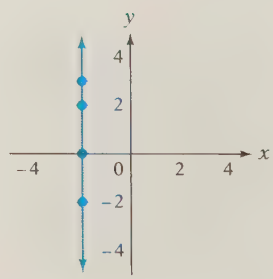
The solutions are plotted in the graph to the right, and a line is drawn through the plotted points. Note that the line is horizontal.

Graph of a Horizontal Line

The graph of $y = b$ is a horizontal line passing through $(0, b)$.

The equation $x = -2$ could be written $x + 0 \cdot y = -2$. Because $0 \cdot y = 0$ for any value of y, the value of x is always -2 no matter what value of y is chosen. For instance, replace y by -2, 0, 2, or 3. In each case, $x = -2$.

$$\begin{aligned} x + 0y &= -2 \\ x + 0(-2) &= -2 \quad (-2, -2) \text{ is a solution.} \\ x + 0(0) &= -2 \quad (-2, 0) \text{ is a solution.} \\ x + 0(2) &= -2 \quad (-2, 2) \text{ is a solution.} \\ x + 0(3) &= -2 \quad (-2, 3) \text{ is a solution.} \end{aligned}$$

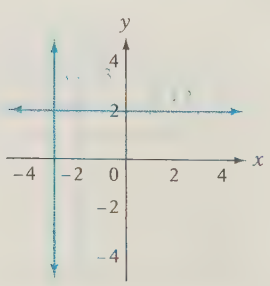


The solutions are plotted in the graph at the right, and a line is drawn through the plotted points. Note that the line is vertical.

Graph of a Vertical Line

The graph of $x = a$ is a vertical line passing through $(a, 0)$.

Graph $x = -3$ and $y = 2$ on the same coordinate grid.



- The graph of $x = -3$ is a vertical line passing through $(-3, 0)$.
- The graph of $y = 2$ is a horizontal line passing through $(0, 2)$.

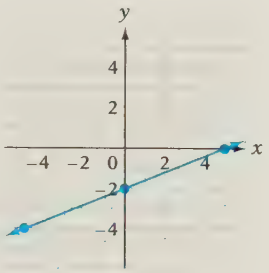
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Example 4 Graph $2x - 5y = 10$.

Solution

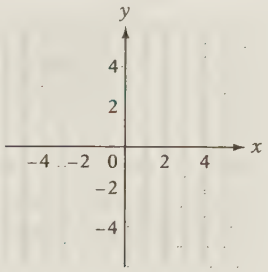
$$2x - 5y = 10$$
$$-5y = -2x + 10$$
$$y = \frac{2}{5}x - 2$$

x	y
0	-2
5	0
-5	-4



You Try It 4 Graph $5x - 2y = 10$.

Your solution

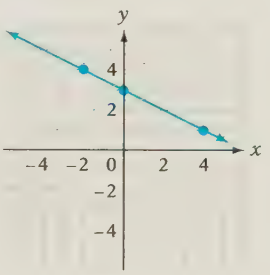


Example 5 Graph $x + 2y = 6$.

Solution

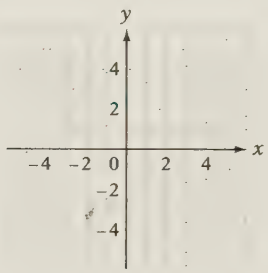
$$x + 2y = 6$$
$$2y = -x + 6$$
$$y = -\frac{1}{2}x + 3$$

x	y
0	3
-2	4
4	1



You Try It 5 Graph $x - 3y = 9$.

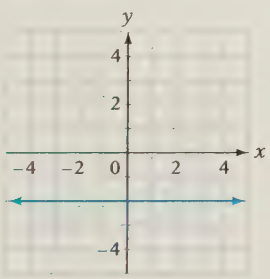
Your solution



Example 6 Graph $y = -2$.

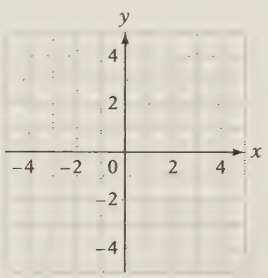
Solution

The graph of an equation of the form $y = b$ is a horizontal line passing through the point $(0, b)$.



You Try It 6 Graph $y = 3$.

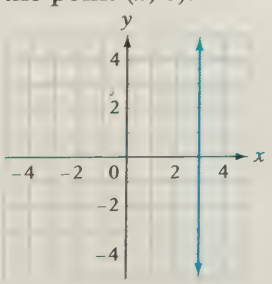
Your solution



Example 7 Graph $x = 3$.

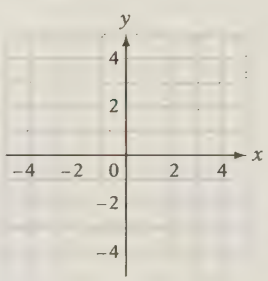
Solution

The graph of an equation of the form $x = a$ is a vertical line passing through the point $(a, 0)$.



You Try It 7 Graph $x = -4$.

Your solution



Solutions on p. S17

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Objective C To solve application problems

There are a variety of applications of linear functions.

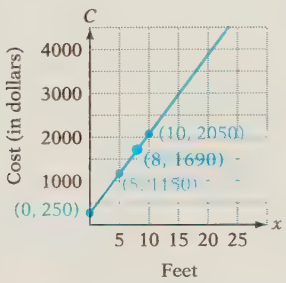
➡ Solve: An installer of marble kitchen countertops charges \$250 plus \$180 per foot of countertop. The equation that describes the total cost, C, to have x feet of countertop installed is $C = 180x + 250$.

a. Graph this equation for $0 \le x \le 25$. (*Note:* In many applications, the domain of the variable is given so that the equation makes sense. For instance, it would not be sensible to have values of x that are less than 0. This would mean negative countertop! The choice of 25 is somewhat arbitrary, but most kitchens have less than 25 ft of counter space.)

b. The point whose coordinates are (8, 1690) is on the graph. Write a sentence that describes this ordered pair.

Solution

a.



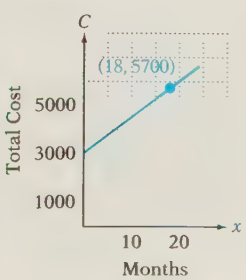
• Choosing $x = 0$, 5, and 10, you find that the corresponding ordered pairs are (0, 250), (5, 1150), and (10, 2050). Plot these points and draw a line through them.

b. The point whose coordinates are (8, 1690) means that 8 ft of countertop costs \$1690 to install.

Example 8

A local car dealer is advertising a 2-year lease for a new Honda Civic. The upfront cost is \$3000 with a monthly lease payment of \$150. The total cost, C, after x months of the lease is given by $C = 150x + 3000$. Graph this equation for $0 \le x \le 24$. The point whose coordinates are (18, 5700) is on the graph. Write a sentence that describes this ordered pair.

Solution

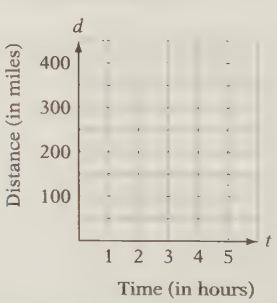


The ordered pair (18, 5700) means that the total cost of the lease payments for 18 months is \$5700.

You Try It 8

A car is traveling at a uniform speed of 40 mph. The distance, d, the car travels in t hours is given by $d = 40t$. Graph this equation for $0 \le t \le 5$. The point whose coordinates are (3, 120) is on the graph. Write a sentence that describes this ordered pair.

Your solution



Solution on p. S17

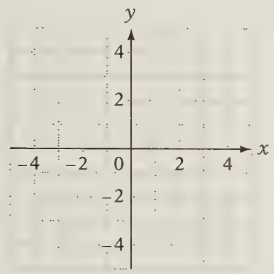
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7.2 Exercises

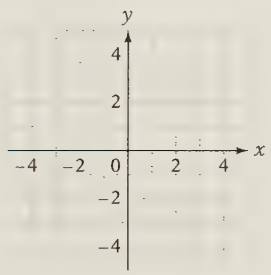
Objective A

Graph.

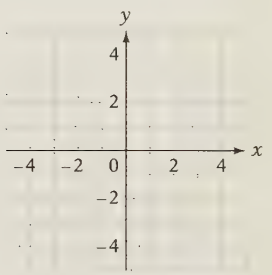
1. $y = 2x - 3$



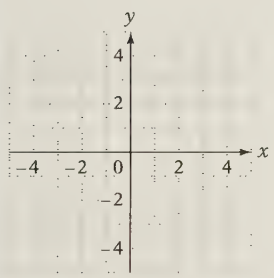
2. $y = -2x + 2$



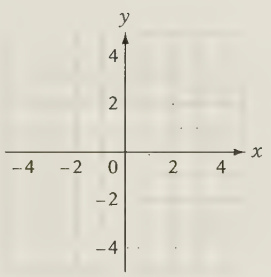
3. $y = \frac{1}{3}x$



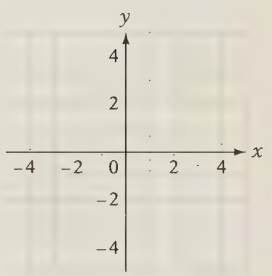
4. $y = -3x$



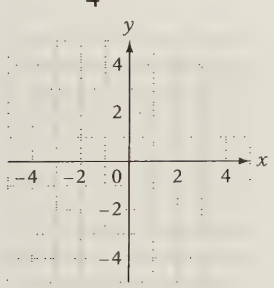
5. $y = \frac{2}{3}x - 1$



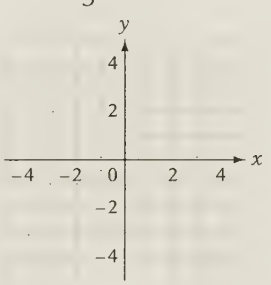
6. $y = \frac{3}{4}x + 2$



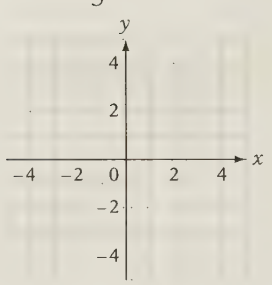
7. $y = -\frac{1}{4}x + 2$



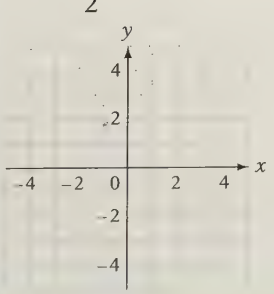
8. $y = -\frac{1}{3}x + 1$



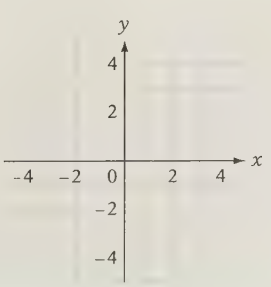
9. $y = -\frac{2}{5}x + 1$



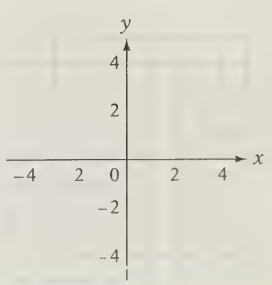
10. $y = -\frac{1}{2}x + 3$



11. $y = 2x - 4$

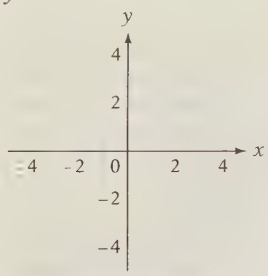


12. $y = 3x - 4$

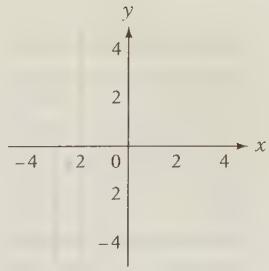


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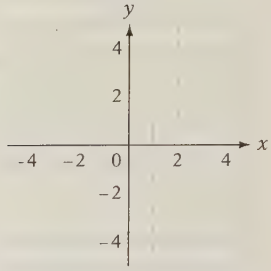
13. $y = x - 3$



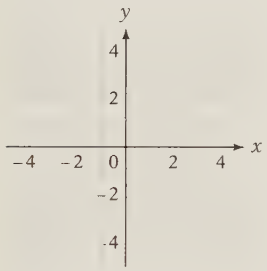
14. $y = x + 2$



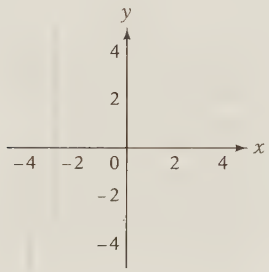
15. $y = -x + 2$



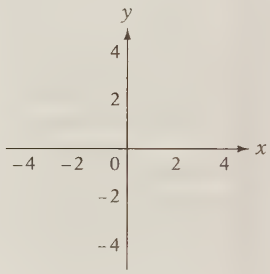
16. $y = -x - 1$



17. $y = -\frac{2}{3}x + 1$



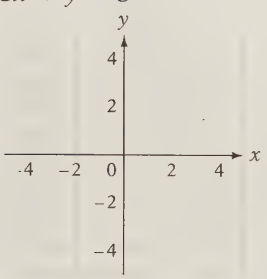
18. $y = 5x - 4$



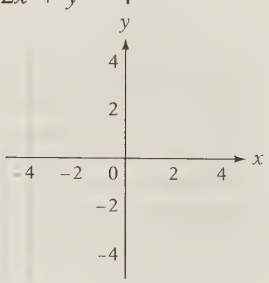
Objective B

Graph.

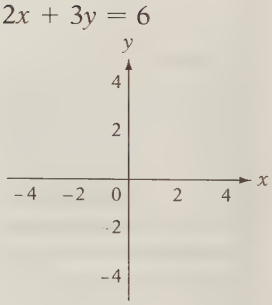
19. $3x + y = 3$



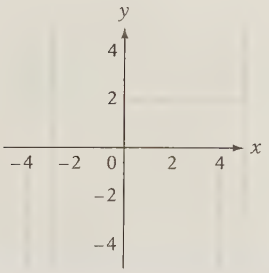
20. $2x + y = 4$



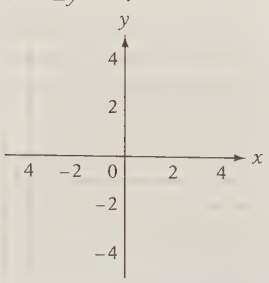
21. $2x + 3y = 6$



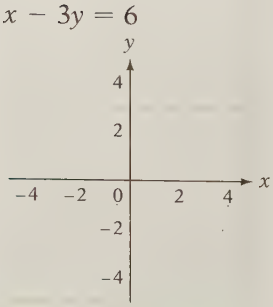
22. $3x + 2y = 4$



23. $x - 2y = 4$

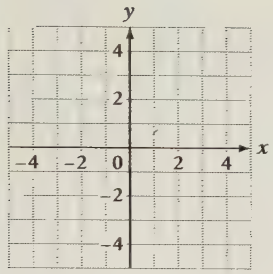


24. $x - 3y = 6$

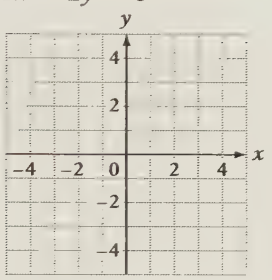


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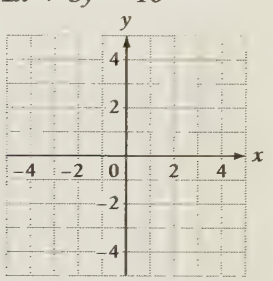
25. $2x - 3y = 6$



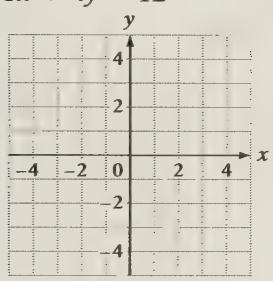
26. $3x - 2y = 8$



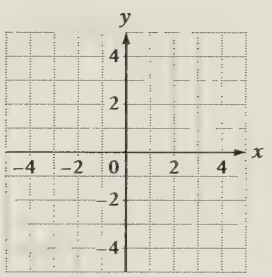
27. $2x + 5y = 10$



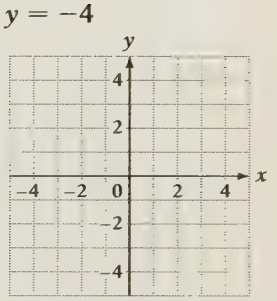
28. $3x + 4y = 12$



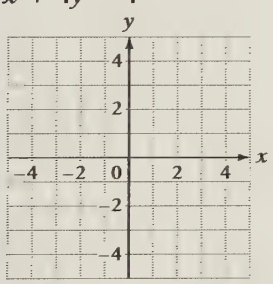
29. $x = 3$



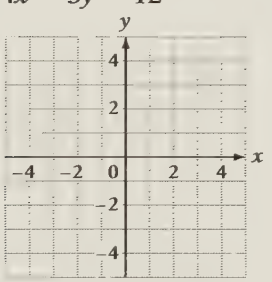
30. $y = -4$



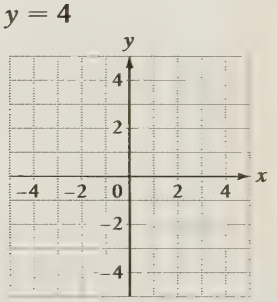
31. $x + 4y = 4$



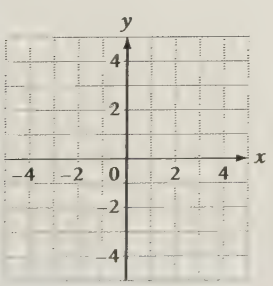
32. $4x - 3y = 12$



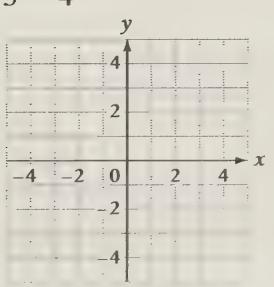
33. $y = 4$



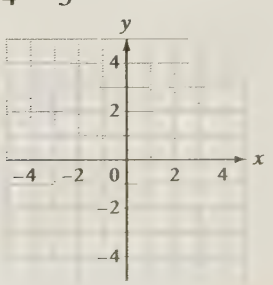
34. $x = -2$



35. $\frac{x}{5} + \frac{y}{4} = 1$



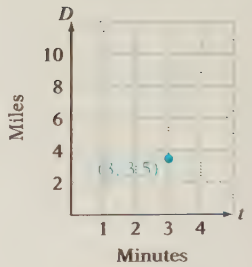
36. $\frac{x}{4} - \frac{y}{3} = 1$



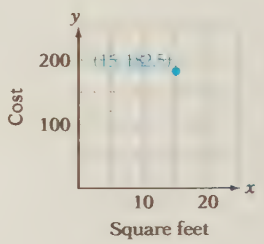
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Objective C Application Problems

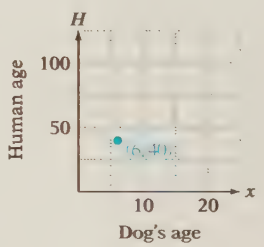
37. A rescue helicopter is rushing at a constant speed of 150 mph to reach several people stranded in the ocean 11 mi away after their boat sank. The rescuers can determine how far they are from the victims using the equation $D = 11 - 2.5t$, where D is the distance in miles and t is the time elapsed in minutes. Graph this equation for $0 \le t \le 4$. The point (3, 3.5) is on the graph. Write a sentence that describes the meaning of this ordered pair.



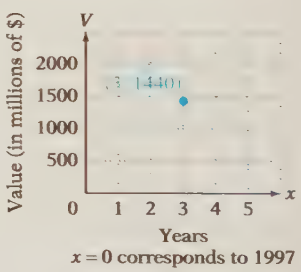
38. A custom-illustrated sign or banner can be commissioned for a cost of \$25 for the material and \$10.50 per square foot for the artwork. The equation that represents this cost is given by $y = 10.50x + 25$, where y is the cost and x is the number of square feet in the sign. Graph this equation for $0 \le x \le 20$. The point (15, 182.5) is on the graph. Write a sentence that describes the meaning of this ordered pair.



39. According to some veterinarians, the age, x, of a dog can be translated to "human years" by using the equation $H = 4x + 16$, where H is the human equivalent age for the dog. Graph this equation for $2 \le x \le 21$. The point whose coordinates are (6, 40) is on this graph. Write a sentence that explains the meaning of this ordered pair.



40. One projection for the total value, V, of stocks that will be bought and sold over the internet for the years 1997 through 2002 is given by the equation $V = 380x + 300$, where $0 \le x \le 5$ and 0 corresponds to the year 1997. Graph the equation. The point whose coordinates are (3, 1440) is on the graph. Write a sentence that describes the meaning of this ordered pair.

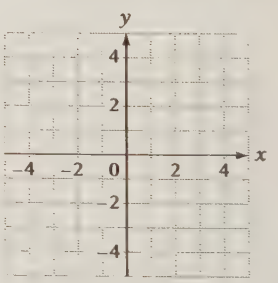


APPLYING THE CONCEPTS

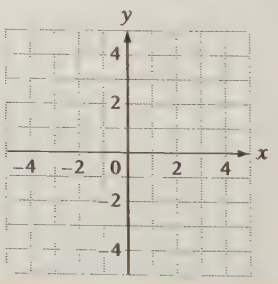
41. For the equation $y = 3x + 2$, when the value of x changes from 1 to 2, does the value of y increase or decrease? What is the change in y? Suppose that the value of x changes from 13 to 14. What is the change in y?

42. For the equation $y = -2x + 1$, when the value of x changes from 1 to 2, does the value of y increase or decrease? What is the change in y? Suppose the value of x changes from 13 to 14. What is the change in y?

43. Graph $y = 2x - 2$, $y = 2x$, and $y = 2x + 3$ in the first coordinate system at the right. What observation can you make about the graphs?



44. Graph $y = x + 3$, $y = 2x + 3$, and $y = -\frac{1}{2}x + 3$ in the second coordinate system at the right. What observation can you make about the graphs?

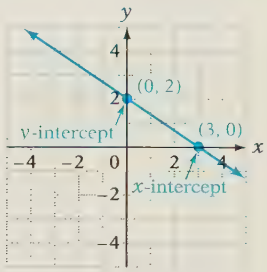


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7.3 Intercepts and Slopes of Straight Lines

Objective A To find the x- and y-intercepts of a straight line

The graph of the equation $2x + 3y = 6$ is shown at the right. The graph crosses the x-axis at the point $(3, 0)$. This point is called the ***x*-intercept.** The graph also crosses the y-axis at the point $(0, 2)$. This point is called the ***y*-intercept.**



➡ Find the x-intercept and the y-intercept of the graph of the equation $2x - 3y = 12$.

TAKE NOTE

To find the x-intercept, let $y = 0$. To find the y-intercept, let $x = 0$.

To find the x-intercept, let $y = 0$. (Any point on the x-axis has y-coordinate 0.)

$$\begin{aligned} 2x - 3y &= 12 \\ 2x - 3(0) &= 12 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

The x-intercept is $(6, 0)$.

To find the y-intercept, let $x = 0$. (Any point on the y-axis has x-coordinate 0.)

$$\begin{aligned} 2x - 3y &= 12 \\ 2(0) - 3y &= 12 \\ -3y &= 12 \\ y &= -4 \end{aligned}$$

The y-intercept is $(0, -4)$.

➡ Find the y-intercept of $y = 3x + 4$.

$y = 3x + 4 = 3(0) + 4 = 4$ • Let $x = 0$.

The y-intercept is $(0, 4)$.

For any equation of the form $y = mx + b$, the y-intercept is $(0, b)$.

Some linear equations can be graphed by finding the x- and y-intercepts and then drawing a line through these two points.

Example 1 Find the x- and y-intercepts for $x - 2y = 4$. Graph the line.

Solution

x-intercept:

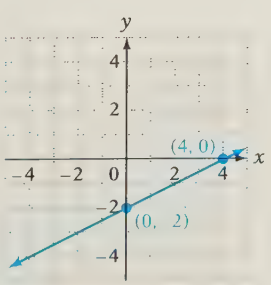
$$\begin{aligned} x - 2y &= 4 \\ x - 2(0) &= 4 \\ x &= 4 \end{aligned}$$

$(4, 0)$

y-intercept:

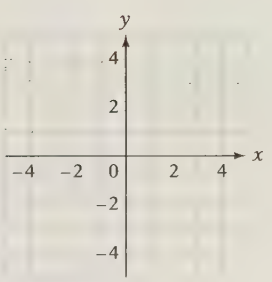
$$\begin{aligned} x - 2y &= 4 \\ 0 - 2y &= 4 \\ -2y &= 4 \\ y &= -2 \end{aligned}$$

$(0, -2)$



You Try It 1 Find the x- and y-intercepts for $y = 2x - 4$. Graph the line.

Your solution



Solution on p. S18

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Objective B To find the slope of a straight line

The graphs of $y = \frac{2}{3}x + 1$ and $y = 2x + 1$ are shown in Figure 1. Each graph crosses the y-axis at the point $(0, 1)$, but the graphs have different slants. The **slope** of a line is a measure of the slant of a line. The symbol for slope is m.

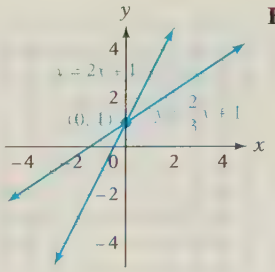


Figure 1

The slope of a line containing two points is the ratio of the change in the y values of the two points to the change in the x values. The line containing the points $(-2, -3)$ and $(6, 1)$ is graphed in Figure 2. The change in the y values is the difference between the two ordinates.

$$\text{Change in } y = 1 - (-3) = 4$$

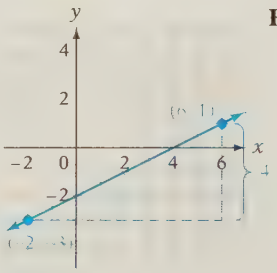


Figure 2

The change in the x values is the difference between the two abscissas (Figure 3).

$$\text{Change in } x = 6 - (-2) = 8$$

$$\text{Slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{4}{8} = \frac{1}{2}$$

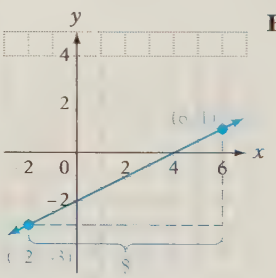


Figure 3

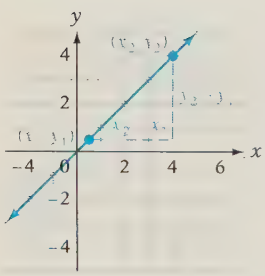


Figure 4

Slope Formula

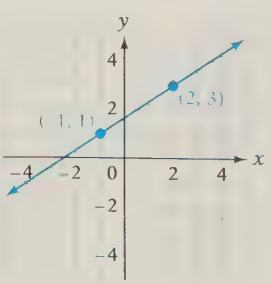
If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are two points on a line and $x_1 \neq x_2$, then $m = \frac{y_2 - y_1}{x_2 - x_1}$ (Figure 4). If $x_1 = x_2$, the slope is undefined.

➡ Find the slope of the line containing the points $(-1, 1)$ and $(2, 3)$.

Let P_1 be $(-1, 1)$ and P_2 be $(2, 3)$. Then, $x_1 = -1$, $y_1 = 1$, $x_2 = 2$, and $y_2 = 3$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{2 - (-1)} = \frac{2}{3}$$

The slope is $\frac{2}{3}$.



Positive slope

A line that slants upward to the right always has a **positive slope.**

TAKE NOTE

Positive slope means that the value of y increases as the value of x increases.

Note that you obtain the same results if the points are named oppositely. Let P_1 be $(2, 3)$ and P_2 be $(-1, 1)$. Then $x_1 = 2$, $y_1 = 3$, $x_2 = -1$, and $y_2 = 1$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{-1 - 2} = \frac{-2}{-3} = \frac{2}{3}$$

The slope is $\frac{2}{3}$.

Therefore, it does not matter which point is named P_1 and which P_2; the slope remains the same.

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TAKE NOTE

Negative slope means that the value of y decreases as x increases. Compare this to positive slope.

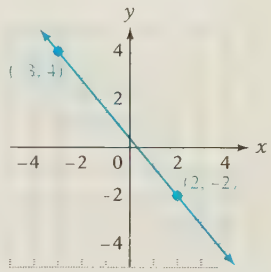
➡ Find the slope of the line containing the points $(-3, 4)$ and $(2, -2)$.

Let P_1 be $(-3, 4)$ and P_2 be $(2, -2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{2 - (-3)} = \frac{-6}{5} = -\frac{6}{5}$$

The slope is $-\frac{6}{5}$.

A line that slants downward to the right always has a **negative slope.**



Negative slope

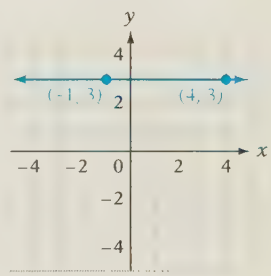
➡ Find the slope of the line containing the points $(-1, 3)$ and $(4, 3)$.

Let P_1 be $(-1, 3)$ and P_2 be $(4, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{4 - (-1)} = \frac{0}{5} = 0$$

The slope is 0.

A horizontal line has **zero slope.**



Zero slope

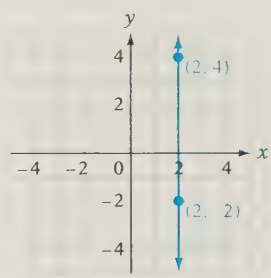
➡ Find the slope of the line containing the points $(2, -2)$ and $(2, 4)$.

Let P_1 be $(2, -2)$ and P_2 be $(2, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{2 - 2} = \frac{6}{0}$$

Division by zero is not defined.

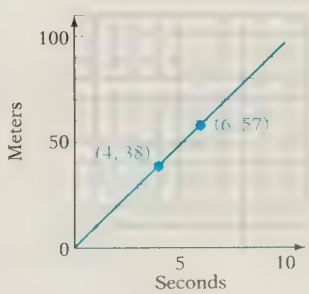
The slope of a vertical line is undefined.



Undefined

There are many applications of the concept of slope. Here are two possibilities.

In 1988, when Florence Griffith-Joyner set the world record for the 100-meter dash, her average rate of speed was approximately 9.5 meters per second. The graph at the right shows the distance she ran during her record-setting run. From the graph, note that after 4 s she had traveled 38 m and that after 6 s she had traveled 57 m. The slope of the line between these two points is

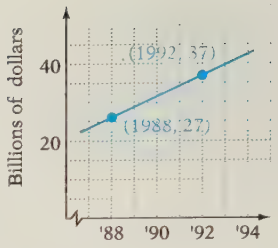


$$m = \frac{57 - 38}{6 - 4} = \frac{19}{2} = 9.5$$

Note that the slope of the line is the same as the rate she was running, 9.5 meters per second. The average speed of an object is related to slope.

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Here is an example of slope taken from economics. According to the Department of Commerce, from 1987 to 1994, U.S. exports of goods to other countries increased at a rate of approximately \$2.5 billion per year. The graph at the right shows the value of exports for each year. From the graph, we learn that in 1988 exports were \$27 billion, and in 1992 exports were \$37 billion. The slope of the line between these two points is



$$m = \frac{37 - 27}{1992 - 1988} = \frac{10}{4} = 2.5$$

Observe that the slope of the line is the same as the rate at which exports increased, \$2.5 billion per year.

In general, any quantity that is expressed by using the word *per* is represented mathematically as slope. In the first example, slope was 9.5 meters *per* second; in the second example, slope was \$2.5 billion *per* year.

Example 2

Find the slope of the line containing the points with coordinates $(-2, -3)$ and $(3, 4)$.

Solution

Let $P_1 = (-2, -3)$ and $P_2 = (3, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-3)}{3 - (-2)} = \frac{7}{5}$$

The slope is $\frac{7}{5}$.

You Try It 2

Find the slope of the line containing the points with coordinates $(-3, 8)$ and $(1, 4)$.

Your solution

Example 3

Find the slope of the line containing the points with coordinates $(-1, 4)$ and $(-1, 0)$.

Solution

Let $P_1 = (-1, 4)$ and $P_2 = (-1, 0)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{-1 - (-1)} = \frac{-4}{0}$$

The slope is undefined.

You Try It 3

Find the slope of the line containing the points with coordinates $(-1, 2)$ and $(4, 2)$.

Your solution

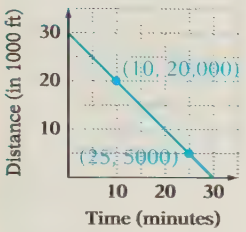
Solutions on p. S18

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Example 4

The graph below shows the height of a plane above an airport during its 30-minute descent from cruising altitude to landing. Find the slope of the line. Write a sentence that explains the meaning of the slope.

Solution



$$m = \frac{5000 - 20{,}000}{25 - 10} = \frac{-15{,}000}{15}$$

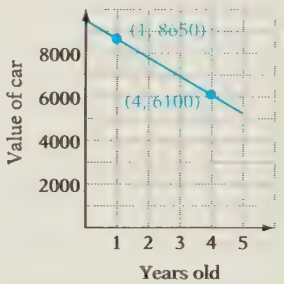
$$= -1000$$

A slope of -1000 means that the height of the plane is *decreasing* at the rate of 1000 ft/min.

You Try It 4

The graph below shows the approximate decline in the value of a used car over a five-year period. Find the slope of the line. Write a sentence that states the meaning of the slope.

Your solution

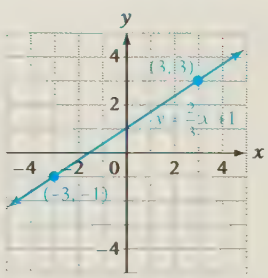


Solution on p. S18

Objective C To graph a line using the slope and the y-intercept



The graph of the equation $y = \frac{2}{3}x + 1$ is shown at the right. The points $(-3, -1)$ and $(3, 3)$ are on the graph. The slope of the line between the two points is



$$m = \frac{3 - (-1)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

Observe that the slope of the line is the coefficient of x in the equation $y = \frac{2}{3}x + 1$. Also recall that the y-intercept is $(0, 1)$, where 1 is the constant term of the equation.

Slope-Intercept Equation of a Line

An equation of the form $y = mx + b$ is called the **slope-intercept** form of a straight line. The slope of the line is m, the coefficient of x. The y-intercept is $(0, b)$, where b is the constant term of the equation.

The following equations are written in slope-intercept form.

$y = 2x - 3$ Slope $= 2$; y-intercept $= (0, -3)$

$y = -x + 2$ Slope $= -1 (-x = -1x)$; y-intercept $= (0, 2)$

$y = \frac{x}{2}$ Slope $= \frac{1}{2}\left(\frac{x}{2} = \frac{1}{2}x\right)$; y-intercept $= (0, 0)$

When an equation of a straight line is written in slope-intercept form, the graph can be drawn using the slope and the y-intercept. First locate the y-intercept. Use the slope to find a second point on the line. Then draw a line through the two points.

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➡ Graph $y = 2x - 3$.

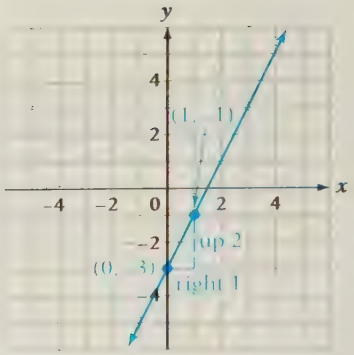
y-intercept $= (0, b) = (0, -3)$

$$m = 2 = \frac{2}{1} = \frac{\text{change in } y}{\text{change in } x}$$

Beginning at the y-intercept, move right 1 unit (change in x) and then up 2 units (change in y).

$(1, -1)$ is a second point on the graph.

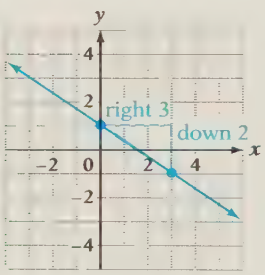
Draw a line through the two points $(0, -3)$ and $(1, -1)$.



Example 5 Graph $y = -\frac{2}{3}x + 1$ by using the slope and y-intercept.

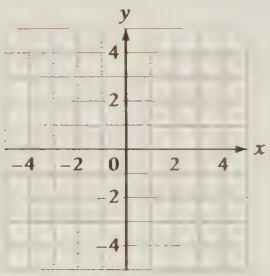
Solution y-intercept $= (0, b) = (0, 1)$

$$m = -\frac{2}{3} = \frac{-2}{3}$$



You Try It 5 Graph $y = -\frac{1}{4}x - 1$ by using the slope and y-intercept.

Your solution



Example 6 Graph $2x - 3y = 6$ by using the slope and y-intercept.

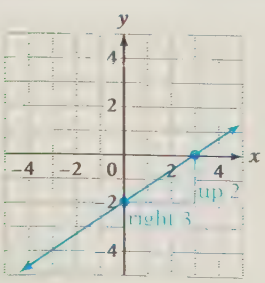
Solution Solve the equation for y.

$$2x - 3y = 6$$

$$-3y = -2x + 6$$

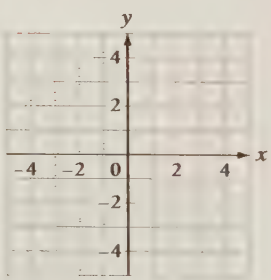
$$y = \frac{2}{3}x - 2$$

y-intercept $= (0, -2)$; $m = \frac{2}{3}$



You Try It 6 Graph $x - 2y = 4$ by using the slope and y-intercept.

Your solution



Solutions on p. S18

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7.3 Exercises

Objective A

Find the x- and y-intercepts.

1. $x - y = 3$

2. $3x + 4y = 12$

3. $y = 3x - 6$

4. $y = 2x + 10$

5. $x - 5y = 10$

6. $3x + 2y = 12$

7. $y = 3x + 12$

8. $y = 5x + 10$

9. $2x - 3y = 0$

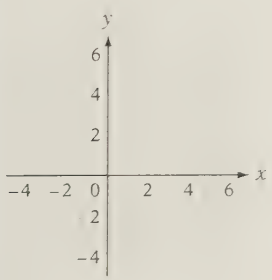
10. $3x + 4y = 0$

11. $y = -\frac{1}{2}x + 3$

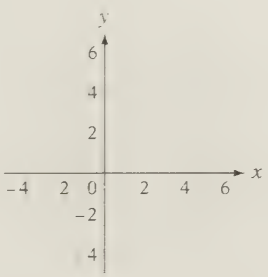
12. $y = \frac{2}{3}x - 4$

Find the x- and y-intercepts and then graph.

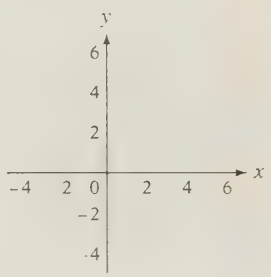
13. $5x + 2y = 10$



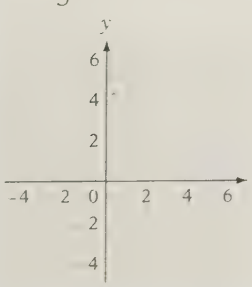
14. $x - 3y = 6$



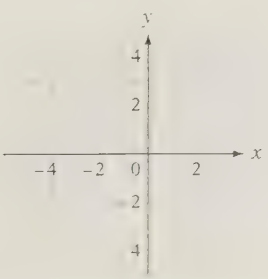
15. $y = \frac{3}{4}x - 3$



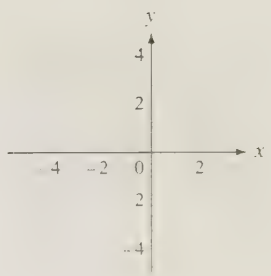
16. $y = \frac{2}{5}x - 2$



17. $5y - 3x = 15$



18. $9y - 4x = 18$



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Objective B

Find the slope of the line containing the given points.

19. $P_1(4, 2), P_2(3, 4)$

20. $P_1(2, 1), P_2(3, 4)$

21. $P_1(-1, 3), P_2(2, 4)$

22. $P_1(-2, 1), P_2(2, 2)$

23. $P_1(2, 4), P_2(4, -1)$

24. $P_1(1, 3), P_2(5, -3)$

25. $P_1(-2, 3), P_2(2, 1)$

26. $P_1(5, -2), P_2(1, 0)$

27. $P_1(8, -3), P_2(4, 1)$

28. $P_1(0, 3), P_2(2, -1)$

29. $P_1(3, -4), P_2(3, 5)$

30. $P_1(-1, 2), P_2(-1, 3)$

31. $P_1(4, -2), P_2(3, -2)$

32. $P_1(5, 1), P_2(-2, 1)$

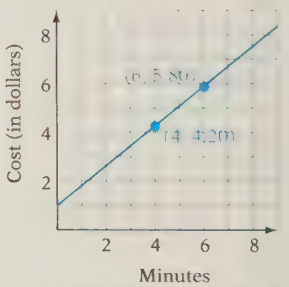
33. $P_1(0, -1), P_2(3, -2)$

34. $P_1(3, 0), P_2(2, -1)$

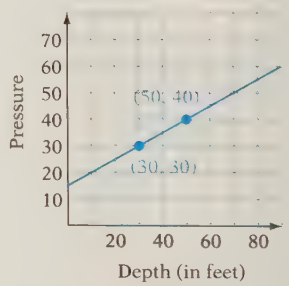
35. $P_1(-2, 3), P_2(1, 3)$

36. $P_1(4, -1), P_2(-3, -1)$

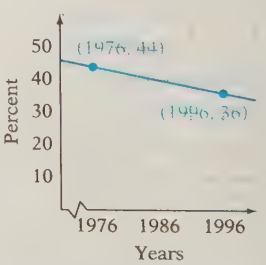
37. The graph at the right shows the cost, in dollars, to make a transatlantic telephone call. Find the slope of the line. Write a sentence that states the meaning of the slope.



38. The pressure, in pounds per square inch, on a diver is shown in the graph on the right. Find the slope of the line. Write a sentence that explains the meaning of the slope.

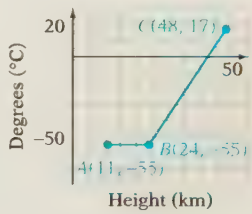


39. One measure of the affordability of a home for a given population is the percent of the population that has the necessary income to purchase a median-priced new home ($136,000 in 1996). The graph at the right shows the decrease in the percent of the population that can afford a new home. Find the slope of the line and write a sentence that states its meaning.



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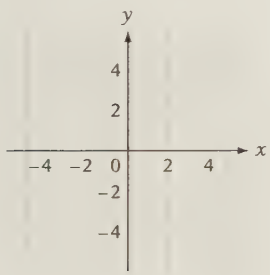
40. The stratosphere extends from approximately 11 km to 50 km above Earth. The graph at the right shows how the temperature in degrees Celsius changes in the stratosphere. Explain the meaning of the horizontal line segment from A to B. Find the slope of the line from B to C and explain its meaning.



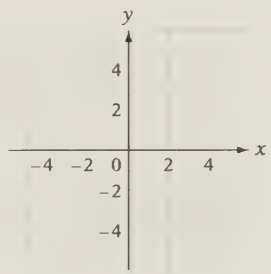
Objective C

Graph by using the slope and y-intercept.

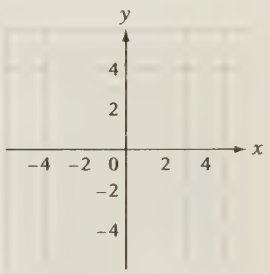
41. $y = 3x + 1$



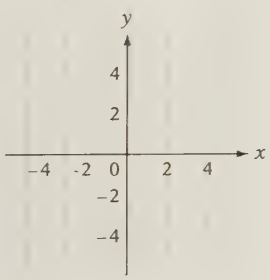
42. $y = -2x - 1$



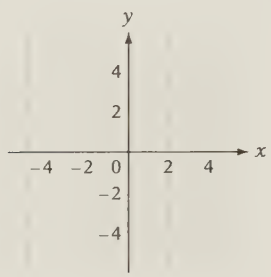
43. $y = \frac{2}{5}x - 2$



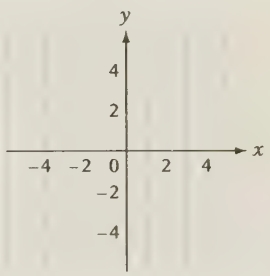
44. $y = \frac{3}{4}x + 1$



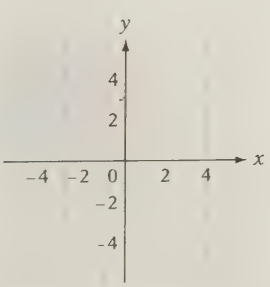
45. $2x + y = 3$



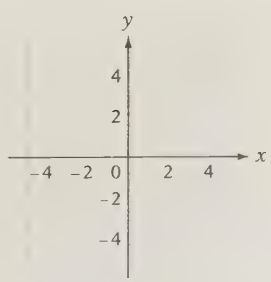
46. $3x - y = 1$



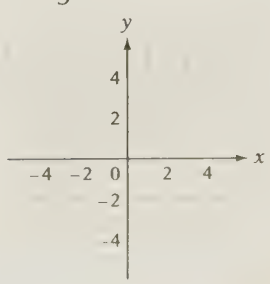
47. $x - 2y = 4$



48. $x + 3y = 6$

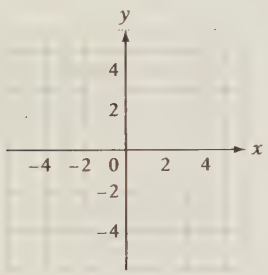


49. $y = \frac{2}{3}x$

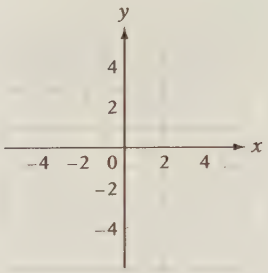


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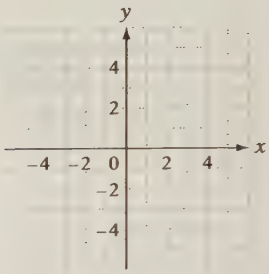
50. $y = \frac{1}{2}x$



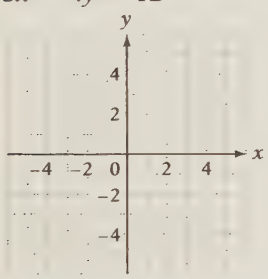
51. $y = -x + 1$



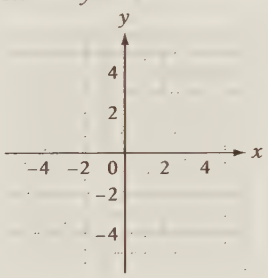
52. $y = -x - 3$



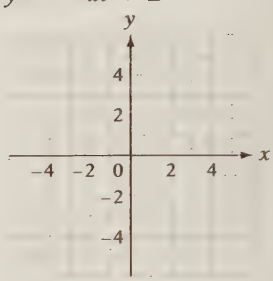
53. $3x - 4y = 12$



54. $5x - 2y = 10$



55. $y = -4x + 2$



APPLYING THE CONCEPTS

56. Do all straight lines have a y-intercept? If not, give an example of one that does not.

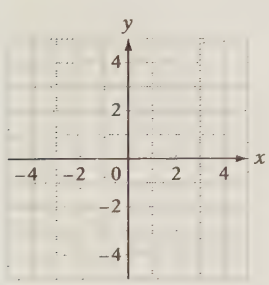
57. If two lines have the same slope and the same y-intercept, must the graphs of the lines be the same? If not, give an example.

58. Draw the graphs of **a.** $\frac{x}{3} + \frac{y}{4} = 1$ **b.** $\frac{x}{2} - \frac{y}{3} = 1$ **c.** $-\frac{x}{4} + \frac{y}{2} = 1$

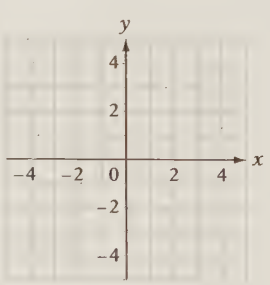
What observations can you make about the x- and y-intercepts of these graphs and the coefficients of x and y?

d. Use this observation to draw the graph of $\frac{x}{4} - \frac{y}{3} = 1$.

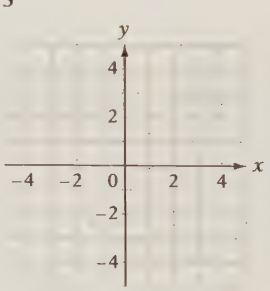
a.



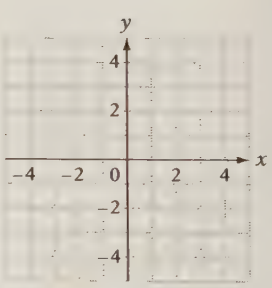
b.



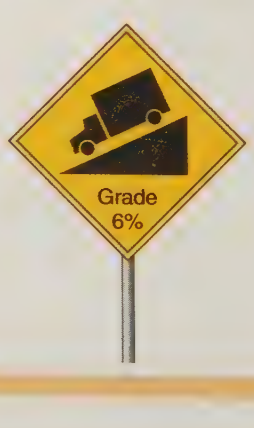
c.



d.



59. What does the highway sign at the right have to do with slope?



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7.4 Equations of Straight Lines

Objective A To find the equation of a line given a point and the slope

In earlier sections, the equation of a line was given and you were asked to determine some properties of the line, such as its intercepts and slope. Here, the process is reversed: Given properties of a line, determine its equation.

If the slope and y-intercept of the line are known, the equation of the line can be determined by using the slope-intercept form of a straight line.

➡ Find the equation of the line with slope $-\frac{1}{2}$ and y-intercept $(0, 3)$.

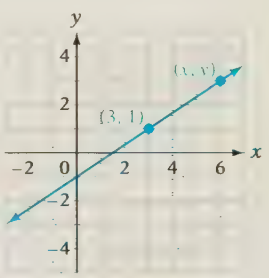
$$y = mx + b \qquad \text{• Use the slope-intercept formula.}$$

$$y = -\frac{1}{2}x + 3 \qquad \text{• } m = -\frac{1}{2};\ (0, b) = (0, 3), \text{ so } b = 3.$$

The equation of the line is $y = -\frac{1}{2}x + 3$.

When the coordinates of a point other than the y-intercept and the slope are known, the equation of the line can be found by using the formula for slope.

Suppose a line passes through the point $(3, 1)$ and has a slope of $\frac{2}{3}$. The equation of the line with these properties is determined by letting (x, y) be the coordinates of an unknown point on the line. Because the slope of the line is known, use the slope formula to write an equation. Then solve for y.



$$\frac{y-1}{x-3} = \frac{2}{3} \qquad \text{• } \frac{y_2 - y_1}{x_2 - x_1} = m;\ m = \frac{2}{3};\ (x_2, y_2) = (x, y);\ (x_1, y_1) = (3, 1)$$

$$\frac{y-1}{x-3}(x-3) = \frac{2}{3}(x-3) \qquad \text{• Multiply each side by } (x-3).$$

$$y - 1 = \frac{2}{3}x - 2 \qquad \text{• Simplify.}$$

$$y = \frac{2}{3}x - 1 \qquad \text{• Solve for } y.$$

The equation of the line is $y = \frac{2}{3}x - 1$.

The same procedure that was used above is used to derive the *point-slope formula*. We use this formula to determine the equation of a line when we are given the coordinates of a point on the line and the slope of the line.

Let (x_1, y_1) be the given coordinates of a point on a line, m the given slope of the line, and (x, y) the coordinates of an unknown point on the line. Then

$$\frac{y - y_1}{x - x_1} = m \qquad \text{• Formula for slope.}$$

$$\frac{y - y_1}{x - x_1}(x - x_1) = m(x - x_1) \qquad \text{• Multiply each side by } x - x_1.$$

$$y - y_1 = m(x - x_1) \qquad \text{• Simplify.}$$

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Point-Slope Formula

If (x_1, y_1) is a point on a line with slope m, then $y - y_1 = m(x - x_1)$.

Find the equation of the line that passes through point $(2, 3)$ and has slope -2.

$y - y_1 = m(x - x_1)$ • Use the point-slope formula.

$y - 3 = -2(x - 2)$ • $m = -2$; $(x_1, y_1) = (2, 3)$

$y - 3 = -2x + 4$ • Solve for y.

$y = -2x + 7$

The equation of the line is $y = -2x + 7$.

Example 1

Find the equation of the line whose slope is $-\frac{2}{3}$ and whose y-intercept is $(0, -1)$.

Solution

Because the slope and y-intercept are known, use the slope-intercept formula, $y = mx + b$.

$y = -\frac{2}{3}x - 1$ • $m = -\frac{2}{3}$; $b = -1$

You Try It 1

Find the equation of the line whose slope is $\frac{5}{3}$ and whose y-intercept is $(0, 2)$.

Your solution

Example 2

Use the point-slope formula to find the equation of the line that passes through the point $(-2, -1)$ and has slope $\frac{3}{2}$.

Solution

$y - y_1 = m(x - x_1)$

$y - (-1) = \frac{3}{2}[x - (-2)]$ • $m = \frac{3}{2}$; $(x_1, y_1) = (-2, -1)$

$y + 1 = \frac{3}{2}(x + 2)$

$y + 1 = \frac{3}{2}x + 3$

$y = \frac{3}{2}x + 2$

The equation of the line is $y = \frac{3}{2}x + 2$.

You Try It 2

Use the point-slope formula to find the equation of the line that passes through the point $(4, -2)$ and has slope $\frac{3}{4}$.

Your solution

Solutions on p. S18

Objective B To find the equation of a line given two points

26 CT

The point-slope formula is used to find the equation of a line when a point on the line and the slope of the line are known. But this formula can also be used to find the equation of a line given two points on the line. In this case,

1. Use the slope formula to determine the slope of the line between the points.
2. Use the point-slope formula, the slope you just calculated, and one of the given points to find the equation of the line.

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➡ Find the equation of the line that passes through the points whose coordinates are $(-3, -1)$ and $(3, 3)$.

Use the slope formula to determine the slope of the line between the points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-1)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$ • $(x_1, y_1) = (-3, -1); (x_2, y_2) = (3, 3)$

Use the point-slope formula, the slope you just calculated, and one of the given points to find the equation of the line.

$$y - y_1 = m(x - x_1)$$ • Point-slope formula

$$y - (-1) = \frac{2}{3}[x - (-3)]$$ • $m = \frac{2}{3}$; $(x_1, y_1) = (-3, -1)$

$$y + 1 = \frac{2}{3}(x + 3)$$

$$y + 1 = \frac{2}{3}x + 2$$

$$y = \frac{2}{3}x + 1$$

TAKE NOTE

You can verify that the equation $y = \frac{2}{3}x + 1$ passes through the points $(-3, -1)$ and $(3, 3)$ by substituting the coordinates of these points into the equation.

$y = \frac{2}{3}x + 1$

-1	$\frac{2}{3}(-3) + 1$
-1	$-2 + 1$

$-1 = -1$ • $(x, y) = (-3, -1)$

$y = \frac{2}{3}x + 1$

3	$\frac{2}{3}(3) + 1$
3	$2 + 1$

$3 = 3$ • $(x, y) = (3, 3)$

The equation of the line that passes through the two points is $y = \frac{2}{3}x + 1$.

Example 3

Find the equation of the line that passes through the points $(-4, 0)$ and $(2, -3)$.

Solution

Find the slope of the line between the two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{2 - (-4)} = \frac{-3}{6} = -\frac{1}{2}$$

Use the point-slope formula.

$$y - y_1 = m(x - x_1)$$ • Point-slope formula

$$y - 0 = -\frac{1}{2}[x - (-4)]$$ • $m = -\frac{1}{2}$; $(x_1, y_1) = (-4, 0)$

$$y = -\frac{1}{2}(x + 4)$$

$$y = -\frac{1}{2}x - 2$$

The equation of the line is $y = -\frac{1}{2}x - 2$.

You Try It 3

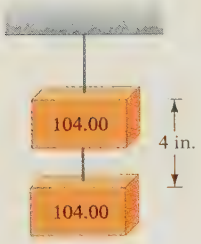
Find the equation of the line that passes through the points $(-6, -1)$ and $(3, 1)$.

Your solution

Solution on p. S18

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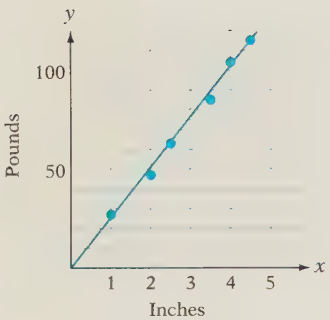
Objective C To solve application problems



A **linear model** is a first-degree equation that is used to describe a relationship between quantities. In many cases, a linear model is used to approximate collected data. The data are graphed as points in a coordinate system, and then a line is drawn that approximates the data. The graph of the points is called a **scatter diagram;** the line is called a **line of best fit.**

Consider an experiment to determine the weight required to stretch a spring a certain distance. Data from such an experiment are shown in the table below.

Distance (in inches)	2.5	4	2	3.5	1	4.5
Weight (in pounds)	63	104	47	85	27	115



The accompanying graph shows the scatter diagram, which is the plotted points, and the line of best fit, which is the line that approximately goes through the plotted points. The equation of the line of best fit is $y = 25.7x - 1.3$, where x is the number of inches the spring is stretched and y is the weight in pounds.

The table below shows the values that the model would predict to the nearest tenth. Good linear models should predict values that are close to the actual values. A more thorough analysis of lines of best fit is undertaken in statistics courses.

Distance, x	2.5	4	2	3.5	1	4.5
Weight predicted using $y = 25.7x - 1.3$	63.0	101.5	50.1	88.7	24.4	114.4

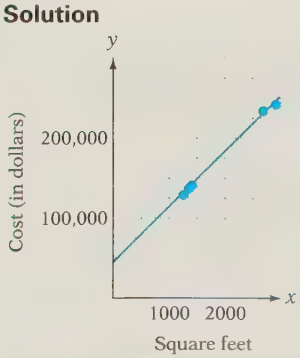
Example 4

The data in the table below show the size of a house in square feet and the cost to build the house. The line of best fit is $y = 70.3x + 41{,}100$, where x is the number of square feet and y is the cost of the house.

Square feet	1250	1400	1348	2675	2900
Cost	128,000	140,000	136,100	233,450	241,500

Graph the data and the line of best fit in the coordinate system below. Write a sentence that describes the meaning of the slope of the line of best fit.

Solution



The slope of the line means that the cost to build the house increases \$70.30 for each additional square foot in the size of the house.

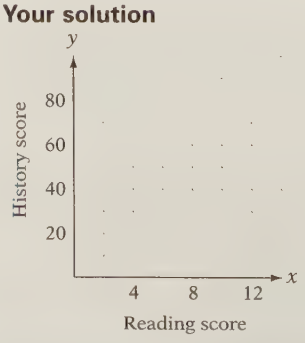
You Try It 4

The data in the table below show a reading test grade and the final exam grade in a history class. The line of best fit is $y = 8.3x - 7.8$, where x is the reading test score and y is the history test score.

Reading	8.5	9.4	10.0	11.4	12.0
History	64	68	76	87	92

Graph the data and the line of best fit in the coordinate system below. Write a sentence that describes the meaning of the slope of the line of best fit.

Your solution



Solution on p. S18

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7.4 Exercises

Objective A

1. Find the equation of the line that contains the point (0, 2) and has slope 2.

2. Find the equation of the line that contains the point (0, −1) and has slope −2.

3. Find the equation of the line that contains the point (−1, 2) and has slope −3.

4. Find the equation of the line that contains the point (2, −3) and has slope 3.

5. Find the equation of the line that contains the point (3, 1) and has slope $\frac{1}{3}$.

6. Find the equation of the line that contains the point (−2, 3) and has slope $\frac{1}{2}$.

7. Find the equation of the line that contains the point (4, −2) and has slope $\frac{3}{4}$.

8. Find the equation of the line that contains the point (2, 3) and has slope $-\frac{1}{2}$.

9. Find the equation of the line that contains the point (5, −3) and has slope $-\frac{3}{5}$.

10. Find the equation of the line that contains the point (5, −1) and has slope $\frac{1}{5}$.

11. Find the equation of the line that contains the point (2, 3) and has slope $\frac{1}{4}$.

12. Find the equation of the line that contains the point (−1, 2) and has slope $-\frac{1}{2}$.

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Objective B

13. Find the equation of the line that passes through the points $(1, -1)$ and $(-2, -7)$.

14. Find the equation of the line that passes through the points $(2, 3)$ and $(3, 2)$.

15. Find the equation of the line that passes through the points $(-2, 1)$ and $(1, -5)$.

16. Find the equation of the line that passes through the points $(-1, -3)$ and $(2, -12)$.

17. Find the equation of the line that passes through the points $(0, 0)$ and $(-3, -2)$.

18. Find the equation of the line that passes through the points $(0, 0)$ and $(-5, 1)$.

19. Find the equation of the line that passes through the points $(2, 3)$ and $(-4, 0)$.

20. Find the equation of the line that passes through the points $(3, -1)$ and $(0, -3)$.

21. Find the equation of the line that passes through the points $(-4, 1)$ and $(4, -5)$.

22. Find the equation of the line that passes through the points $(-5, 0)$ and $(10, -3)$.

23. Find the equation of the line that passes through the points $(-2, 1)$ and $(2, 4)$.

24. Find the equation of the line that passes through the points $(3, -2)$ and $(-3, -3)$.

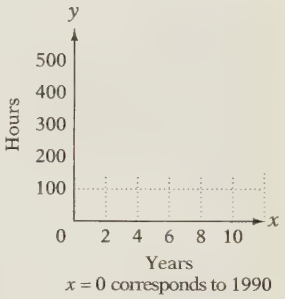
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Objective C Application Problems

25. The data in the table below are estimates that a study projected for the number of hours, on average, a person will spend watching "basic cable" television channels each year. The line of best fit is $y = 12.9x + 332$, where x is the year (with $x = 0$ corresponding to 1990) and y is the number of hours per person.

Year, x	5	6	7	8	9
Hours per Person, y	398	408	420	435	449

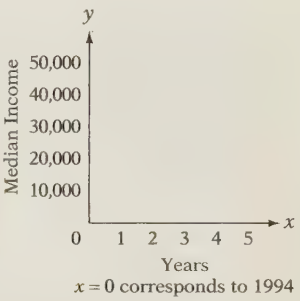
Graph the data and the line of best fit in the coordinate system at the right. Write a sentence that describes the meaning of the slope of the line of best fit.



26. The data in the table below show the projected median annual income for a four-person family during different years. The line of best fit is $y = 1460x + 47{,}016$, where x is the year (with $x = 0$ corresponding to 1994) and y is the annual income.

Year, x	0	1	2	3	4
Annual Income, y	47,012	48,452	49,987	51,406	52,843

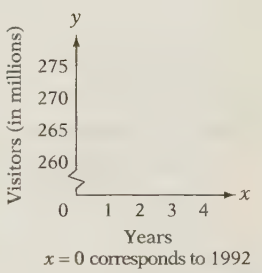
Graph the data and the line of best fit in the coordinate system at the right. Write a sentence that describes the meaning of the slope of the line of best fit.



27. The data in the table below show the number of visitors (in millions) to U.S. national parks during different years. The line of best fit is $y = -2.1x + 275$, where x is the year (with $x = 0$ corresponding to 1992) and y is the number of visitors in millions.

Year, x	0	1	2	3	4
Visitors (in millions), y	274.7	273.1	268.6	269.6	265.8

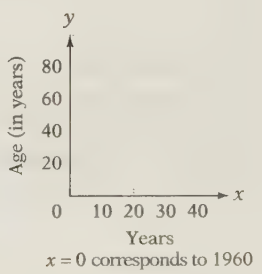
Graph the data and the line of best fit in the coordinate system at the right. Write a sentence that describes the meaning of the slope of the line of best fit.



28. The data in the table below show the average lifetime for women in the United States during different years. The line of best fit is $y = 0.2x + 73$, where x is the year (with $x = 0$ corresponding to 1960) and y is the length of the lifetime in years.

Year, x	0	10	20	30	40
Life Expectancy (in years), y	73.1	74.7	77.4	78.8	81.2

Graph the data and the line of best fit in the coordinate system at the right. Write a sentence that describes the meaning of the slope of the line of best fit.



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APPLYING THE CONCEPTS

In Exercises 29 to 32, the first two given points are on a line. Determine whether the third point is on the line.

29. $(-3, 2), (4, 1); (-1, 0)$

30. $(2, -2), (3, 4); (-1, 5)$

31. $(-3, -5), (1, 3); (4, 9)$

32. $(-3, 7), (0, -2); (1, -5)$

33. If $(-2, 4)$ are the coordinates of a point on the line whose equation is $y = mx + 1$, what is the slope of the line?

34. If $(3, 1)$ are the coordinates of a point on the line whose equation is $y = mx - 3$, what is the slope of the line?

35. If $(0, -3)$, $(6, -7)$, and $(3, n)$ are coordinates of points on the same line, determine n.

36. If $(-4, 11)$, $(2, -4)$, and $(6, n)$ are coordinates of points on the same line, determine n.

The formula $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$, where $x_1 \neq x_2$, is called the **two-point formula** for a straight line. This formula can be used to find the equation of a line given two points. Use this formula for Exercises 37 and 38.

37. Find the equation of the line passing through $(-2, 3)$ and $(4, -1)$.

38. Find the equation of the line passing through $(3, -1)$ and $(4, -3)$.

39. Explain why the condition $x_1 \neq x_2$ is placed on the two-point formula given above.

40. Explain how the two-point formula given above can be derived from the point-slope formula.

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Focus on Problem Solving

Counterexamples

Some of the exercises in this text ask you to determine whether a statement is true or false. For instance, the statement "every real number has a reciprocal" is false because 0 is a real number and 0 does not have a reciprocal.

Finding an example, such as 0 has no reciprocal, to show that a statement is not always true is called "finding a counterexample." A counterexample is an example that shows that a statement is not always true.

Consider the statement "the product of two numbers is greater than either factor." A counterexample to this statement is the factors $\frac{2}{3}$ and $\frac{3}{4}$. The product of these numbers is $\frac{1}{2}$, and $\frac{1}{2}$ is smaller than $\frac{2}{3}$ or $\frac{3}{4}$. There are many other counterexamples to the given statement.

Here are some counterexamples to the statement "the square of a number is always larger than the number."

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad \text{but} \quad \frac{1}{4} < \frac{1}{2} \qquad 1^2 = 1 \quad \text{but} \quad 1 = 1$$

For each of the next five statements, find at least one counterexample to show that the statement, or conjecture, is false.

1. The product of two integers is always a positive number.
2. The sum of two prime numbers is never a prime number.
3. For all real numbers, $|x + y| = |x| + |y|$.
4. If x and y are nonzero real numbers and $x > y$, then $x^2 > y^2$.
5. The quotient of any two nonzero real numbers is less than either one of the numbers.

When a problem is posed, it may not be known whether the statement is true or false. For instance, Christian Goldbach (1690–1764) stated that every even integer greater than 2 can be written as the sum of two prime numbers. No one has been able to find a counterexample to this statement, but neither has anyone been able to prove that it is always true.

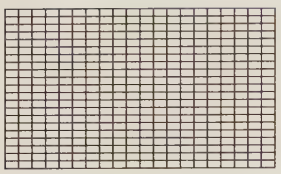
In the next five exercises, answer true if the statement is always true. If there is an instance when the statement is false, give a counterexample.

6. The reciprocal of a positive number is always smaller than the number.
7. If $x < 0$, then $|x| = -x$.
8. For any two real numbers x and y, $x + y > x - y$.
9. For any positive integer n, $n^2 + n + 17$ is a prime number.
10. The list of numbers 1, 11, 111, 1111, 11111, ... contains infinitely many composite numbers. (*Hint:* A number is divisible by 3 if the sum of the digits of the number is divisible by 3.)

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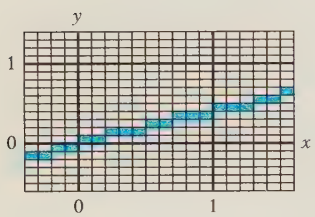
Projects and Group Activities

Graphing Linear Equations with a Graphing Utility



A computer or graphing calculator screen is divided into *pixels*. There are approximately 6000 to 790,000 pixels available on the screen (depending on the computer or calculator). The greater the number of pixels, the smoother a graph will appear. A portion of a screen is shown at the left. Each little rectangle represents one pixel.

The graphing utilities that are used by computers or calculators to graph an equation do basically what we have shown in the text: They choose values of x and, for each, calculate the corresponding value of y. The pixel corresponding to the ordered pair is then turned on. The graph is jagged because pixels are much larger than the dots we draw on paper.



The graph of $y = 0.45x$ is shown at the left as the calculator drew it (jagged). The x- and y-axes have been chosen so that each pixel represents $\frac{1}{10}$ of a unit. Consider the region of the graph where $x = 1$, 1.1, and 1.2.

The corresponding values of y are 0.45, 0.495, and 0.54. Because the y-axis is in tenths, the numbers 0.45, 0.495, and 0.54 are rounded to the nearest tenth before plotting. Rounding 0.45, 0.495, and 0.54 to the nearest tenth results in 0.5 for each number. Thus the ordered pairs (1, 0.45), (1.1, 0.495), and (1.2, 0.54) are graphed as (1, 0.5), (1.1, 0.5), and (1.2, 0.5). These points appear as three illuminated horizontal pixels. The graph of the line appears horizontal. However, if you use the TRACE feature of the calculator (see the appendix), the actual y-coordinate for each value of x is displayed.

TAKE NOTE

Xmin and Xmax are the smallest and largest values of x that will be shown on the screen. Ymin and Ymax are the smallest and largest values of y that will be shown on the screen.

Here are the keystrokes to graph $y = \frac{2}{3}x + 1$. First the equation is entered. Then the domain (Xmin to Xmax) and the range (Ymin to Ymax) are entered. This is called the **viewing window.** By changing the keystrokes 2 [X,T,Θ,n] [÷] 3 [+] 1 (use [X/Θ/T/n] for the Sharp EL-9600 or use [X,Θ,T] for the Casio CFX-9850), you can graph different equations.

TI-83

[Y=] [CLEAR] 2 [X,T,Θ,n] [÷] 3 [+] 1 [WINDOW] [(−)] 10 [ENTER] 10 [ENTER] 1 [ENTER] [(−)] 10 [ENTER] 10 [ENTER] 1 [ENTER] [GRAPH]

SHARP EL-9600

[Y=] [CL] 2 [X/Θ/T/n] [÷] 3 [+] 1 [WINDOW] [(−)] 10 [ENTER] 10 [ENTER] 1 [ENTER] [(−)] 10 [ENTER] 10 [ENTER] 1 [ENTER] [GRAPH]

CASIO CFX-9850

[MENU] 5 [F2] [F1] 2 [X,Θ,T] [÷] 3 [+] 1 [EXE] [SHIFT] F3 [(−)] 10 [EXE] 10 [EXE] 1 [EXE] [(−)] 10 [EXE] 10 [EXE] 1 [EXE] [EXIT] [F6]

1. $y = 2x + 1$ — For $2x$, you may enter $2 \times x$ or just $2x$. The times sign $\times$ is not necessary on many graphing calculators.

2. $y = -\frac{1}{2}x - 2$ — Use the [(−)] key to enter a negative sign.

3. $3x + 2y = 6$ — Solve for y. Then enter the equation.

4. $4x + 3y = 75$ — You must adjust the viewing window.

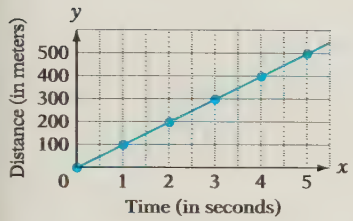
Suggestion: Xmin = −25, Xmax = 25, Xscl = 5, Ymin = −35, Ymax = 35, Yscl = 5. See the appendix for assistance.

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Graphs of Motion

A graph can be useful in analyzing the motion of a body. For example, consider an airplane in uniform motion traveling at 100 m/s. The table at the right shows the distance, in meters, traveled by the plane at the end of each of five one-second intervals.

Time (in seconds)	*Distance (in meters)*
0	0
1	100
2	200
3	300
4	400
5	500



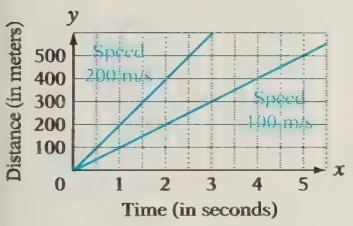
These data can be graphed on a rectangular coordinate system and a straight line drawn through the points plotted. The travel time is shown along the horizontal axis, and the distance traveled by the plane is shown along the vertical axis. (Note that the units along the two axes are not the same length.)

To write the equation for the line just graphed, use the coordinates of any two points on the line to find the slope. The y-intercept is (0, 0).

Let $(x_1, y_1) = (1, 100)$ and $(x_2, y_2) = (2, 200)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{200 - 100}{2 - 1} = 100$$

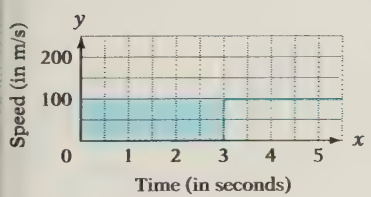
$$y = mx + b$$
$$y = 100x + 0$$
$$y = 100x$$



Note that the slope of the line, 100, is equal to the speed, 100 m/s. *The slope of a distance-time graph represents the speed of the object.*

The distance-time graphs for two planes are shown at the left. One plane is traveling at 100 m/s, and the other is traveling at 200 m/s. The slope of the line representing the faster plane is greater than the slope of the line representing the slower plane.

In the speed-time graph at the left, the time a plane has been flying at 100 m/s is shown along the horizontal axis and its speed is shown along the vertical axis. Because the speed is constant, the graph is a horizontal line.



The area between the horizontal line graphed and the horizontal axis is equal to the distance traveled by the plane up to that time. For example, the area of the shaded region on the graph is

$$\text{Length} \cdot \text{width} = (3\text{ s})(100\text{ m/s}) = 300\text{ m}$$

The distance traveled by the plane in 3 s is equal to 300 m.

1. A car in uniform motion is traveling at 20 m/s.
   - **a.** Prepare a distance-time graph for the car for 0 s to 5 s.
   - **b.** Find the slope of the line.
   - **c.** Find the equation of the line.
   - **d.** Prepare a speed-time graph for the car for 0 s to 5 s.
   - **e.** Find the distance traveled by the car after 3 s.
2. One car in uniform motion is traveling at 10 m/s. A second car in uniform motion is traveling at 15 m/s.
   - **a.** Prepare one distance-time graph for both cars for 0 s to 5 s.
   - **b.** Find the slope of each line.
   - **c.** Find the equation of each line graphed.
   - **d.** Assuming that the cars started at the same point at 0 s, find the distance between the cars at the end of 5 s.
3. **a.** In a distance-time graph, is it possible for the graph to be a horizontal line?
   - **b.** What does a horizontal line reveal about the motion of the object during that time period?

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Chapter Summary

Key Words

A *rectangular coordinate system* is formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line.

The number lines that make up a rectangular coordinate system are called the *coordinate axes* or simply *axes*.

The *origin* is the point of intersection of the two coordinate axes.

A rectangular coordinate system divides the plane into four regions called *quadrants*.

An *ordered pair* (x, y) is used to locate a point in a plane.

The first number in an ordered pair is called the *abscissa* or x-coordinate.

The second number in an ordered pair is called the *ordinate* or y-coordinate.

The *coordinates* of a point are the numbers in the ordered pair that is associated with the point.

A *relation* is any set of ordered pairs.

A *function* is a relation in which no two ordered pairs with the same first coordinates have different second coordinates.

A function designated by $f(x)$ is written in *functional notation*. The *value of the function* at x is $f(x)$.

An equation of the form $y = mx + b$, where m and b are constants, is a *linear equation in two variables* or a *linear function*.

The point at which a graph crosses the x-axis is called the *x-intercept*.

The point at which a graph crosses the y-axis is called the *y-intercept*.

The *slope* of a line is the measure of the slant of a line. The symbol for slope is m.

A line that slants upward to the right has a *positive slope*.

A line that slants downward to the right has a *negative slope*.

A horizontal line has *zero slope*.

The slope of a vertical line is *undefined*.

Essential Rules

Slope of a straight line

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept form of a straight line

$$y = mx + b$$

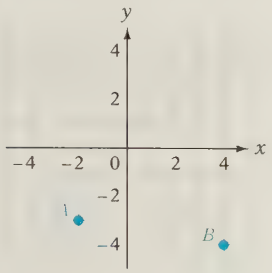
Point-slope formula

$$y - y_1 = m(x - x_1)$$

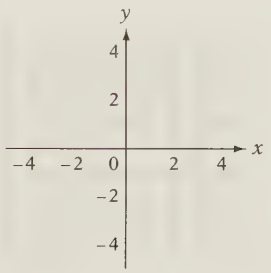
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Chapter Review

1. **a.** Graph the ordered pairs $(-2, 4)$ and $(3, -2)$.
b. Name the abscissa of point A.
c. Name the ordinate of point B.



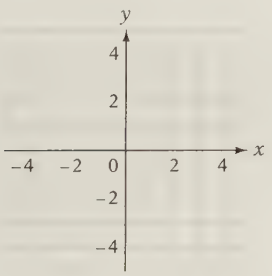
2. Graph the ordered-pair solutions of $y = -\frac{1}{2}x - 2$ when $x \in \{-4, -2, 0, 2\}$.



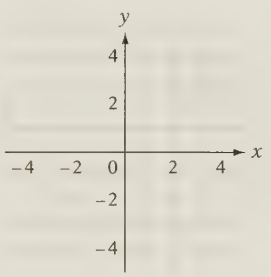
3. Determine the equation of the line that passes through the points $(-1, 3)$ and $(2, -5)$.

4. Determine the equation of the line that passes through the point $(6, 1)$ and has slope $-\frac{5}{2}$.

5. Graph $y = \frac{1}{4}x + 3$.



6. Graph $5x + 3y = 15$.



7. Determine the equation of the line that passes through the point $(-3, 4)$ and has slope $\frac{2}{3}$.

8. Given $f(x) = x^2 - 2$, find $f(-1)$.

9. Determine the equation of the line that passes through the points $(-2, 5)$ and $(4, 1)$.

10. Does $y = -x + 3$, where $x \in \{-2, 0, 3, 5\}$, define y as a function of x?

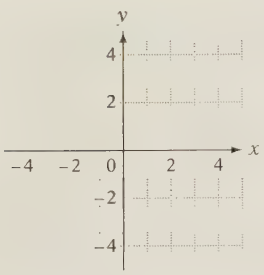
11. Find the slope of the line containing the points $(9, 8)$ and $(-2, 1)$.

12. Find the x- and y-intercepts of $3x - 2y = 24$.

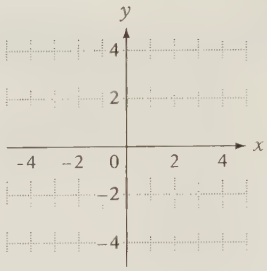
13. Find the slope of the line containing the points $(-2, -3)$ and $(4, -3)$.

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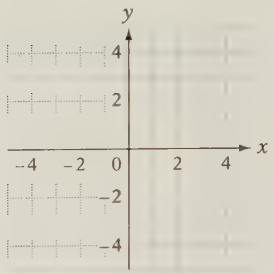
14. Graph the line that has slope $\frac{1}{2}$ and y-intercept $(0, -1)$.



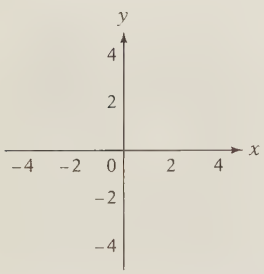
15. Graph $x = -3$.



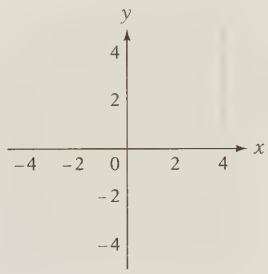
16. Graph the line that has slope $-\frac{2}{3}$ and y-intercept $(0, 2)$.



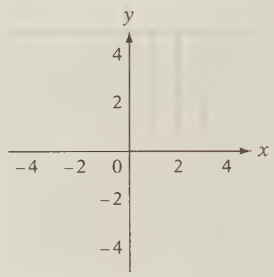
17. Graph $y = -2x - 1$.



18. Graph the line that has slope 2 and y-intercept $(0, -4)$.



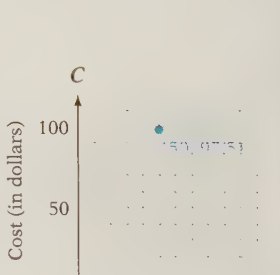
19. Graph $3x - 2y = -6$.



20. The height and weight of 8 seventh-grade students are shown in the following table. Write a relation where the first coordinate is height, in inches, and the second coordinate is weight, in pounds. Is the relation a function?

Height	55	57	53	57	60	61	58	54
Weight	95	101	94	98	100	105	97	95

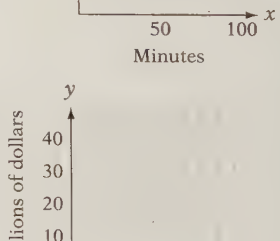
21. An on-line research service charges a monthly access fee of \$75 plus \$.45 per minute to use the service. An equation that represents the monthly cost to use this service is $C = 0.45x + 75$, where C is the monthly cost and x is the number of minutes. Graph this equation for $0 \le x \le 100$. The point $(50, 97.5)$ is on the graph. Write a sentence that describes the meaning of this ordered pair.



22. The data in the table below show the amount spent on health care in the United States. The line of best fit for this data is $y = 3.85x + 15.78$, where $x = 0$ corresponds to the year 1990. Graph the data and the line of best fit in the coordinate system at the right. How much is the amount spent on health care increasing each year?

Year	1990	1992	1994	1996	1998
Amount spent (billions)	14.9	22.7	30.9	36.0	42.1*

*Projected

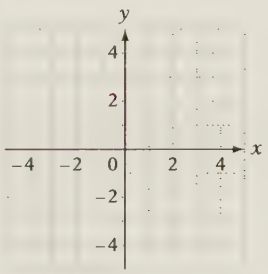


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Chapter Test

1. Find the ordered-pair solution of $2x - 3y = 15$ corresponding to $x = 3$.

2. Graph the ordered-pair solutions of $y = -\frac{3}{2}x + 1$ for $x \in \{-2, 0, 4\}$.



3. Does $y = \frac{1}{2}x - 3$ define y as a function of x for $x \in \{-2, 0, 4\}$?

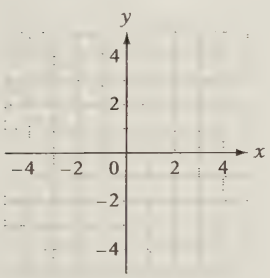
4. Given $f(t) = t^2 + t$, find $f(2)$.

5. Given $f(x) = x^2 - 2x$, find $f(-1)$.

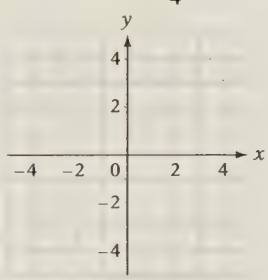
6. The distance a house is from a fire station and the amount of damage that the house sustained in a fire are given in the following table. Write a relation where the first coordinate of the ordered pair is the distance in miles from the fire station and the second coordinate is the amount of damage in thousands of dollars. Is the relation a function?

Distance	3.5	4.0	5.2	5.0	4.0	6.3	5.4
Damage	25	30	45	38	42	12	34

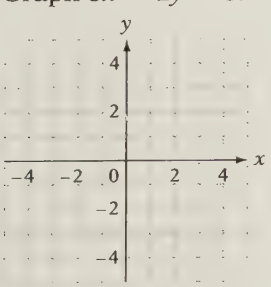
7. Graph $y = 3x + 1$.



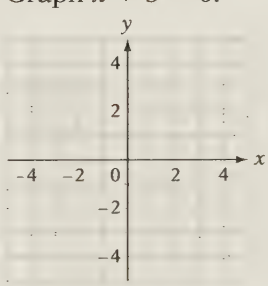
8. Graph $y = -\frac{3}{4}x + 3$.



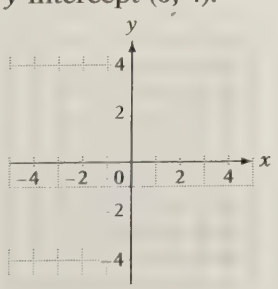
9. Graph $3x - 2y = 6$.



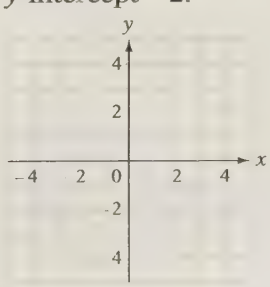
10. Graph $x + 3 = 0$.



11. Graph the line that has slope $-\frac{2}{3}$ and y-intercept $(0, 4)$.



12. Graph the line that has slope 2 and y-intercept -2.



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13. The equation for the speed of a ball that is thrown straight up with an initial speed of 128 ft/s is $v = 128 - 32t$, where v is the speed of the ball after t seconds. Graph this equation for $0 \le t \le 4$. The point whose coordinates are (1, 96) is on the graph in Figure 1. Write a sentence that describes this ordered pair.

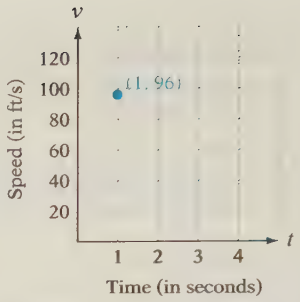


Figure 1

14. The graph in Figure 2 shows the increase in the cost of tuition for a college for the years 1989 through 1994 (with 1989 as 0). Find the slope of the line. Write a sentence that states the meaning of the slope.

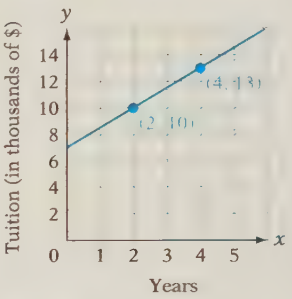


Figure 2

15. The data in the following table show the number of mutual fund companies for selected years between 1983 and 1993. The line of best fit is $y = 330x + 1016$, where x is the year (with 1983 as 0) and y is the number of mutual fund companies.

Year, x	0 (1983)	3 (1986)	5 (1988)	8 (1991)	10 (1993)
No. of companies, y	1020	2000	2670	3660	4320

Graph the data and the line of best fit in the coordinate system in Figure 3. Write a sentence that describes the meaning of the slope of the line of best fit.

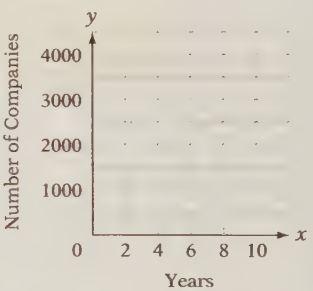


Figure 3

16. Find the x- and y-intercepts for $6x - 4y = 12$.

17. Find the x- and y-intercepts for $y = \frac{1}{2}x + 1$.

18. Find the slope of the line containing the points (2, −3) and (4, 1).

19. Find the slope of the line containing the points (3, −4) and (1, −4).

20. Find the slope of the line containing the points (−5, 2) and (−5, 7).

21. Find the slope of the line whose equation is $2x + 3y = 6$.

22. Find the equation of the line that contains the point (0, −1) and has slope 3.

23. Find the equation of the line that contains the point (−3, 1) and has slope $\frac{2}{3}$.

24. Find the equation of the line that passes through the points (5, −4) and (−3, 1).

25. Find the equation of the line that passes through the points (−2, 0) and (5, −2).

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Cumulative Review

1. Simplify: $12 - 18 \div 3 \cdot (-2)^2$

2. Evaluate $\frac{a-b}{a^2-c}$ when $a = -2$, $b = 3$, and $c = -4$.

3. Given $f(x) = \frac{2}{x-1}$, find $f(-2)$.

4. Solve: $2x - \frac{2}{3} = \frac{7}{3}$

5. Solve: $3x - 2[x - 3(2 - 3x)] = x - 7$

6. Write $6\frac{2}{3}\%$ as a fraction.

7. Simplify: $(-2x^2y)^3(2xy^2)^2$

8. Simplify: $\frac{-15x^7}{5x^5}$

9. Divide: $(x^2 - 4x - 21) \div (x - 7)$

10. Factor: $5x^2 + 15x + 10$

11. Factor: $x(a + 2) + y(a + 2)$

12. Solve: $x(x - 2) = 8$

13. Simplify: $\frac{x^5y^3}{x^2 - x - 6} \cdot \frac{x^2 - 9}{x^2y^4}$

14. Simplify: $\frac{3x}{x^2 + 5x - 24} - \frac{9}{x^2 + 5x - 24}$

15. Solve: $3 - \frac{1}{x} = \frac{5}{x}$

16. Solve $4x - 5y = 15$ for y.

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17. Find the ordered-pair solution of $y = 2x - 1$ corresponding to $x = -2$.

18. Find the slope of the line that contains the points $(2, 3)$ and $(-2, 3)$.

19. Find the equation of the line that contains the point $(2, -1)$ and has slope $\frac{1}{2}$.

20. Find the equation of the line that contains the point $(0, 2)$ and has slope -3.

21. Find the equation of the line that contains the point $(-1, 0)$ and has slope 2.

22. Find the equation of the line that contains the point $(6, 1)$ and has slope $\frac{2}{3}$.

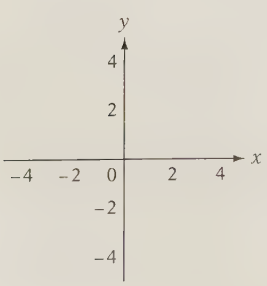
23. A suit that regularly sells for \$89 is on sale for 30% off the regular price. Find the sale price.

24. The first angle of a triangle is 3° more than the measure of the second angle. The third angle is 5° more than twice the measure of the second angle. Find the measure of each angle.

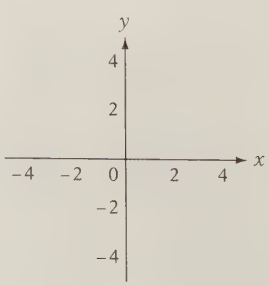
25. The real estate tax for a home that costs \$50,000 is \$625. At this rate, what is the value of a home for which the real estate tax is \$1375?

26. An electrician requires 6 h to wire a garage. An apprentice can do the same job in 10 h. How long would it take to wire the garage if both the electrician and the apprentice were working?

27. Graph $y = \frac{1}{2}x - 1$.



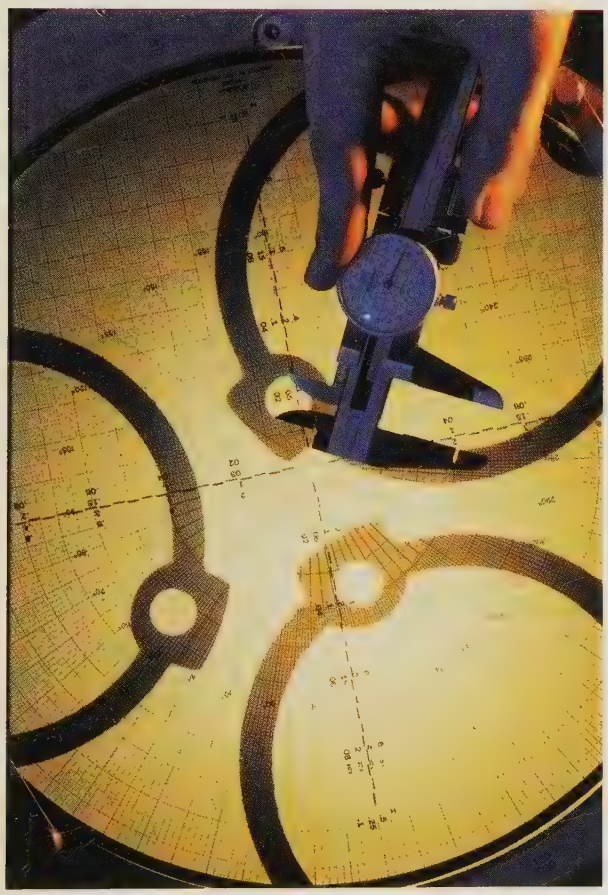
28. Graph the line that has slope $-\frac{2}{3}$ and y-intercept 2.



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CHAPTER

8 Systems of Linear Equations



A mechanical engineer designs engines and motors. It is part of the mechanical engineer's job to determine the size and the layout of pipes that are needed to transport water or air or any other material from one place to another within a motor. In these designs, the temperatures and pressures of the materials being transported must be taken into consideration. Therefore, calculations performed by a mechanical engineer normally require equations that have more than one variable.

Objectives

Section 8.1

To solve a system of linear equations by graphing

Section 8.2

To solve a system of linear equations by the substitution method

To solve investment problems

Section 8.3

To solve a system of linear equations by the addition method

Section 8.4

To solve rate-of-wind or -current problems

To solve application problems using two variables

Input-Output Analysis



The economies of the industrial nations are very complex; they comprise hundreds of different industries, and each industry supplies other industries with goods and services needed in the production process. For example, the steel industry requires coal to produce steel, and the coal industry requires steel (in the form of machinery) to mine and transport coal.

Wassily Leontief, a Russian-born economist, developed a method of describing mathematically the interactions of an economic system. His technique was to examine various sectors of an economy (steel industry, oil, farms, autos, and so on) and determine how each sector interacted with the others. More than five hundred sectors of the economy were studied.

The interaction of each sector with the others was written as a series of equations. This series of equations is called a *system of equations*. Using a computer, economists searched for a solution to the system of equations that would determine the output levels various sectors would have to meet to satisfy the requests from other sectors. The method is called input-output analysis.

Input-output analysis has many applications. For example, it is used today to predict the production needs of large corporations and to determine the effect of price changes on the economy. In recognition of the importance of his ideas, Leontief was awarded the Nobel Prize in Economics in 1973.

This chapter begins the study of systems of equations.

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8.1 Solving Systems of Linear Equations by Graphing

Objective A To solve a system of linear equations by graphing

Equations considered together are called a **system of equations.** A system of equations is shown at the right.

$$\begin{aligned} 2x + 3y &= 2 \\ 3x - 5y &= 22 \end{aligned}$$

A **solution of a system of equations in two variables** is an ordered pair that is a solution of each equation of the system.

Is $(4, -2)$ a solution of the following system of equations?

$$\begin{aligned} 2x + 3y &= 2 \\ 3x - 5y &= 22 \end{aligned}$$

$2x + 3y = 2$	
$2(4) + 3(-2)$	2
$8 + (-6)$	2
$2 = 2$	

$3x - 5y = 22$	
$3(4) - 5(-2)$	22
$12 - (-10)$	22
$22 = 22$	

Yes, because $(4, -2)$ is a solution of each equation, it is the solution of the system. However, $(7, -4)$ is not a solution, because

$2x + 3y = 2$	
$2(7) + 3(-4)$	2
$14 + (-12)$	2
$2 = 2$	

$(7, -4)$ is a solution.

$3x - 5y = 22$	
$3(7) - 5(-4)$	22
$21 - (-20)$	22
$41 \neq 22$	

$(7, -4)$ is not a solution.

Is $(3, -3)$ a solution of the following system of equations?

$$\begin{aligned} 2x + y &= 3 \\ x + y &= 1 \end{aligned}$$

$2x + y = 3$	
$2(3) + (-3)$	3
$6 + (-3)$	3
$3 = 3$	

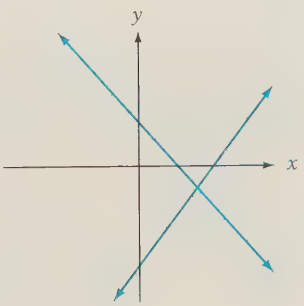
$x + y = 1$	
$3 + (-3)$	1
$0 \neq 1$	

Because $(3, -3)$ is not a solution of each equation, $(3, -3)$ is not a solution of the system of equations.

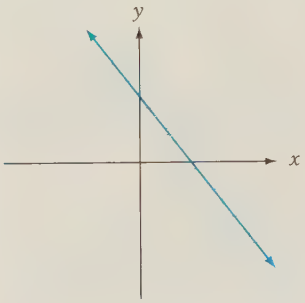
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Graphing the equations in a system of linear equations is one method of finding a solution of the system of equations. The lines can intersect at one point, the lines can intersect at infinitely many points (the graphs are the same line), or the lines can be parallel and not intersect at all.

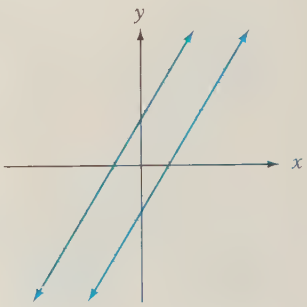
Such systems are called **independent, dependent, and inconsistent,** respectively.



Independent:
one solution



Dependent:
infinitely many solutions



Inconsistent:
no solutions

POINT OF INTEREST

The "Projects and Group Activities" at the end of this chapter discusses using a calculator to approximate the solution of a system of equations.

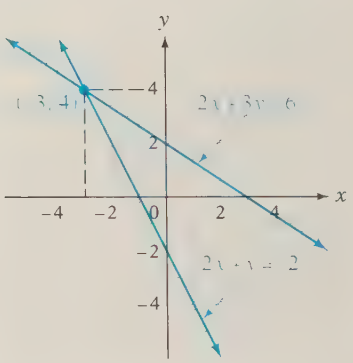
➡ Solve by graphing: $2x + 3y = 6$
$2x + y = -2$

Graph each line.

Find the point of intersection.

$(-3, 4)$ is a solution of each equation. The system of equations is *independent*.

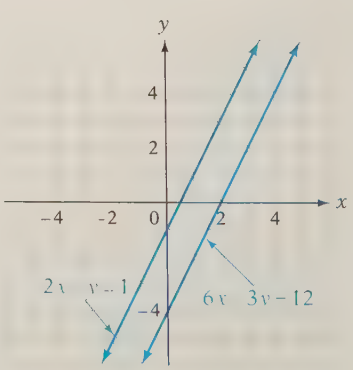
The solution is $(-3, 4)$.



➡ Solve by graphing: $2x - y = 1$
$6x - 3y = 12$

Graph each line.

The lines are parallel and therefore do not intersect. The system of equations is *inconsistent* and has no solution.

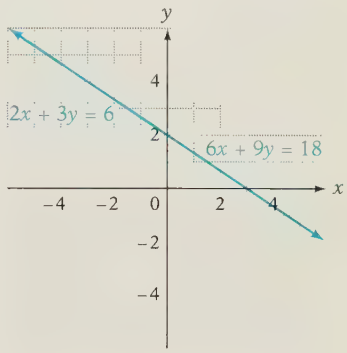


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When a system of equations is *dependent*, the graphs of the two equations are the same line. Therefore, the lines intersect at infinitely many points. The solutions of the system of equations are the ordered pairs that satisfy either one (and hence both) of the two equations of the system of equations.

➡ Solve by graphing: $2x + 3y = 6$
$6x + 9y = 18$

Graph each line.



The two equations represent the same line. The system of equations is dependent, so there are an infinite number of solutions. The solutions are the ordered pairs that satisfy both equations. The solutions are stated by using the ordered pairs of one of the equations. Therefore, the solutions are the ordered pairs that satisfy the equation $2x + 3y = 6$.

By choosing values of x, we can find some specific ordered-pair solutions. For example, when $x = -3$, 0, and 6, three of the infinite solutions of the system of equations are $(-3, 4)$, $(0, 2)$, and $(6, -2)$.

Example 1

Is $(1, -3)$ a solution of the following system?

$3x + 2y = -3$
$x - 3y = 6$

Solution

$3x + 2y = -3$	
$3 \cdot 1 + 2(-3)$	-3
$3 + (-6)$	-3
$-3 = -3$	

$x - 3y = 6$	
$1 - 3(-3)$	6
$1 - (-9)$	6
$10 \neq 6$	

No, $(1, -3)$ is not a solution of the system of equations.

You Try It 1

Is $(-1, -2)$ a solution of the following system?

$2x - 5y = 8$
$-x + 3y = -5$

Your solution

Solution on p. S19

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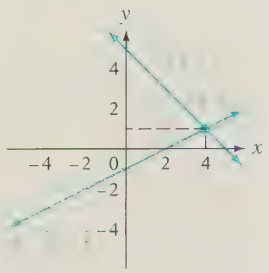
Example 2

Solve by graphing:

$x - 2y = 2$

$x + y = 5$

Solution



The solution is (4, 1).

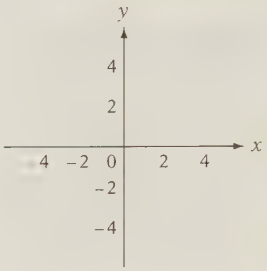
You Try It 2

Solve by graphing:

$x + 3y = 3$

$-x + y = 5$

Your solution



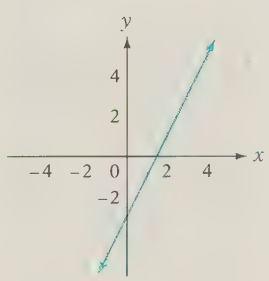
Example 3

Solve by graphing:

$4x - 2y = 6$

$y = 2x - 3$

Solution



The solutions are the ordered pairs that satisfy the equation $y = 2x - 3$.

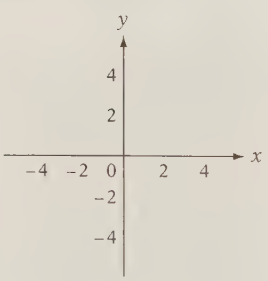
You Try It 3

Solve by graphing:

$y = 3x - 1$

$6x - 2y = -6$

Your solution



Solutions on p. S19

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8.1 Exercises

Objective A

1. Is (2, 3) a solution of this system?
$$3x + 4y = 18$$
$$2x - y = 1$$

2. Is (2, −1) a solution of this system?
$$x - 2y = 4$$
$$2x + y = 3$$

3. Is (1, −2) a solution of this system?
$$3x - y = 5$$
$$2x + 5y = -8$$

4. Is (−1, −1) a solution of this system?
$$x - 4y = 3$$
$$3x + y = 2$$

5. Is (4, 3) a solution of this system?
$$5x - 2y = 14$$
$$x + y = 8$$

6. Is (2, 5) a solution of this system?
$$3x + 2y = 16$$
$$2x - 3y = 4$$

7. Is (−1, 3) a solution of this system?
$$4x - y = -5$$
$$2x + 5y = 13$$

8. Is (4, −1) a solution of this system?
$$x - 4y = 9$$
$$2x - 3y = 11$$

9. Is (0, 0) a solution of this system?
$$4x + 3y = 0$$
$$2x - y = 1$$

10. Is (2, 0) a solution of this system?
$$3x - y = 6$$
$$x + 3y = 2$$

11. Is (2, −3) a solution of this system?
$$y = 2x - 7$$
$$3x - y = 9$$

12. Is (−1, −2) a solution of this system?
$$3x - 4y = 5$$
$$y = x - 1$$

13. Is (5, 2) a solution of this system?
$$y = 2x - 8$$
$$y = 3x - 13$$

14. Is (−4, 3) a solution of this system?
$$y = 2x + 11$$
$$y = 5x - 19$$

15. Is (−2, −3) a solution of this system?
$$3x - 4y = 6$$
$$2x - 7y = 17$$

16. Is (0, 0) a solution of this system?
$$y = 2x$$
$$3x + 5y = 0$$

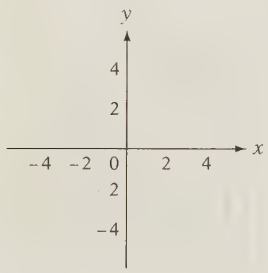
17. Is (0, −3) a solution of this system?
$$4x - 3y = 9$$
$$2x + 5y = 15$$

18. Is (4, 0) a solution of this system?
$$2x + 3y = 8$$
$$x - 5y = 4$$

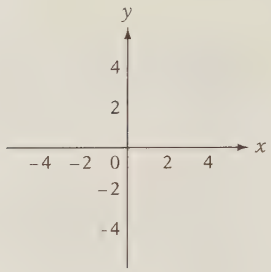
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Solve by graphing.

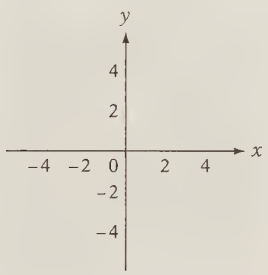
19. $x - y = 3$
$x + y = 5$



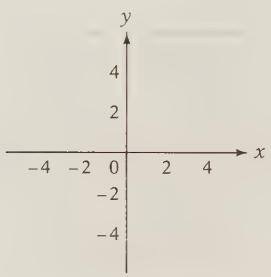
20. $2x - y = 4$
$x + y = 5$



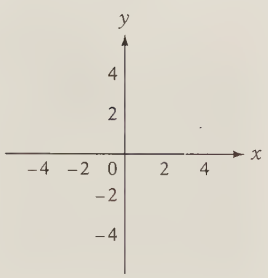
21. $x + 2y = 6$
$x - y = 3$



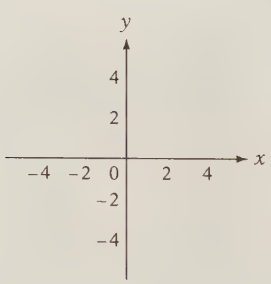
22. $3x - y = 3$
$2x + y = 2$



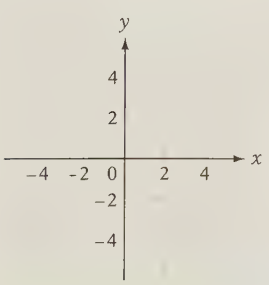
23. $3x - 2y = 6$
$y = 3$



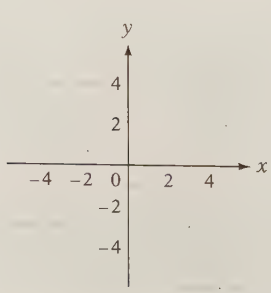
24. $x = 2$
$3x + 2y = 4$



25. $x = 3$
$y = -2$

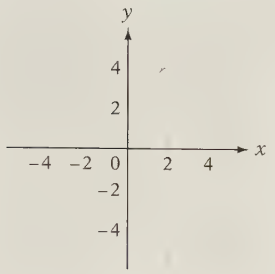


26. $x + 1 = 0$
$y - 3 = 0$

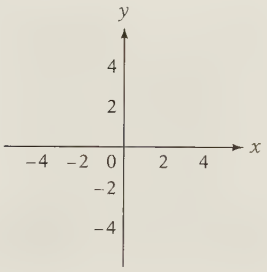


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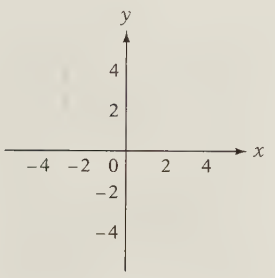
27. $y = 2x - 6$
$x + y = 0$



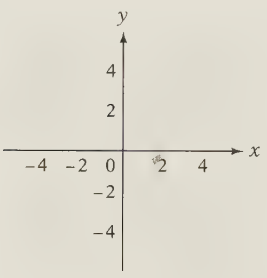
28. $5x - 2y = 11$
$y = 2x - 5$



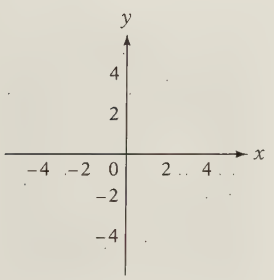
29. $2x + y = -2$
$6x + 3y = 6$



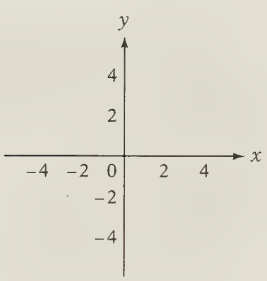
30. $x + y = 5$
$3x + 3y = 6$



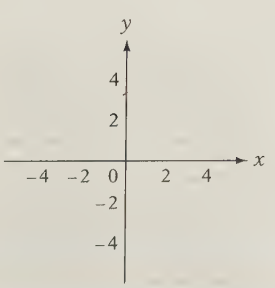
31. $y = 2x - 2$
$4x - 2y = 4$



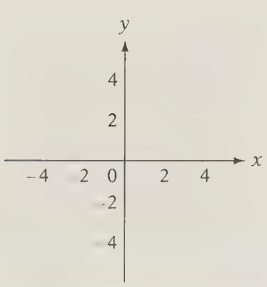
32. $y = -\frac{1}{3}x + 1$
$2x + 6y = 6$



33. $x - y = 5$
$2x - y = 6$

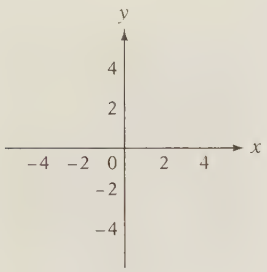


34. $5x - 2y = 10$
$3x + 2y = 6$

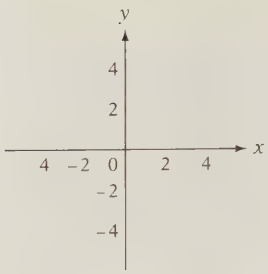


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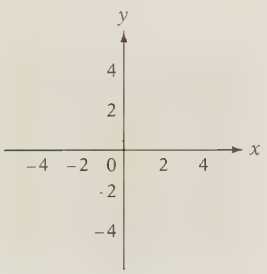
35. $3x + 4y = 0$
$2x - 5y = 0$



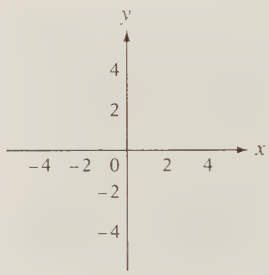
36. $2x - 3y = 0$
$y = -\frac{1}{3}x$



37. $x - 3y = 3$
$2x - 6y = 12$



38. $4x + 6y = 12$
$6x + 9y = 18$



APPLYING THE CONCEPTS

39. Determine whether the statement is always true, sometimes true, or never true.
- **a.** A solution of a system of two equations with two variables is a point in the plane.
- **b.** Two parallel lines have the same slope.
- **c.** Two different lines with the same y-intercept are parallel.
- **d.** Two different lines with the same slope are parallel.

40. Explain how you can determine from the graph of a system of two equations in two variables whether it is an independent system of equations.

41. Explain how you can determine from the graph of a system of two equations in two variables whether it is an inconsistent system of equations.

42. Write a system of equations that has $(-2, 4)$ as its only solution.

43. Write a system of equations for which there is no solution.

44. Write a system of equations that is a dependent system of equations.

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8.2 Solving Systems of Linear Equations by the Substitution Method

Objective A To solve a system of linear equations by the substitution method

28 CT

A graphical solution of a system of equations is based on approximating the coordinates of a point of intersection. An algebraic method called the **substitution method** can be used to find an exact solution of a system of equations.

➡ Solve by the substitution method: (1) $2x + 5y = -11$
(2) $y = 3x - 9$

Equation (2) states that $y = 3x - 9$. Substitute $3x - 9$ for y in Equation (1). Then solve for x.

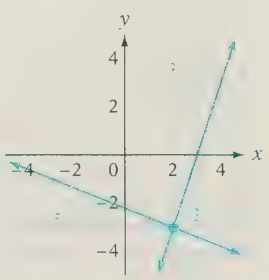
$2x + 5y = -11$ • This is Equation (1).
$2x + 5(3x - 9) = -11$ • From Equation (2), substitute $3x - 9$ for y.
$2x + 15x - 45 = -11$ • Solve for x.
$17x - 45 = -11$
$17x = 34$
$x = 2$

Now substitute the value of x into Equation (2) and solve for y.

$y = 3x - 9$ • This is Equation (2).
$y = 3(2) - 9$ • Substitute 2 for x.
$y = 6 - 9 = -3$

The solution is the ordered pair $(2, -3)$.

The graph of the equations in this system of equations is shown at the right. Note that the lines intersect at the point whose coordinates are $(2, -3)$, which is the algebraic solution we determined by the substitution method.



To solve a system of equations by the substitution method, we may need to solve one of the equations in the system of equations for one of its variables. For instance, the first step in solving the system of equations

(1) $x + 2y = -3$
(2) $2x - 3y = 5$

is to solve an equation of the system for one of its variables. Either equation can be used.

Solving Equation (1) for x:

$x + 2y = -3$
$x = -2y - 3$

Solving Equation (2) for x:

$2x - 3y = 5$
$2x = 3y + 5$
$x = \frac{3y + 5}{2}$

Because solving Equation (1) for x does not result in fractions, it is the easier of the two equations to use.

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Here is the solution of the system of equations given on the previous page.

➡ Solve by the substitution method: (1) $x + 2y = -3$
(2) $2x - 3y = 5$

To use the substitution method, we must solve an equation for one of its variables. Equation (1) is used here because solving it for x does not result in fractions.

$$x + 2y = -3$$

(3) $$x = -2y - 3$$ • Solve for x. This is Equation (3).

Now substitute $-2y - 3$ for x in Equation (2) and solve for y.

$$2x - 3y = 5$$ • This is Equation (2).

$$2(-2y - 3) - 3y = 5$$ • From Equation (3), substitute $-2y - 3$ for x.

$$-4y - 6 - 3y = 5$$ • Solve for y.

$$-7y - 6 = 5$$

$$-7y = 11$$

$$y = \frac{11}{7}$$

Substitute the value of y into Equation (3) and solve for x.

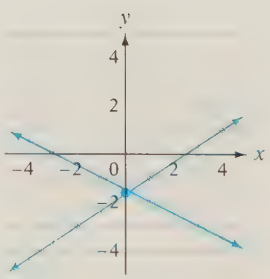
$x = -2y - 3$ • This is Equation (3).

$= -2\left(-\frac{11}{7}\right) - 3$ • Substitute $-\frac{11}{7}$ for y.

$= \frac{22}{7} - 3 = \frac{22}{7} - \frac{21}{7} = \frac{1}{7}$

The solution is $\left(\frac{1}{7}, -\frac{11}{7}\right)$.

The graph of the system of equations given above is shown at the right. It would be difficult to determine the exact solution of this system of equations from the graphs of the equations.



➡ Solve by the substitution method: (1) $y = 3x - 1$
(2) $y = -2x - 6$

$$y = -2x - 6$$

$$3x - 1 = -2x - 6$$ • Substitute $3x - 1$ for y in Equation (2).

$$5x = -5$$ • Solve for x.

$$x = -1$$

Substitute this value of x into Equation (1) or Equation (2) and solve for y. Equation (1) is used here.

$$y = 3x - 1$$

$$y = 3(-1) - 1 = -4$$

The solution is $(-1, -4)$.

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The substitution method can be used to analyze inconsistent and dependent systems of equations.

➡ Solve by the substitution method: (1) $2x + 3y = 3$

(2) $y = -\frac{2}{3}x + 3$

$$2x + 3y = 3$$ • This is Equation (1).

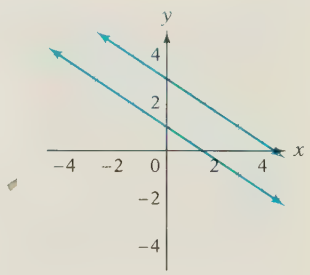
$$2x + 3\left(-\frac{2}{3}x + 3\right) = 3$$ • From Equation (2), replace y with $-\frac{2}{3}x + 3$.

$$2x - 2x + 9 = 3$$ • Solve for x.

$$9 = 3$$ • This is not a true equation.

Because $9 = 3$ is not a true equation, the system of equations has no solution.

Solving Equation (1) above for y, we have $y = -\frac{2}{3}x + 1$. Comparing this with Equation (2) reveals that the slopes are equal and the y-intercepts are different. The graphs of the equations that make up this system of equations are parallel and thus never intersect. Because the graphs do not intersect, there are no solutions of the system of equations. The system of equations is inconsistent.



➡ Solve by the substitution method: (1) $x = 2y + 3$

(2) $4x - 8y = 12$

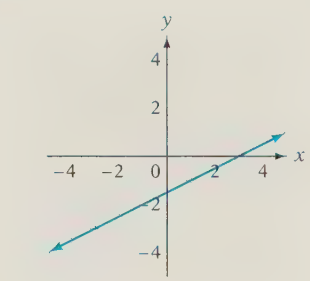
$$4x - 8y = 12$$ • This is Equation (2).

$$4(2y + 3) - 8y = 12$$ • From Equation (1), replace x by $2y + 3$.

$$8y + 12 - 8y = 12$$ • Solve for y.

$$12 = 12$$ • This is a true equation.

The true equation $12 = 12$ indicates that any ordered pair (x, y) that satisfies one equation of the system satisfies the other equation. Therefore, the system of equations has an infinite number of solutions. The solutions are the ordered pairs (x, y) that are solutions of $x = 2y + 3$.



If we write Equation (1) and Equation (2) in slope-intercept form, we have

$$x = 2y + 3 \qquad 4x - 8y = 12$$
$$-2y = -x + 3 \qquad -8y = -4x + 12$$
$$y = \frac{1}{2}x - \frac{3}{2} \qquad y = \frac{1}{2}x - \frac{3}{2}$$

The slope-intercept forms of the equations are the same, and therefore the graphs are the same. If we graph these two equations, we essentially graph one over the other. Accordingly, the graphs intersect at an infinite number of points.

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Example 1 Solve by substitution:

$$(1)\quad 3x + 4y = -2$$
$$(2)\quad -x + 2y = 4$$

Solution Solve Equation (2) for x.

$$-x + 2y = 4$$
$$-x = -2y + 4$$
$$x = 2y - 4$$

Substitute in Equation (1).

$$(1)\quad 3x + 4y = -2$$
$$3(2y - 4) + 4y = -2$$
$$6y - 12 + 4y = -2$$
$$10y - 12 = -2$$
$$10y = 10$$
$$y = 1$$

Substitute in Equation (2).

$$(2)\quad -x + 2y = 4$$
$$-x + 2(1) = 4$$
$$-x + 2 = 4$$
$$-x = 2$$
$$x = -2$$

The solution is $(-2, 1)$.

You Try It 1 Solve by substitution:

$$(1)\quad 7x - y = 4$$
$$(2)\quad 3x + 2y = 9$$

Your solution

Example 2 Solve by substitution:

$$4x + 2y = 5$$
$$y = -2x + 1$$

Solution

$$4x + 2y = 5$$
$$4x + 2(-2x + 1) = 5$$
$$4x - 4x + 2 = 5$$
$$2 = 5$$

This is not a true equation. The system of equations is inconsistent and therefore does not have a solution.

You Try It 2 Solve by substitution:

$$3x - y = 4$$
$$y = 3x + 2$$

Your solution

Example 3 Solve by substitution:

$$y = 3x - 2$$
$$6x - 2y = 4$$

Solution

$$6x - 2y = 4$$
$$6x - 2(3x - 2) = 4$$
$$6x - 6x + 4 = 4$$
$$4 = 4$$

This is a true equation. The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $y = 3x - 2$.

You Try It 3 Solve by substitution:

$$y = -2x + 1$$
$$6x + 3y = 3$$

Your solution

Solutions on p. S19

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Objective B To solve investment problems

The annual simple interest that an investment earns is given by the equation $Pr = I$, where P is the principal, or the amount invested, r is the simple interest rate, and I is the simple interest.

For instance, if you invest \$750 at a simple interest rate of 6%, then the interest earned after one year is calculated as follows.

$$\begin{aligned} Pr &= I \\ 750(0.06) &= I \quad \text{• Replace } P \text{ by 750 and } r \text{ by 0.06 (6\%).} \\ 45 &= I \quad \text{• Simplify.} \end{aligned}$$

The amount of interest earned is \$45.

➡ A medical lab technician decided to open an Individual Retirement Account (IRA) by placing \$2000 in two simple-interest accounts. On one account, a corporate bond fund, the annual simple interest rate is 7.5%. On the second account, a real estate investment trust, the annual simple interest rate is 9%. If the technician's annual earnings were \$168 per year from these two investments, how much was invested in each account?

Strategy for Solving Simple-Interest Investment Problems

1. For each amount invested, use the equation $Pr = I$. Write a numerical or variable expression for the principal, the interest rate, and the interest earned.

Amount invested at 7.5%: x
Amount invested at 9%: y

	Principal, P	·	*Interest rate, r*	=	*Interest earned, I*
Amount at 7.5%	x	·	0.075	=	$0.075x$
Amount at 9%	y	·	0.09	=	$0.09y$

2. Write a system of equations. One equation will express the relationship among the amounts invested. The second equation will express the relationship among the interest earned by each investment.

The total amount invested is \$2000: $x + y = 2000$
The total annual interest earned is \$168: $0.075x + 0.09y = 168$

Solve the system of equations.

(1) $x + y = 2000$
(2) $0.075x + 0.09y = 168$

Solve Equation (1) for y and substitute into Equation (2).

(3) $y = -x + 2000$

$$\begin{aligned} 0.075x + 0.09(-x + 2000) &= 168 \\ 0.075x - 0.09x + 180 &= 168 \\ -0.015x &= -12 \\ x &= 800 \end{aligned}$$

Substitute the value of x into Equation (3) and solve for y.

$y = -x + 2000$
$y = -800 + 2000 = 1200$

The amount invested at 7.5% is \$800. The amount invested at 9% is \$1200.

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Example 4

A hair stylist invested $2000 at an annual simple interest rate of 5.2%. How much additional money must be invested at an annual simple interest rate of 7.2% so that the total interest earned is 6.4% of the total investment?

Strategy

Amount invested at 5.2%: 2000
Amount invested at 7.2%: x
Total amount invested: y

	Principal	*Rate*	*Interest*
Amount at 5.2%	2000	0.052	0.052(2000)
Amount at 7.2%	x	0.072	$0.072x$
Amount at 6.4%	y	0.064	$0.064y$

The total amount invested (y) is $2000 more than the amount invested at 7.2% (x):

$y = x + 2000$

The sum of the interest earned at 5.2% and the interest earned at 7.2% equals the interest earned at 6.4%:

$0.052(2000) + 0.072x = 0.064y$

Solution

(1) $y = x + 2000$

(2) $0.052(2000) + 0.072x = 0.064y$

Replace y in Equation (2) by $x + 2000$ from Equation (1). Then solve for x.

$$0.052(2000) + 0.072x = 0.064(x + 2000)$$
$$104 + 0.072x = 0.064x + 128$$
$$0.008x = 24$$
$$x = 3000$$

$3000 must be invested at an annual simple interest rate of 7.2%.

You Try It 4

The manager of a city's investment income wished to place $330,000 in two simple-interest accounts. The first account earns 6.5% annual interest and the second account earns 4.5%. How much should be invested in each account so that both accounts earn the same annual interest?

Your strategy

Your solution

Solution on p. S19

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8.2 Exercises

Objective A

Solve by substitution.

1. $2x + 3y = 7$
$x = 2$

2. $y = 3$
$3x - 2y = 6$

3. $y = x - 3$
$x + y = 5$

4. $y = x + 2$
$x + y = 6$

5. $x = y - 2$
$x + 3y = 2$

6. $x = y + 1$
$x + 2y = 7$

7. $y = 4 - 3x$
$3x + y = 5$

8. $y = 2 - 3x$
$6x + 2y = 7$

9. $x = 3y + 3$
$2x - 6y = 12$

10. $x = 2 - y$
$3x + 3y = 6$

11. $3x + 5y = -6$
$x = 5y + 3$

12. $y = 2x + 3$
$4x - 3y = 1$

13. $3x + y = 4$
$4x - 3y = 1$

14. $x - 4y = 9$
$2x - 3y = 11$

15. $3x - y = 6$
$x + 3y = 2$

16. $4x - y = -5$
$2x + 5y = 13$

17. $3x - y = 5$
$2x + 5y = -8$

18. $3x + 4y = 18$
$2x - y = 1$

19. $4x + 3y = 0$
$2x - y = 0$

20. $5x + 2y = 0$
$x - 3y = 0$

21. $2x - y = 2$
$6x - 3y = 6$

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22. $3x + y = 4$
$9x + 3y = 12$

23. $x = 3y + 2$
$y = 2x + 6$

24. $x = 4 - 2y$
$y = 2x - 13$

25. $y = 2x + 11$
$y = 5x - 19$

26. $y = 2x - 8$
$y = 3x - 13$

27. $y = -4x + 2$
$y = -3x - 1$

28. $x = 3y + 7$
$x = 2y - 1$

29. $x = 4y - 2$
$x = 6y + 8$

30. $x = 3 - 2y$
$x = 5y - 10$

Objective B Application Problems

31. An investment of \$3500 is divided between two simple-interest accounts. On one account, the annual simple interest rate is 5%, and on the second account the annual simple interest rate is 7.5%. How much should be invested in each account so that the total interest from the two accounts is \$215?

32. A mortgage broker purchased two trust deeds for a total of \$250,000. One trust deed earns 7% simple annual interest whereas the second one earns 8% simple annual interest. If the total annual interest from the two trust deeds is \$18,500, what was the purchase price of each trust deed?

33. When Sara Whitehorse changed jobs, she rolled over the \$6000 in her retirement account into two simple-interest accounts. On one account, the annual simple interest rate is 9%; on the second account, the annual simple interest rate is 6%. How much must be invested in each account if the accounts earn the same amount of annual interest?

34. An animal trainer decided to take the \$15,000 won on a game show and deposit it in two simple-interest accounts. Part of the winnings were placed in a 7% annual simple-interest account and the remainder was used to purchase a government bond that earns 6.5% annual simple interest. The amount of interest earned for one year was \$1020. How much was invested in each account?

35. A police officer has chosen a high-yield stock fund that earns 8% annual simple interest for part of a \$6000 investment. The remaining portion is used to purchase a preferred stock that earns 11% annual simple interest. How much should be invested in each account so that the amount earned on the 8% account is twice the amount earned on the 11% account?

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36. To plan for the purchase of a new car, a deposit was made into an account that earns 7% annual simple interest. Another deposit, $1500 less than the first deposit, was placed in a 9% annual simple-interest certificate of deposit. The total interest earned on both accounts for one year was $505. How much money was deposited in the certificate of deposit?

37. The Pacific Investment Group invested some money in a certificate of deposit (CD) that earns 6.5% annual simple interest. Twice the amount invested at 6.5% was invested in a second CD that earns 8.5% annual simple interest. If the total annual interest earned from the two investments was $4935, how much was invested at 6.5%?

38. A corporation gave a university $300,000 to support product safety research. The university deposited some of the money in a 10% simple-interest account and the remainder in an 8.5% simple-interest account. How much should be deposited in each account so that the annual interest earned is $28,500?

39. Ten coworkers formed an investment club, and each deposited $2000 in the club's account. They decided to take the total amount and invest some of it in preferred stock that pays 8% annual simple interest and the remainder in a municipal bond that pays 7% annual simple interest. The amount of interest earned each year from the investments was $1520. How much was invested in each?

40. A financial consultant advises a client to invest part of $30,000 in municipal bonds that earn 6.5% annual simple interest and the remainder of the money in 8.5% corporate bonds. How much should be invested in each so that the total annual interest earned each year is $2190?

41. Alisa Rhodes placed some money in a real estate investment trust that earns 7.5% annual simple interest. A second investment, which was one-half the amount placed in the real estate investment trust, was used to purchase a trust deed that earns 9% annual simple interest. If the total annual interest earned from the two investments was $900, how much was invested in the trust deed?

APPLYING THE CONCEPTS

For what value of k does the system of equations have no solution?

42.
$$\begin{aligned} 2x - 3y &= 7 \\ kx - 3y &= 4 \end{aligned}$$

43.
$$\begin{aligned} 8x - 4y &= 1 \\ 2x - ky &= 3 \end{aligned}$$

44.
$$\begin{aligned} x &= 4y + 4 \\ kx - 8y &= 4 \end{aligned}$$

45. Describe in your own words the process of solving a system of equations by the substitution method.

46. When you solve a system of equations by the substitution method, how do you determine whether the system of equations is dependent?

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47. The following was offered as a solution to the system of equations

(1) $y = \frac{1}{2}x + 2$

(2) $2x + 5y = 10$

$$2x + 5y = 10 \qquad \text{• Equation (2)}$$

$$2x + 5\left(\frac{1}{2}x + 2\right) = 10 \qquad \text{• Substitute } \frac{1}{2}x + 2 \text{ for } y.$$

$$2x + \frac{5}{2}x + 10 = 10 \qquad \text{• Solve for } x.$$

$$\frac{9}{2}x = 0$$

$$x = 0$$

At this point the student stated that because $x = 0$, the system of equations has no solution. If this assertion is correct, is the system of equations independent, dependent, or inconsistent? If the assertion is not correct, what is the correct solution?

48. When you solve a system of equations by the substitution method, how do you determine whether the system of equations is inconsistent?

49. A sales representative invests in a stock paying 9% dividends. A research consultant invests \$5000 more than the sales representative in bonds paying 8% annual simple interest. The research consultant's income from the investment is equal to the sales representative's. Find the amount of the research consultant's investment.

50. A plant manager invested \$3000 more in stocks than in bonds. The stocks paid 8% annual simple interest, and the bonds paid 9.5% annual simple interest. Both investments yielded the same income. Find the total annual interest received on both investments.

51. The exercises in this objective were based on annual *simple* interest, r, which means that the amount of interest earned after one year is given by $I = Pr$. For *compound* interest, the interest earned for a certain period of time (usually daily or monthly) is added to the principal before the interest for the next period is calculated. The compound interest earned in one year is given by the formula $I = P\left[\left(1 + \frac{r}{n}\right)^n - 1\right]$, where n is the number of times per year that interest is compounded. For instance, if interest is compounded daily, then $n = 365$; if interest is compounded monthly, then $n = 12$. Suppose an investment of \$5000 is made into three different accounts. The first account earns 8% annual simple interest, the second earns 8% compounded monthly ($n = 12$), and the third earns 8% compounded daily ($n = 365$). Find the amount of interest earned from each account.

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8.3 Solving Systems of Equations by the Addition Method

Objective A *To solve a system of linear equations by the addition method*

Another method of solving a system of equations is called the **addition method.** This method is based on the Addition Property of Equations.

Note, for the system of equations at the right, the effect of adding Equation (2) to Equation (1). Because $2y$ and $-2y$ are opposites, adding the equations results in an equation with only one variable.

$$\begin{array}{rl} (1) & 5x + 2y = 11 \\ (2) & 3x - 2y = 13 \\ \hline & 8x + 0y = 24 \\ & 8x = 24 \end{array}$$

Solving $8x = 24$ for x gives the first coordinate of the ordered-pair solution of the system of equations.

$$\frac{8x}{8} = \frac{24}{8}$$
$$x = 3$$

The second coordinate is found by substituting the value of x into Equation (1) or Equation (2) and then solving for y. Equation (1) is used here.

$$\begin{array}{rl} (1) & 5x + 2y = 11 \\ & 5(3) + 2y = 11 \\ & 15 + 2y = 11 \\ & 2y = -4 \\ & y = -2 \end{array}$$

The solution is $(3, -2)$.

Sometimes adding the two equations does not eliminate one of the variables. In this case, use the Multiplication Property of Equations to rewrite one or both of the equations such that the coefficients of one of the variables are opposites.

➡ Solve by the addition method:

$$\begin{array}{rl} (1) & 4x + 3y = -1 \\ (2) & 2x - 5y = 19 \end{array}$$

Multiply Equation (2) by -2. The coefficients of x will then be opposites.

$$-2(2x - 5y) = -2 \cdot 19 \quad \text{• Multiply Equation (2) by } -2.$$
$$(3) \quad -4x + 10y = -38 \quad \text{• Simplify. This is Equation (3).}$$

Add Equation (1) to Equation (3). Then solve for y.

$$\begin{array}{rll} (1) & 4x + 3y = -1 & \\ (3) & -4x + 10y = -38 & \text{• Note that the coefficients of } x \text{ are opposites.} \\ \hline & 13y = -39 & \text{• Add the two equations.} \\ & y = -3 & \text{• Solve for } y. \end{array}$$

Substitute the value of y into Equation (1) or Equation (2) and solve for x. Equation (1) will be used here.

$$\begin{array}{rll} (1) & 4x + 3y = -1 & \\ & 4x + 3(-3) = -1 & \text{• Substitute } -3 \text{ for } y. \\ & 4x - 9 = -1 & \text{• Solve for } x. \\ & 4x = 8 & \\ & x = 2 & \end{array}$$

The solution is $(2, -3)$.

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Sometimes each equation of the system of equations must be multiplied by a constant so that the coefficients of one of the variable terms are opposites.

➡ Solve by the addition method: (1) $3x + 7y = 2$
(2) $5x - 3y = -26$

To eliminate x, multiply Equation (1) by 5 and Equation (2) by -3. Note at the right how the constants are chosen.

$$5(3x + 7y) = 5 \cdot 2$$
$$-3(5x - 3y) = -3(-26)$$

• The negative is used so that the coefficients will be opposites.

$$\begin{array}{rl} 15x + 35y = 10 & \bullet \text{ 5 times Equation (1).} \\ -15x + 9y = 78 & \bullet \text{ } -3 \text{ times Equation (2).} \\ \hline 44y = 88 & \bullet \text{ Add the equations.} \\ y = 2 & \bullet \text{ Solve for } y. \end{array}$$

Substitute the value of y into Equation (1) or Equation (2) and solve for x. Equation (1) will be used here.

$$\begin{array}{rll} (1) & 3x + 7y = 2 & \\ & 3x + 7(2) = 2 & \bullet \text{ Substitute 2 for } y. \\ & 3x + 14 = 2 & \bullet \text{ Solve for } x. \\ & 3x = -12 & \\ & x = -4 & \end{array}$$

The solution is $(-4, 2)$.

For the previous system of equations, the value of x was determined by substitution. This value can also be determined by eliminating y from the system.

$$\begin{array}{rl} 9x + 21y = 6 & \bullet \text{ 3 times Equation (1).} \\ 35x - 21y = -182 & \bullet \text{ 7 times Equation (2).} \\ \hline 44x = -176 & \bullet \text{ Add the equations.} \\ x = -4 & \bullet \text{ Solve for } x. \end{array}$$

Note that this is the same value of x as was determined by using substitution.

➡ Solve by the addition method: (1) $5x - 2y = -4$
(2) $3y = 7x + 5$

Rewrite Equation (2) in the form $Ax + By = C$.

$$\begin{array}{rll} (2) & 3y = 7x + 5 & \bullet \text{ Subtract } 7x \text{ from each side of the equation. This is Equation (3).} \\ (3) & -7x + 3y = 5 & \end{array}$$

Eliminate x or y. We will eliminate x by using Equations (1) and (3).

$$\begin{array}{rl} 35x - 14y = -28 & \bullet \text{ 7 times Equation (1).} \\ -35x + 15y = 25 & \bullet \text{ 5 times Equation (3).} \\ \hline y = -3 & \bullet \text{ Add the equations.} \end{array}$$

Substitute the value of y into Equation (1) or Equation (2) and solve for x. Equation (1) will be used here.

$$\begin{array}{rll} (1) & 5x - 2y = -4 & \\ & 5x - 2(-3) = -4 & \bullet \text{ Substitute } -3 \text{ for } y. \\ & 5x + 6 = -4 & \bullet \text{ Solve for } x. \\ & 5x = -10 & \\ & x = -2 & \end{array}$$

The solution is $(-2, -3)$.

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➡ Solve by the addition method: (1) $2x + y = 2$
(2) $4x + 2y = -5$

Eliminate y. Multiply Equation (1) by -2.

(1) $-2(2x + y) = -2 \cdot 2$ • −2 times Equation (1).
(3) $-4x - 2y = -4$ • This is Equation (3).

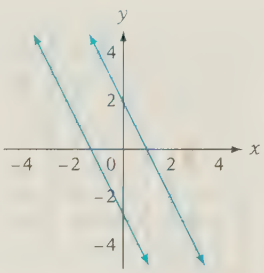
Add Equation (2) to Equation (3) and solve for x.

$$\begin{array}{rl} (3) & -4x - 2y = -4 \\ (2) & \underline{4x + 2y = -5} \\ & 0x + 0y = -9 \\ & 0 = -9 \end{array}$$

• Add Equation (2) to Equation (3).
• This is not a true equation.

The system of equations is inconsistent and therefore does not have a solution.

The graphs of the two equations in the system of equations above are shown at the right. Note that the graphs are parallel and therefore do not intersect. Thus the system of equations has no solutions.



Example 1

Solve by the addition method:
(1) $2x + 4y = 7$
(2) $5x - 3y = -2$

Solution

Eliminate x.

$5(2x + 4y) = 5 \cdot 7$ • 5 times Equation (1).
$-2(5x - 3y) = -2(-2)$ • −2 times Equation (2).

$$\begin{array}{rl} & 10x + 20y = 35 \\ & -10x + 6y = 4 \\ \hline & 26y = 39 \\ & y = \dfrac{39}{26} = \dfrac{3}{2} \end{array}$$

• Add the equations.
• Solve for y.

Substitute $\frac{3}{2}$ for y in Equation (1).

$$\begin{array}{rll} (1) & 2x + 4y = 7 & \\ & 2x + 4\left(\dfrac{3}{2}\right) = 7 & \bullet \text{ Replace } y \text{ by } \dfrac{3}{2}. \\ & 2x + 6 = 7 & \bullet \text{ Solve for } x. \\ & 2x = 1 & \\ & x = \dfrac{1}{2} & \end{array}$$

The solution is $\left(\frac{1}{2}, \frac{3}{2}\right)$.

You Try It 1

Solve by the addition method:
(1) $x - 2y = 1$
(2) $2x + 4y = 0$

Your solution

Solution on p. S20

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Example 2

Solve by the addition method:

$$(1) \quad 6x + 9y = 15$$
$$(2) \quad 4x + 6y = 10$$

Solution

Eliminate x.

$$4(6x + 9y) = 4 \cdot 15$$ • 4 times Equation (1).
$$-6(4x + 6y) = -6 \cdot 10$$ • −6 times Equation (2).

$$24x + 36y = 60$$
$$-24x - 36y = -60$$
$$0x + 0y = 0$$ • Add the equations.
$$0 = 0$$

The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $6x + 9y = 15$.

You Try It 2

Solve by the addition method:

$$2x - 3y = 4$$
$$-4x + 6y = -8$$

Your solution

Example 3

Solve by the addition method:

$$(1) \quad 2x = y + 8$$
$$(2) \quad 3x + 2y = 5$$

Solution

Write Equation (1) in the form $Ax + By = C$.

$$2x = y + 8$$
$$(3) \quad 2x - y = 8$$ • This is Equation (3).

Eliminate y.

$$2(2x - y) = 2 \cdot 8$$ • 2 times Equation (3).
$$3x + 2y = 5$$ • This is Equation (2).

$$4x - 2y = 16$$
$$3x + 2y = 5$$
$$7x = 21$$ • Add the equations.
$$x = 3$$

Replace x in Equation (1).

$$(1) \quad 2x = y + 8$$
$$2 \cdot 3 = y + 8$$ • Replace x by 3.
$$6 = y + 8$$
$$-2 = y$$

The solution is $(3, -2)$.

You Try It 3

Solve by the addition method:

$$4x + 5y = 11$$
$$3y = x + 10$$

Your solution

Solutions on p. S20

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8.3 Exercises

Objective A

Solve by the addition method.

1. $x + y = 4$
$x - y = 6$

2. $2x + y = 3$
$x - y = 3$

3. $x + y = 4$
$2x + y = 5$

4. $x - 3y = 2$
$x + 2y = -3$

5. $2x - y = 1$
$x + 3y = 4$

6. $x - 2y = 4$
$3x + 4y = 2$

7. $4x - 5y = 22$
$x + 2y = -1$

8. $3x - y = 11$
$2x + 5y = 13$

9. $2x - y = 1$
$4x - 2y = 2$

10. $x + 3y = 2$
$3x + 9y = 6$

11. $4x + 3y = 15$
$2x - 5y = 1$

12. $3x - 7y = 13$
$6x + 5y = 7$

13. $2x - 3y = 1$
$4x - 6y = 2$

14. $2x + 4y = 6$
$3x + 6y = 9$

15. $5x - 2y = -1$
$x + 3y = -5$

16. $4x - 3y = 1$
$8x + 5y = 13$

17. $5x + 7y = 10$
$3x - 14y = 6$

18. $7x + 10y = 13$
$4x + 5y = 6$

19. $3x - 2y = 0$
$6x + 5y = 0$

20. $5x + 2y = 0$
$3x + 5y = 0$

21. $2x - 3y = 16$
$3x + 4y = 7$

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22. $3x + 4y = 10$
$4x + 3y = 11$

23. $5x + 3y = 7$
$2x + 5y = 1$

24. $-2x + 7y = 9$
$3x + 2y = -1$

25. $7x - 2y = 13$
$5x + 3y = 27$

26. $12x + 5y = 23$
$2x - 7y = 39$

27. $8x - 3y = 11$
$6x - 5y = 11$

28. $4x - 8y = 36$
$3x - 6y = 27$

29. $5x + 15y = 20$
$2x + 6y = 8$

30. $y = 2x - 3$
$3x + 4y = -1$

31. $3x = 2y + 7$
$5x - 2y = 13$

32. $2y = 4 - 9x$
$9x - y = 25$

33. $2x + 9y = 16$
$5x = 1 - 3y$

34. $3x - 4 = y + 18$
$4x + 5y = -21$

35. $2x + 3y = 7 - 2x$
$7x + 2y = 9$

36. $5x - 3y = 3y + 4$
$4x + 3y = 11$

APPLYING THE CONCEPTS

37. Describe in your own words the process of solving a system of equations by the addition method.

38. The point of intersection of the graphs of the equations $Ax + 2y = 2$ and $2x + By = 10$ is $(2, -2)$. Find A and B.

39. The point of intersection of the graphs of the equations $Ax - 4y = 9$ and $4x + By = -1$ is $(-1, -3)$. Find A and B.

40. For what value of k is the system of equations dependent?

a. $2x + 3y = 7$
$4x + 6y = k$

b. $y = \frac{2}{3}x - 3$
$y = kx - 3$

c. $x = ky - 1$
$y = 2x + 2$

41. For what value of k is the system of equations independent?

a. $x + y = 7$
$kx + y = 3$

b. $x + 2y = 4$
$kx + 3y = 2$

c. $2x + ky = 1$
$x + 2y = 2$

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8.4 Application Problems in Two Variables

Objective A To solve rate-of-wind or -current problems

Motion problems that involve an object moving with or against a wind or current normally require two variables to solve.

➡ Flying with the wind, a small plane can fly 600 mi in 3 h. Against the wind, the plane can fly the same distance in 4 h. Find the rate of the plane in calm air and the rate of the wind.

Strategy for Solving Rate-of-Wind or -Current Problems

Choose one variable to represent the rate of the object in calm conditions and a second variable to represent the rate of the wind or current. Using these variables, express the rate of the object with and against the wind or current. Use the equation $d = rt$ to write expressions for the distance traveled by the object. The results can be recorded in a table.

Rate of plane in calm air: p
Rate of wind: w

	Rate	·	*Time*	=	*Distance*
With the wind	$p + w$	·	3	=	$3(p + w)$
Against the wind	$p - w$	·	4	=	$4(p - w)$

Determine how the expressions for distance are related.

The distance traveled with the wind is 600 mi. $3(p + w) = 600$
The distance traveled against the wind is 600 mi. $4(p - w) = 600$

Solve the system of equations.

$$\begin{aligned} 3(p + w) &= 600 \\ 4(p - w) &= 600 \end{aligned} \quad \rightarrow \quad \begin{aligned} \frac{1}{3} \cdot 3(p + w) &= \frac{1}{3} \cdot 600 \\ \frac{1}{4} \cdot 4(p - w) &= \frac{1}{4} \cdot 600 \end{aligned} \quad \rightarrow \quad \begin{aligned} p + w &= 200 \\ p - w &= 150 \end{aligned}$$

$$\begin{aligned} 2p &= 350 \\ p &= 175 \end{aligned}$$

$$\begin{aligned} p + w &= 200 \\ 175 + w &= 200 \\ w &= 25 \end{aligned}$$

The rate of the plane in calm air is 175 mph.
The rate of the wind is 25 mph.

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Example 1

A 450-mile trip from one city to another takes 3 h when a plane is flying with the wind. The return trip, against the wind, takes 5 h. Find the rate of the plane in still air and the rate of the wind.

Strategy

Rate of the plane in still air: p
Rate of the wind: w

	Rate	*Time*	*Distance*
With wind	$p + w$	3	$3(p + w)$
Against wind	$p - w$	5	$5(p - w)$

The distance traveled with the wind is 450 mi. The distance traveled against the wind is 450 mi.

Solution

$$3(p + w) = 450 \quad \frac{1}{3} \cdot 3(p + w) = \frac{1}{3} \cdot 450$$

$$5(p - w) = 450 \quad \frac{1}{5} \cdot 5(p - w) = \frac{1}{5} \cdot 450$$

$$p + w = 150$$
$$p - w = 90$$

$$2p = 240$$
$$p = 120$$

$$p + w = 150$$
$$120 + w = 150$$
$$w = 30$$

The rate of the plane in still air is 120 mph.
The rate of the wind is 30 mph.

You Try It 1

A canoeist paddling with the current can travel 15 mi in 3 h. Against the current, it takes 5 h to travel the same distance. Find the rate of the current and the rate of the canoeist in calm water.

Your strategy

Your solution

Solution on p. S20

Objective B To solve application problems using two variables



The application problems in this section are varieties of those problems solved earlier in the text. Each of the strategies for the problems in this section will result in a system of equations.

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POINT OF INTEREST

The Babylonians had a method for solving a system of equations. Here is an adaptation of a problem from an ancient (around 1500 B.C.) Babylonian text. "There are two silver blocks. The sum of $\frac{1}{7}$ of the first block and $\frac{1}{11}$ of the second block is one sheqel (a weight). The first block diminished by $\frac{1}{7}$ of its weight equals the second diminished by $\frac{1}{11}$ of its weight. What are the weights of the two blocks?"

A jeweler purchased 5 oz of a gold alloy and 20 oz of a silver alloy for a total cost of \$540. The next day, at the same prices per ounce, the jeweler purchased 4 oz of the gold alloy and 25 oz of the silver alloy for a total cost of \$450. Find the cost per ounce of the gold and silver alloys.

Strategy for Solving an Application Problem in Two Variables

Choose one variable to represent one of the unknown quantities and a second variable to represent the other unknown quantity. Write numerical or variable expressions for all the remaining quantities. These results can be recorded in two tables, one for each of the conditions.

Cost per ounce of gold: g
Cost per ounce of silver: s

First day:

	Amount	·	*Unit Cost*	=	*Value*
Gold	5	·	g	=	$5g$
Silver	20	·	s	=	$20s$

Second day:

	Amount	·	*Unit Cost*	=	*Value*
Gold	4	·	g	=	$4g$
Silver	25	·	s	=	$25s$

Determine a system of equations. Each table will give one equation of the system.

The total value of the purchase on the first day was \$540. $5g + 20s = 540$

The total value of the purchase on the second day was \$450. $4g + 25s = 450$

Solve the system of equations.

$$\begin{aligned} 5g + 20s &= 540 \\ 4g + 25s &= 450 \end{aligned}$$

$$\begin{aligned} 4(5g + 20s) &= 4 \cdot 540 \\ -5(4g + 25s) &= -5 \cdot 450 \end{aligned}$$

$$\begin{aligned} 20g + 80s &= 2160 \\ -20g - 125s &= -2250 \\ \hline -45s &= -90 \\ s &= 2 \end{aligned}$$

$$\begin{aligned} 5g + 20s &= 540 \\ 5g + 20(2) &= 540 \quad \bullet\ s = 2 \\ 5g + 40 &= 540 \\ 5g &= 500 \\ g &= 100 \end{aligned}$$

The cost per ounce of the gold alloy was \$100.
The cost per ounce of the silver alloy was \$2.

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Example 2

A store owner purchased 20 incandescent light bulbs and 30 fluorescent bulbs for a total cost of $40. A second purchase, at the same prices, included 30 incandescent bulbs and 10 fluorescent bulbs for a total cost of $25. Find the cost of an incandescent bulb and of a fluorescent bulb.

Strategy

Cost of an incandescent bulb: b
Cost of a fluorescent bulb: f

First purchase:

	Amount	*Unit Cost*	*Value*
Incandescent	20	b	$20b$
Fluorescent	30	f	$30f$

Second purchase:

	Amount	*Unit Cost*	*Value*
Incandescent	30	b	$30b$
Fluorescent	10	f	$10f$

The total of the first purchase was $40.
The total of the second purchase was $25.

Solution

$$\begin{aligned} 20b + 30f &= 40 \\ 30b + 10f &= 25 \end{aligned} \qquad \begin{aligned} 3(20b + 30f) &= 3 \cdot 40 \\ -2(30b + 10f) &= -2 \cdot 25 \end{aligned}$$

$$\begin{aligned} 60b + 90f &= 120 \\ -60b - 20f &= -50 \\ \hline 70f &= 70 \\ f &= 1 \end{aligned}$$

$$\begin{aligned} 20b + 30f &= 40 \\ 20b + 30(1) &= 40 \\ 20b &= 10 \\ b &= \frac{1}{2} \end{aligned}$$

The cost of an incandescent bulb was $.50.
The cost of a fluorescent bulb was $1.00.

You Try It 2

A citrus grower purchased 25 orange trees and 20 grapefruit trees for $290. The next week, at the same prices, the grower bought 20 orange trees and 30 grapefruit trees for $330. Find the cost of an orange tree and the cost of a grapefruit tree.

Your strategy

Your solution

Solution on p. S21

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8.4 Exercises

Objective A Application Problems

1. A whale swimming against an ocean current traveled 60 mi in 2 h. Swimming in the opposite direction, with the current, the whale was able to travel the same distance in 1.5 h. Find the speed of the whale in calm water and the rate of the ocean current.
2. A plane flying with the jet stream flew from Los Angeles to Chicago, a distance of 2250 mi, in 5 h. Flying against the jet stream, the plane could fly only 1750 mi in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.
3. A rowing team rowing with the current traveled 40 km in 2 h. Rowing against the current, the team could travel only 16 km in 2 h. Find the rate of rowing in calm water and the rate of the current.
4. The bird capable of the fastest flying speed is the swift. A swift flying with the wind to a favorite feeding spot traveled 26 mi in 0.2 h. On returning, now against the wind, the swift was able to travel only 16 mi in the same amount of time. What is the rate of the swift in calm air and what was the rate of the wind?
5. A private Learjet 31A transporting passengers was flying with a tailwind and traveled 1120 mi in 2 h. Flying against the wind on the return trip the jet was able to travel only 980 mi in 2 h. Find the speed of the jet in calm air and the rate of the wind.
6. A plane flying with a tailwind flew 300 mi in 2 h. Against the wind, it took 3 h to travel the same distance. Find the rate of the plane in calm air and the rate of the wind.
7. A Boeing Apache Longbow military helicopter traveling directly into a strong headwind was able to travel 450 mi in 2.5 h. The return trip, now with a tailwind, took 1 h 40 min. Find the speed of the helicopter in calm air and the rate of the wind.
8. A seaplane pilot flying with the wind flew from an ocean port to a lake, a distance of 240 mi, in 2 h. Flying against the wind, the trip from the lake to the ocean port took 2 h 40 min. Find the rate of the plane in calm air and the rate of the wind.
9. Rowing with the current, a canoeist paddled 14 mi in 2 h. Against the current, the canoeist could paddle only 10 mi in the same amount of time. Find the rate of the canoeist in calm water and the rate of the current.

Objective B Application Problems

10. A computer on-line service charges one hourly price for regular use but a higher hourly rate for designated "premium" areas. One customer was charged $28 after spending 2 h in premium areas and 9 regular hours; another spent 3 h in the premium areas and 6 regular hours and was charged $27. What does the service charge per hour for regular and premium services?
11. A baker purchased 12 lb of wheat flour and 15 lb of rye flour for a total cost of $18.30. A second purchase, at the same prices, included 15 lb of wheat flour and 10 lb of rye flour. The cost of the second purchase was $16.75. Find the cost per pound of the wheat flour and of the rye flour.

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12. An investor owned 300 shares of an oil company and 200 shares of a movie company. The quarterly dividend from the two stocks was \$165. After the investor sold 100 shares of the oil company and bought an additional 100 shares of the movie company, the quarterly dividend became \$185. Find the dividend per share for each stock.

13. The charge for 25 min of prime time and 35 min of nonprime time to a customer for using a computerized financial news network was \$10.75. A second customer used the system for 30 min of prime time and 45 min of nonprime time for a cost of \$13.35. Find the cost per minute for using the financial news network during prime time and during nonprime time.

14. A stocker at a grocery store gets paid a standard hourly rate for her day hours but a higher hourly rate for any hours she works during the night shift. One week she worked 17 daytime hours and 8 nighttime hours and earned \$191. The next week she earned \$219 for a total of 12 daytime and 15 nighttime hours. What is the rate she is being paid for day hours and what is the rate for nighttime hours?

15. The employees of a hardware store ordered lunch from a local delicatessen. The lunch consisted of 4 submarine sandwiches and 7 orders of french fries for a total cost of \$23.30. The next day, the employees ordered 5 submarine sandwiches and 5 orders of french fries totaling \$25.75. What does the delicatessen charge for a submarine sandwich and for an order of french fries?

APPLYING THE CONCEPTS

16. Two angles are supplementary. The larger angle is 15° more than twice the measure of the smaller angle. Find the measure of the two angles. (Supplementary angles are two angles whose sum is 180°.)

17. The value of the nickels and dimes in a coin bank is \$.25. If the number of nickels and the number of dimes were doubled, the value of the coins would be \$.50. How many nickels and how many dimes are in the bank?

18. An investor has \$5000 to invest in two accounts. The first account earns 8% annual simple interest and the second account earns 10% annual simple interest. How much money should be invested in each account so that the annual simple interest earned is \$600?

19. Solve the following problem, which dates from a Chinese manuscript called the Jinzhang that is approximately 2100 years old. "The price of 1 acre of good land is 300 pieces of gold; the price of 7 acres of bad land is 500 pieces of gold. One has purchased altogether 100 acres. The price was 10,000 pieces of gold. How much good land and how much bad land was bought?" Adapted from *A History of Mathematics, An Introduction*, Victor J. Katz (New York: HarperCollins, 1993, p. 15).

20. A coin bank contains only nickels or dimes, but there are no more than 27 coins. The value of the coins is \$2.10. How many different combinations of nickels and dimes could be in the coin bank?

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Focus on Problem Solving

Using a Table and Searching for a Pattern

Consider the numbers 10, 12, and 28 and the sum of the proper factors (the natural number factors less than the number) of those numbers.

$10: 1 + 2 + 5 = 8 \quad 12: 1 + 2 + 3 + 4 + 6 = 16 \quad 28: 1 + 2 + 4 + 7 + 14 = 28$

10 is called a *deficient number* because the sum of its proper factors is less than the number ($8 < 10$). 12 is called an *abundant number* because the sum of its proper factors is greater than the number ($16 > 12$), and 28 is called a *perfect number* because the sum of its proper divisors equals the number ($28 = 28$).

Our goal for this "Focus on Problem Solving" is to try to find a method that will determine whether a number is deficient, abundant, or perfect without having to first find all the factors and then add them up. We will use a table and search for a pattern.

Before we begin, recall that a prime number is a number greater than 1 whose only factors are itself and 1, and each natural number greater than 1 has a unique prime factorization. For instance, the prime factorization of 36 is given by $36 = 2^2 \cdot 3^2$. Note that the proper factors of 36 (1, 2, 3, 4, 6, 9, 12, 18) can be represented in terms of the same prime numbers.

$1 = 2^0, \quad 2 = 2^1, \quad 3 = 3^1, \quad 4 = 2^2, \quad 6 = 2 \cdot 3, \quad 9 = 3^2, \quad 12 = 2^2 \cdot 3, \quad 18 = 2 \cdot 3^2$

Now let us consider a trial problem of determining whether 432 is deficient, abundant, or perfect.

TAKE NOTE

This table contains the factor $432 = 2^4 \cdot 3^3$, which is not a proper factor.

We write the prime factorization of 432 as $2^4 \cdot 3^3$ and place the factors of 432 in a table as shown at the right. This table contains *all* the factors of 432 represented in terms of the prime number factors. The sum of each column is shown at the bottom.

	1	$1 \cdot 3$	$1 \cdot 3^2$	$1 \cdot 3^3$
	2	$2 \cdot 3$	$2 \cdot 3^2$	$2 \cdot 3^3$
	2^2	$2^2 \cdot 3$	$2^2 \cdot 3^2$	$2^2 \cdot 3^3$
	2^3	$2^3 \cdot 3$	$2^3 \cdot 3^2$	$2^3 \cdot 3^3$
	2^4	$2^4 \cdot 3$	$2^4 \cdot 3^2$	$2^4 \cdot 3^3$
Sum	31	$31 \cdot 3$	$31 \cdot 3^2$	$31 \cdot 3^3$

Here is the calculation of the sum for the column headed by $1 \cdot 3$.

$$1 \cdot 3 + 2 \cdot 3 + 2^2 \cdot 3 + 2^3 \cdot 3 + 2^4 \cdot 3 = (1 + 2 + 2^2 + 2^3 + 2^4)3 = 31(3)$$

For the column headed by $1 \cdot 3^2$ there is a similar situation.

$$1 \cdot 3^2 + 2 \cdot 3^2 + 2^2 \cdot 3^2 + 2^3 \cdot 3^2 + 2^4 \cdot 3^2 = (1 + 2 + 2^2 + 2^3 + 2^4)3^2 = 31(3^2)$$

The sum of *all* the factors (including 432) is the sum of the last row.

$$\text{Sum of all factors} = 31 + 31 \cdot 3 + 31 \cdot 3^2 + 31 \cdot 3^3 = 31(1 + 3 + 3^2 + 3^3) = 31(40) = 1240$$

To find the sum of the *proper* factors, we must subtract 432 from 1240; we get 808 (see "Take Note"). Thus, 432 is abundant.

We now look for some pattern for the sum of all the factors. Note that

$$\underbrace{1 + 2 + 2^2 + 2^3 + 2^4}_{\text{Sum of left column}} = \underbrace{1 + 3 + 3^2 + 3^3}_{\text{Sum of top row}}$$

$$31(40) = 1240$$

This suggests that the sum of the proper factors can be found by finding the sum of all the prime power factors for each prime, multiplying those numbers, and then subtracting the original number. Although we have not proved this for all cases, it is a true statement.

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For instance, to find the sum of the proper factors of 3240, first find the prime factorization.

$$3240 = 2^3 \cdot 3^4 \cdot 5$$

Now find the following sums:

$$1 + 2 + 2^2 + 2^3 = 15 \qquad 1 + 3 + 3^2 + 3^3 + 3^4 = 121 \qquad 1 + 5 = 6$$

The sum of the proper factors $= (15)(121)6 - 3240 = 7650$.

Determine whether the number is deficient, abundant, or perfect.

1. 200 **2.** 3125 **3.** 8128 **4.** 10,000

5. Is a prime number deficient, abundant, or perfect?

Projects and Group Activities

Find a Pattern

The "Focus on Problem Solving" involved finding the sum of $1 + 3 + 3^2$ and $1 + 2 + 2^2 + 2^3$. For sums that contain a larger number of terms, it may be difficult or time consuming to try to evaluate the sum. Perhaps there is a pattern for these sums that can be used to calculate them without having to evaluate each exponential expression and then add the results.

Look at the following from the calculation of the sum of the factors of 3240.

$$1 + 2 + 2^2 + 2^3 = 1 + 2 + 4 + 8 = 15$$

$$\frac{2^{3+1} - 1}{2 - 1} = \frac{2^4 - 1}{1} = 16 - 1 = 15$$

$$1 + 3 + 3^2 + 3^3 + 3^4 = 1 + 3 + 9 + 27 + 81 = 121$$

$$\frac{3^{4+1} - 1}{3 - 1} = \frac{3^5 - 1}{2} = \frac{243 - 1}{2} = \frac{242}{2} = 121$$

Consider another sum of this type.

$$1 + 7 + 7^2 + 7^3 + 7^4 + 7^5 = 1 + 7 + 49 + 343 + 2401 + 16{,}807 = 19{,}608$$

$$\frac{7^{5+1} - 1}{7 - 1} = \frac{7^6 - 1}{6} = \frac{117{,}649 - 1}{6} = \frac{117{,}648}{6} = 19{,}608$$

On the basis of these examples shown above, make a conjecture as to the value of $1 + n + n^2 + n^3 + n^4 + \cdots + n^k$, where n and k are natural numbers greater than 1.

Solving a System of Equations with a Scientific Calculator

By using the addition method, it is possible to solve the system of equations.

$$ax + by = c$$
$$dx + ey = f$$



The solution is

$$x = \frac{ce - bf}{ae - bd} \quad \text{and} \quad y = \frac{af - cd}{ae - bd} \quad \text{where } ae - bd \neq 0$$

Using this solution, a system of two equations in two variables can be solved with a calculator. It is helpful to observe that the denominators for both expressions are identical. The calculation for the denominator is done first and then stored in the calculator's memory. If the value of the denominator is zero, the system of equations is dependent or inconsistent, and this calculator method cannot be used.

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Solve: $2x - 5y = 9$
$4x + 3y = 2$

Make a list of the values of a, b, c, d, e, and f.

$$a = 2 \quad b = -5 \quad c = 9$$
$$d = 4 \quad e = 3 \quad f = 2$$

Calculate the denominator $D = ae - bd$.

2 [×] 3 [−] 5 [+/−] [×] 4 [=] [M+][1]

Find x. Replace the variables in $x = \dfrac{ce - bf}{ae - bd}$ by their values.

[(] 9 [×] 3 [−] 5 [+/−] [×] 2 [)] [÷] [MR] [=]

The result should be approximately 1.423077.

Find y. Replace the variables in $y = \dfrac{af - cd}{ae - bd}$ by their values.

[(] 2 [×] 2 [−] 9 [×] 4 [)] [÷] [MR] [=]

The result should be approximately −1.230769.

The approximate solution is the ordered pair (1.423077, −1.230769). The result is an approximation because the calculator display is an approximation to the exact result.

Solve by using a scientific calculator.

1. $3.29x + 4.17y = -2.34$
$0.34x - 0.17y = 0.1$

2. $27x + 17y = 50$
$4.5x - 9.2y = 18.3$

3. $y = \dfrac{2}{3}x - 4$
$2x + y = 5$

4. $y = \dfrac{7}{4}x + 2$
$y = \dfrac{2}{3}x - 3$

5. Use the addition method to show that the solution of the system of equations $ax + by = c$, $dx + ey = f$ is given by $x = \dfrac{ce - bf}{ae - bd}$ and $y = \dfrac{af - cd}{ae - bd}$, where $ae - bd \neq 0$.

Solving a System of Equations with a Graphing Calculator



A graphing calculator can also be used to approximate the solution of a system of equations in two variables. Graph each equation of the system of equations, and then approximate the coordinates of the point of intersection. The process by which you approximate the solution depends on the model of calculator you have. In all cases, however, you must first solve each equation in the system of equations for y.

Solve: $2x - 5y = 9$
$4x + 3y = 2$

Solve each equation for y.

$2x - 5y = 9$
$-5y = -2x + 9$
$y = \dfrac{2}{5}x - \dfrac{9}{5}$

$4x + 3y = 2$
$3y = -4x + 2$
$y = -\dfrac{4}{3}x + \dfrac{2}{3}$

[1]Some calculators use STO to store a result and use RCL to recall the result.

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TI-83 Press [Y=]. Enter one equation as Y1 and the other equation as Y2. Press [GRAPH]. The intersection point must be shown in the viewing window. If it is not shown, adjust the viewing window. Press [2nd] CALC 5 [ENTER] [ENTER] [ENTER].

The point of intersection will show on the bottom of the screen as X = 1.4230769 Y = −1.230769.

Sharp EL-9600 Press [Y=]. Enter one equation as Y1 and the other equation as Y2. Press [GRAPH]. The point of intersection must be shown on the display. If it is not shown, adjust the viewing window. Press [2ndF] CALC 2.

The point of intersection will show on the bottom of the screen as X = 1.4230769 Y = −1.230769.

Casio CFX 9850 Press [MENU] 5. Enter one equation as Y1 and the other equation as Y2. Press [F6]. The intersection point must be shown in the viewing window. If it is not shown, adjust the viewing window. Press [SHIFT] G-SOLV [F5].

The point of intersection will show on the bottom of the screen as X = 1.4230769 Y = −1.230769.

Each of these calculators has a built-in method of solving a system of equations. For calculators without those methods, use the ZOOM and TRACE features of the calculator. Graph the two equations and then use the arrow keys to place the cursor near the intersection point. Then ZOOM in on the point of intersection, and use TRACE to determine the approximate coordinates. Each time you ZOOM in on the point of intersection, the solution of the system of equations becomes more accurate (up to the limits of the calculator).

Solve by using a graphing calculator.

1. $4x - 5y = 8$
$5x + 7y = 7$

2. $3x + 2y = 11$
$7x - 6y = 13$

3. $x = 3y + 2$
$y = 4x - 2$

4. $x = 2y - 5$
$x = 3y + 2$

Chapter Summary

Key Words

Equations considered together are called a *system of equations*.

A *solution of a system of equations in two variables* is an ordered pair that is a solution of each equation of the system.

An *independent system of equations* has one solution.

A *dependent system of equations* has an infinite number of solutions.

An *inconsistent system of equations* has no solution.

Essential Rules

A system of equations can be solved by the *graphing* method, the *substitution* method, or the *addition* method.

Annual Simple-Interest Equation

$I = Pr$, where I is the annual simple interest, P is the principal, and r is the annual interest rate as a decimal.

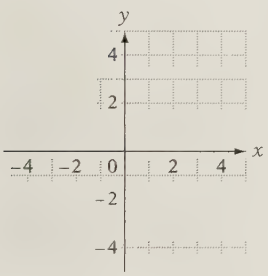
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Chapter Review

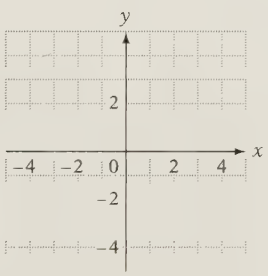
1. Is $(-1, -3)$ a solution of this system of equations?
$$5x + 4y = -17$$
$$2x - y = 1$$

2. Is $(-2, 0)$ a solution of this system of equations?
$$-x + 9y = 2$$
$$6x - 4y = 12$$

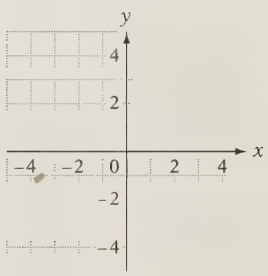
3. Solve by graphing:
$$3x - y = 6$$
$$y = -3$$



4. Solve by graphing:
$$4x - 2y = 8$$
$$y = 2x - 4$$



5. Solve by graphing:
$$x + 2y = 3$$
$$y = -\frac{1}{2}x + 1$$



6. Solve by substitution:
$$4x + 7y = 3$$
$$x = y - 2$$

7. Solve by substitution:
$$6x - y = 0$$
$$7x - y = 1$$

8. Solve by the addition method:
$$3x + 8y = -1$$
$$x - 2y = -5$$

9. Solve by the addition method:
$$6x + 4y = -3$$
$$12x - 10y = -15$$

10. Solve by substitution:
$$12x - 9y = 18$$
$$y = \frac{4}{3}x - 3$$

11. Solve by substitution:
$$8x - y = 2$$
$$y = 5x + 1$$

12. Solve by the addition method:
$$4x - y = 9$$
$$2x + 3y = -13$$

13. Solve by the addition method:
$$5x + 7y = 21$$
$$20x + 28y = 63$$

14. Solve by substitution:
$$4x + 3y = 12$$
$$y = -\frac{4}{3}x + 4$$

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15. Solve by substitution:
$7x + 3y = -16$
$x - 2y = 5$

16. Solve by the addition method:
$3x + y = -2$
$-9x - 3y = 6$

17. Solve by the addition method:
$6x - 18y = 7$
$9x + 24y = 2$

18. A sculling team rowing with the current went 24 mi in 2 h. Rowing against the current, the sculling team went 18 mi in 3 h. Find the rate of the sculling team in calm water and the rate of the current.

19. An investor bought 1500 shares of stock, some at \$6 per share and the rest at \$25 per share. If \$12,800 worth of stock was purchased, how many shares of each kind did the investor buy?

20. A flight crew flew 420 km in 3 h with a tailwind. Flying against the wind, the flight crew flew 440 km in 4 h. Find the rate of the flight crew in calm air and the rate of the wind.

21. A small plane flying with the wind flew 360 mi in 3 h. Against a headwind, the plane took 4 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.

22. A small wood carving company mailed 190 advertisements, some requiring \$.25 postage and others \$.45. The total cost for mailing was \$59.50. Find the number of advertisements mailed at each rate.

23. A Terra Cotta Art Center receives an annual income of \$915 from two simple-interest investments. One investment, in a corporate bond fund, earns 8.5% annual simple interest. The second investment, in a real estate investment trust, earns 7% annual simple interest. If the total amount invested in the two accounts is \$12,000, how much is invested in each account?

24. A silo contains a mixture of lentils and corn. If 50 bushels of lentils were added, there would be twice as many bushels of lentils as of corn; if 150 bushels of corn were added instead, there would be the same amount of corn as of lentils. How many bushels of each were originally in the silo?

25. Mosher Children's Hospital received a \$300,000 donation that it invested in two simple-interest accounts, one earning 5.4% and the other earning 6.6%. If each account earned the same amount of annual interest, how much was invested in each account?

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Chapter Test

1. Is $(-2, 3)$ a solution of this system?

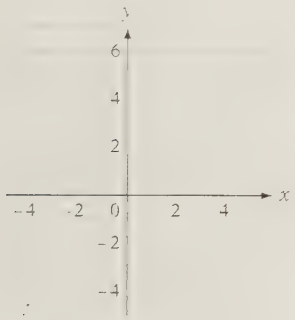
$$2x + 5y = 11$$
$$x + 3y = 7$$

2. Is $(1, -3)$ a solution of this system?

$$3x - 2y = 9$$
$$4x + y = 1$$

3. Solve by graphing:

$$3x + 2y = 6$$
$$5x + 2y = 2$$



4. Solve by substitution:

$$4x - y = 11$$
$$y = 2x - 5$$

5. Solve by substitution:

$$x = 2y + 3$$
$$3x - 2y = 5$$

6. Solve by substitution:

$$3x + 5y = 1$$
$$2x - y = 5$$

7. Solve by substitution:

$$3x - 5y = 13$$
$$x + 3y = 1$$

8. Solve by substitution:

$$2x - 4y = 1$$
$$y = \frac{1}{2}x + 3$$

9. Solve by the addition method:

$$4x + 3y = 11$$
$$5x - 3y = 7$$

10. Solve by the addition method:

$$2x - 5y = 6$$
$$4x + 3y = -1$$

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11. Solve by the addition method:
$x + 2y = 8$
$3x + 6y = 24$

12. Solve by the addition method:
$7x + 3y = 11$
$2x - 5y = 9$

13. Solve by the addition method:
$5x + 6y = -7$
$3x + 4y = -5$

14. With the wind, a plane flies 240 mi in 2 h. Against the wind, the plane requires 3 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.

15. For the first performance of a play in a community theater, 50 reserved-seat tickets and 80 general-admission tickets were sold. The total receipts were \$980. For the second performance, 60 reserved-seat tickets and 90 general-admission tickets were sold. The total receipts were \$1140. Find the price of a reserved-seat ticket and the price of a general-admission ticket.

16. Bernardo Community Library received a \$28,000 donation that it invested in two simple-interest accounts, one earning 7.6% and the other earning 6.4%. If each account earned the same amount of annual interest, how much was invested in each account?

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Cumulative Review

1. Evaluate $\frac{a^2 - b^2}{2a}$ when $a = 4$ and $b = -2$.

2. Solve: $-\frac{3}{4}x = \frac{9}{8}$

3. Given $f(x) = x^2 + 2x - 1$, find $f(2)$.

4. Simplify: $(2a^2 - 3a + 1)(2 - 3a)$

5. Simplify: $\frac{(-2x^2y)^4}{-8x^3y^2}$

6. Simplify: $(4b^2 - 8b + 4) \div (2b - 3)$

7. Simplify: $\frac{8x^{-2}y^5}{-2xy^4}$

8. Factor: $4x^2y^4 - 64y^2$

9. Solve: $(x - 5)(x + 2) = -6$

10. Simplify: $\frac{x^2 - 6x + 8}{2x^3 + 6x^2} \div \frac{2x - 8}{4x^3 + 12x^2}$

11. Simplify: $\frac{x - 1}{x + 2} + \frac{2x + 1}{x^2 + x - 2}$

12. Simplify: $\dfrac{x + 4 - \frac{7}{x - 2}}{x + 8 + \frac{21}{x - 2}}$

13. Solve: $\frac{x}{2x - 3} + 2 = \frac{-7}{2x - 3}$

14. Solve $A = P + Prt$ for r.

15. Find the x- and y-intercepts for $2x - 3y = 12$.

16. Find the slope of the line that passes through the points $(2, -3)$ and $(-3, 4)$.

17. Find the equation of the line that passes through the point $(-2, 3)$ and has slope $-\frac{3}{2}$.

18. Is $(2, 0)$ a solution of this system?

$$5x - 3y = 10$$
$$4x + 7y = 8$$

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19. Solve by substitution:
$3x - 5y = -23$
$x + 2y = -4$

20. Solve by the addition method:
$5x - 3y = 29$
$4x + 7y = -5$

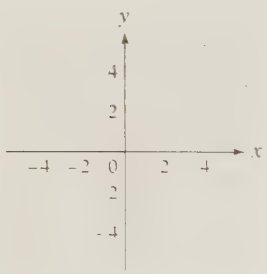
21. A total of \$8750 is invested in two simple-interest accounts. On one account, the annual simple interest rate is 9.6%; on the second account, the annual simple interest rate is 7.2%. How much should be invested in each account so that both accounts earn the same interest?

22. A passenger train leaves a train depot $\frac{1}{2}$ h after a freight train leaves the same depot. The freight train is traveling 8 mph slower than the passenger train. Find the rate of each train if the passenger train overtakes the freight train in 3 h.

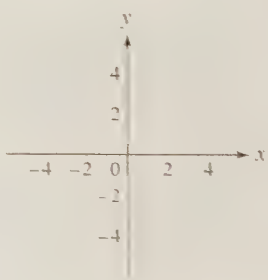
23. The length of each side of a square is extended 4 in. The area of the resulting square is 144 in^2. Find the length of a side of the original square.

24. A plane can travel 160 mph in calm air. Flying with the wind, the plane can fly 570 mi in the same amount of time as it takes to fly 390 mi against the wind. Find the rate of the wind.

25. Graph $2x - 3y = 6$.



26. Solve by graphing: $3x + 2y = 6$
$3x - 2y = 6$



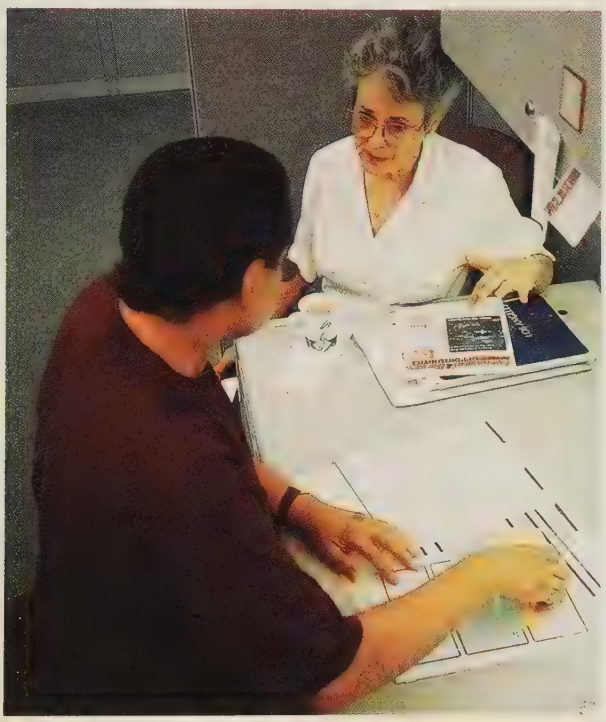
27. With the current, a motorboat can travel 48 mi in 3 h. Against the current, the boat requires 4 h to travel the same distance. Find the rate of the boat in calm water.

28. A child adds 8 g of sugar to a 50-gram serving of a breakfast cereal that is 25% sugar. What is now the percent concentration of sugar in the mixture? Round to the nearest tenth of a percent.

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CHAPTER

9 Inequalities



The human resources department of a company is involved in all areas of a company's dealings and relationships with its employees. One member of this department is the employee benefits manager, who is responsible for the selection and administration of such company programs as retirement packages, profit-sharing plans, and group medical insurance. Because a benefits officer must compare benefits packages, an understanding of inequalities is important.

Objectives

Section 9.1

To write a set using the roster method

To write a set using set-builder notation

To graph an inequality on the number line

Section 9.2

To solve an inequality using the Addition Property of Inequalities

To solve an inequality using the Multiplication Property of Inequalities

To solve application problems

Section 9.3

To solve general inequalities

To solve application problems

Section 9.4

To graph an inequality in two variables

Calculations of Pi



There are many early references to estimated values for pi. One of the earliest is from the Rhind Papyrus, which was found in Egypt in the 1800s. Scientists have estimated that these tablets were written around 1600 B.C. The Rhind Papyrus contains the estimate 3.1604 for pi.

One of the most famous calculations of pi occurred around 240 B.C. and was performed by Archimedes. The calculation was based on finding the perimeter of inscribed and circumscribed six-sided polygons (hexagons). Once the perimeter for a hexagonal figure was calculated, known formulas could be used to calculate the perimeter of polygons with twice that number of sides. Continuing in this way, Archimedes calculated the perimeters for polygons with 12, 24, 48, and 96 sides.

His calculations resulted in a value of pi between $3\frac{10}{71}$ and $3\frac{1}{7}$. You might recognize $3\frac{1}{7}$ or $\frac{22}{7}$, as an approximation for pi still used today.

Calculations to improve the accuracy of pi continued. One French mathematician, relying on Archimedes's method, estimated pi by using a polygon of 393,216 sides. A mathematician from the Netherlands estimated pi by using a polygon with over one million sides.

Around the 1650s, new mathematical methods were developed to estimate the value of pi. These methods started yielding estimates of pi that were accurate to over 70 places. By the 1850s, an estimate for pi was accurate to 200 places.

Today, thanks to more refined mathematical methods and computers, estimates of the value of pi now exceed one million places.

In 1914 an issue of *Scientific American* contained the following short note:

> "See, I have a rhyme assisting my feeble brain,
> its tasks oftimes resisting."

Can you see what this note has to do with the estimates for the value of pi?

(Each word length represents a digit in the approximation 3.141592653579.)

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9.1 Sets

Objective A To write a set using the roster method

Recall that a *set* is a collection of objects, which are called the *elements* of the set.

The **roster method** of writing a set encloses a list of the elements in braces.

The set of the last three letters of the alphabet is written {x, y, z}.

The set of the positive integers less than 5 is written {1, 2, 3, 4}.

➡ Use the roster method to write the set of integers between 0 and 10.

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

• A set can be designated by a capital letter. Note that 0 and 10 are not elements of the set.

➡ Use the roster method to write the set of natural numbers.

$A = \{1, 2, 3, 4, \ldots\}$

• The three dots mean that the pattern of numbers continues without end.

The **empty set,** or **null set,** is the set that contains no elements. The symbol $\varnothing$ or $\{\ \}$ is used to represent the empty set.

The set of people who have run a 2-minute mile is the empty set.

The **union** of two sets, written $A \cup B$, is the set that contains the elements of A and the elements of B.

➡ Find $A \cup B$, given $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$.

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

• The union of *A* and *B* contains all the elements of *A* and all the elements of *B*. Any elements that are in both *A* and *B* are listed only once.

The **intersection** of two sets, written $A \cap B$, is the set that contains the elements that are common to both A and B.

➡ Find $A \cap B$, given $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$.

$A \cap B = \{3,4\}$

• The intersection of *A* and *B* contains the elements common to *A* and *B*.

Example 1
Use the roster method to write the set of the odd positive integers less than 12.

Solution
$A = \{1, 3, 5, 7, 9, 11\}$

You Try It 1
Use the roster method to write the set of the odd negative integers greater than -10.

Your solution

Solution on p. S21

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Example 2
Use the roster method to write the set of the even positive integers.

Solution
$A = \{2, 4, 6, \ldots\}$

You Try It 2
Use the roster method to write the set of the odd positive integers.

Your solution

Example 3
Find $D \cup E$, given
$D = \{6, 8, 10, 12\}$ and
$E = \{-8, -6, 10, 12\}$.

Solution
$D \cup E = \{-8, -6, 6, 8, 10, 12\}$

You Try It 3
Find $A \cup B$, given
$A = \{-2, -1, 0, 1, 2\}$ and
$B = \{0, 1, 2, 3, 4\}$.

Your solution

Example 4
Find $A \cap B$, given
$A = \{5, 6, 9, 11\}$ and
$B = \{5, 9, 13, 15\}$.

Solution
$A \cap B = \{5, 9\}$

You Try It 4
Find $C \cap D$, given
$C = \{10, 12, 14, 16\}$ and
$D = \{10, 16, 20, 26\}$.

Your solution

Example 5
Find $A \cap B$, given
$A = \{1, 2, 3, 4\}$ and
$B = \{8, 9, 10, 11\}$.

Solution
$A \cap B = \varnothing$

You Try It 5
Find $A \cap B$, given
$A = \{-5, -4, -3, -2\}$ and
$B = \{2, 3, 4, 5\}$.

Your solution

Solutions on p. S21

Objective B To write a set using set-builder notation



POINT OF INTEREST

The symbol $\in$ was first used in the book *Arithmeticae Principia*, published in 1889. It was the first letter of the Greek word $\varepsilon\sigma\tau\iota$ which means "is." The symbols for union and intersection were also introduced at that time.

Another method of representing sets is called **set-builder notation.** Using set-builder notation, the set of all positive integers less than 10 is represented as

$\{x \mid x < 10, x \in \text{positive integers}\}$, which is read "the set of all x such that x is less than 10 and x is an element of the positive integers."

➡ Use set-builder notation to write the set of real numbers greater than 4.

$\{x \mid x > 4, x \in \text{real numbers}\}$

• "$x \in$ real numbers" is read "x is an element of the real numbers."

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Example 6
Use set-builder notation to write the set of negative integers greater than −100.

Solution
$\{x|x > -100, x \in \text{negative integers}\}$

You Try It 6
Use set-builder notation to write the set of positive even integers less than 59.

Your solution

Example 7
Use set-builder notation to write the set of real numbers less than 60.

Solution
$\{x|x < 60, x \in \text{real numbers}\}$

You Try It 7
Use set-builder notation to write the set of real numbers greater than −3.

Your solution

Solutions on p. S21

Objective C To graph an inequality on the number line

30 CT

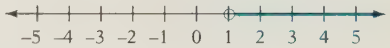
An expression that contains the symbol $>$, $<$, $\geq$ (is greater than or equal to), or $\leq$ (is less than or equal to) is called an **inequality.** An inequality expresses the relative order of two mathematical expressions. The expressions can be either numerical or variable.

$4 > 2$
$3x \leq 7$
$x^2 - 2x > y + 4$
} Inequalities

An **inequality** can be graphed on the number line.

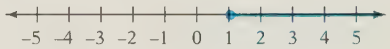
Graph: $x > 1$

The graph is the real numbers greater than 1. The circle at 1 indicates that 1 is not included in the graph.



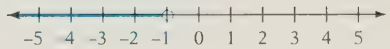
Graph: $x \geq 1$

The dot at 1 indicates that 1 is included in the solution set.



Graph: $-1 > x$

$-1 > x$ is equivalent to $x < -1$. The numbers less than -1 are to the left of -1 on the number line.



TAKE NOTE

For the remainder of this section, all variables will represent real numbers. Using this convention, the expression $\{x|x > 4, x \in \text{real numbers}\}$ will be written as $\{x|x > 4\}$, as shown in the example at the right.

The union of two sets is the set that contains all the elements of each set.

Graph: $\{x|x > 4\} \cup \{x|x < 1\}$

The graph is the numbers greater than 4 and the numbers less than 1.

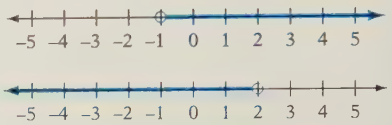


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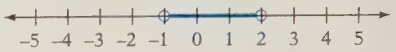
The intersection of two sets is the set that contains the elements common to both sets.

➡ Graph: $\{x|x > -1\} \cap \{x|x < 2\}$

The graphs of $\{x|x > -1\}$ and $\{x|x < 2\}$ are shown at the right.



The graph of $\{x|x > -1\} \cap \{x|x < 2\}$ is the numbers between -1 and 2.



Example 8

Graph: $x < 5$

Solution

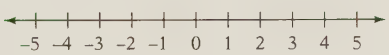
The graph is the numbers less than 5.



You Try It 8

Graph: $-2 < x$

Your solution

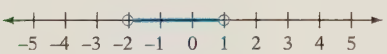


Example 9

Graph: $\{x|x > -2\} \cap \{x|x < 1\}$

Solution

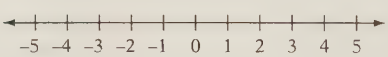
The graph is the numbers between -2 and 1.



You Try It 9

Graph: $\{x|x > -1\} \cup \{x|x < -3\}$

Your solution



Example 10

Graph: $\{x|x \leq 5\} \cup \{x|x \geq -3\}$

Solution

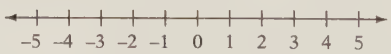
The graph is the real numbers.



You Try It 10

Graph: $\{x|x < 2\} \cup \{x| \geq -2\}$

Your solution



Example 11

Graph: $\{x|x > 3\} \cup \{x|x < 1\}$

Solution

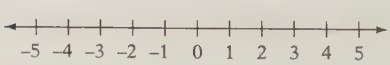
The graph is the numbers greater than 3 or the numbers less than 1.



You Try It 11

Graph: $\{x|x \leq 4\} \cap \{x|x \geq -4\}$

Your solution



Solutions on p. S21

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9.1 Exercises

Objective A

Use the roster method to write the set.

1. the integers between 15 and 22
2. the integers between −10 and −4
3. the odd integers between 8 and 18
4. the even integers between −11 and −1
5. the letters of the alphabet between a and d
6. the letters of the alphabet between p and v

Find $A \cup B$.

7. $A = \{3, 4, 5\}$ $B = \{4, 5, 6\}$
8. $A = \{-3, -2, -1\}$ $B = \{-2, -1, 0\}$
9. $A = \{-10, -9, -8\}$ $B = \{8, 9, 10\}$
10. $A = \{a, b, c\}$ $B = \{x, y, z\}$
11. $A = \{a, b, d, e\}$ $B = \{c, d, e, f\}$
12. $A = \{m, n, p, q\}$ $B = \{m, n, o\}$
13. $A = \{1, 3, 7, 9\}$ $B = \{7, 9, 11, 13\}$
14. $A = \{-3, -2, -1\}$ $B = \{-1, 1, 2\}$

Find $A \cap B$.

15. $A = \{3, 4, 5\}$ $B = \{4, 5, 6\}$
16. $A = \{-4, -3, -2\}$ $B = \{-6, -5, -4\}$
17. $A = \{-4, -3, -2\}$ $B = \{2, 3, 4\}$
18. $A = \{1, 2, 3, 4\}$ $B = \{1, 2, 3, 4\}$
19. $A = \{a, b, c, d, e\}$ $B = \{c, d, e, f, g\}$
20. $A = \{m, n, o, p\}$ $B = \{k, l, m, n\}$

Objective B

Use set-builder notation to write the set.

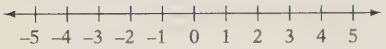
21. the negative integers greater than −5
22. the positive integers less than 5
23. the integers greater than 30
24. the integers less than −70
25. the even integers greater than 5
26. the odd integers less than −2
27. the real numbers greater than 8
28. the real numbers less than 57

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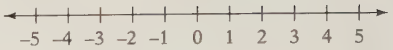
Objective C

Graph.

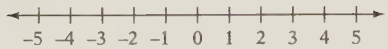
29. $x > 2$



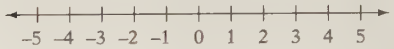
30. $x \geq -1$



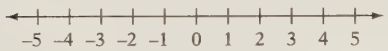
31. $0 \geq x$



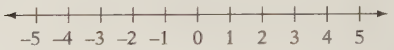
32. $4 > x$



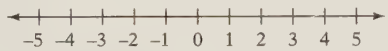
33. $\{x \mid x > -2\} \cup \{x \mid x < -4\}$



34. $\{x \mid x > 4\} \cup \{x \mid x < -2\}$



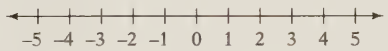
35. $\{x \mid x > -2\} \cap \{x < 4\}$



36. $\{x \mid x > -3\} \cap \{x \mid x < 3\}$



37. $\{x \mid x \geq -2\} \cup \{x \mid x < 4\}$



38. $\{x \mid x > 0\} \cup \{x \mid x \leq 4\}$



APPLYING THE CONCEPTS

39. Explain how to find the union of two sets.

40. Explain how to find the intersection of two sets.

41. Determine whether the statement is always true, sometimes true, or never true.
- **a.** Given that $a > 0$ and $b < 0$, then $ab > 0$.
- **b.** Given that $a < 0$, then $a^2 > 0$.
- **c.** Given that $a > 0$ and $b < 0$, then $a^2 > b$.

42. By trying various sets, make a conjecture as to whether the union of two sets is
- **a.** a commutative operation
- **b.** an associative operation

43. By trying various sets, make a conjecture as to whether the intersection of two sets is
- **a.** a commutative operation
- **b.** an associative operation

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9.2 The Addition and Multiplication Properties of Inequalities

Objective A To solve an inequality using the Addition Property of Inequalities

The **solution set of an inequality** is a set of numbers, each element of which, when substituted for the variable, results in a true inequality.

The inequality at the right is true if the variable is replaced by 7, 9.3, or $\frac{15}{2}$.

$$x + 5 > 8$$

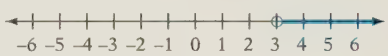
$$\left.\begin{array}{l} 7 + 5 > 8 \\ 9.3 + 5 > 8 \\ \frac{15}{2} + 5 > 8 \end{array}\right\} \text{True inequalities}$$

The inequality $x + 5 > 8$ is false if the variable is replaced by 2, 1.5, or $-\frac{1}{2}$.

$$\left.\begin{array}{l} 2 + 5 > 8 \\ 1.5 + 5 > 8 \\ -\frac{1}{2} + 5 > 8 \end{array}\right\} \text{False inequalities}$$

There are many values of the variable x that will make the inequality $x + 5 > 8$ true. The solution set of $x + 5 > 8$ is any number greater than 3.

At the right is the graph of the solution set of $x + 5 > 8$.



In solving an inequality, the goal is to rewrite the given inequality in the form *variable* $>$ *constant* or *variable* $<$ *constant*. The Addition Property of Inequalities is used to rewrite an inequality in this form.

Addition Property of Inequalities

The same term can be added to each side of an inequality without changing the solution set of the inequality.

If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

The Addition Property of Inequalities also holds true for an inequality containing the symbol $\geq$ or $\leq$.

The Addition Property of Inequalities is used when, in order to rewrite an inequality in the form *variable* $>$ *constant* or *variable* $<$ *constant*, we must remove a term from one side of the inequality. Add the opposite of that term to each side of the inequality.

Solve: $x - 4 < -3$

$$\begin{array}{rll} x - 4 < -3 & & \\ x - 4 + 4 < -3 + 4 & \bullet & \text{Add 4 to each side of the inequality.} \\ x < 1 & \bullet & \text{Simplify.} \end{array}$$

At the right is the graph of the solution set of $x - 4 < -3$.



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Because subtraction is defined in terms of addition, the Addition Property of Inequalities allows the same term to be subtracted from each side of an inequality.

Solve: $5x - 6 \le 4x - 4$

$$5x - 6 \le 4x - 4$$

$$5x - 4x - 6 \le 4x - 4x - 4$$ • Subtract $4x$ from each side of the inequality.

$$x - 6 \le -4$$ • Simplify.

$$x - 6 + 6 \le -4 + 6$$ • Add 6 to each side of the inequality.

$$x \le 2$$ • Simplify.

Example 1
Solve and graph the solution set of $3 < x + 5$.

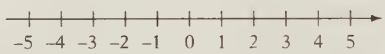
Solution

$$3 < x + 5$$
$$3 - 5 < x + 5 - 5$$
$$-2 < x$$



You Try It 1
Solve and graph the solution set of $x + 2 < -2$.

Your solution



Example 2
Solve: $7x - 14 \le 6x - 16$

Solution

$$7x - 14 \le 6x - 16$$
$$7x - 6x - 14 \le 6x - 6x - 16$$
$$x - 14 \le -16$$
$$x - 14 + 14 \le -16 + 14$$
$$x \le -2$$

You Try It 2
Solve: $5x + 3 > 4x + 5$

Your solution

Solutions on pp. S21–S22

Objective B To solve an inequality using the Multiplication Property of Inequalities

30 CT

In solving an inequality, the goal is to rewrite the given inequality in the form *variable* > *constant* or *variable* < *constant*. The Multiplication Property of Inequalities is used when, in order to rewrite an inequality in this form, we must remove a coefficient from one side of the inequality.

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Multiplication Property of Inequalities

Each side of an inequality can be multiplied by the same positive number without changing the solution set of the inequality.

If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.

If each side of an inequality is multiplied by the same negative number and the inequality symbol is reversed, then the solution set of the inequality is not changed.

If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

TAKE NOTE

Any time an inequality is multiplied or divided by a negative number, the inequality symbol must be reversed. Compare the next two examples.

$2x < -4$
$\frac{2x}{2} < \frac{-4}{2}$ Divide each side by *positive* 2. Inequality *is not* reversed.
$x < -2$

$-2x < 4$
$\frac{-2x}{-2} > \frac{4}{-2}$ Divide each side by *negative* 2. Inequality *is* reversed.
$x > -2$

$5 > 4$
$5(2) > 4(2)$ • Multiply by *positive* 2.
$10 > 8$ • Still a true inequality

$6 < 9$
$6(-3) > 9(-3)$ • Multiply by *negative* 3 and *reverse* the inequality.
$-18 > -27$ • Still a true inequality

The Multiplication Property of Inequalities also holds true for an inequality containing the symbol $\geq$ or $\leq$.

➡ Solve $-\frac{3}{2}x \leq 6$ and graph the solution set.

$$-\frac{3}{2}x \leq 6$$

$$-\frac{2}{3}\left(-\frac{3}{2}x\right) \geq -\frac{2}{3}(6)$$

$$x \geq -4$$

• Multiply each side of the inequality by $-\frac{2}{3}$. Because $-\frac{2}{3}$ is a negative number, the inequality symbol must be reversed.



• Graph $x \geq -4$.

Because division is defined in terms of multiplication, the Multiplication Property of Inequalities allows each side of an inequality to be divided by a nonzero constant.

➡ Solve: $-4 < 6x$

$$-4 < 6x$$

$$\frac{-4}{6} < \frac{6x}{6}$$

• Divide each side of the inequality by 6.

$$-\frac{2}{3} < x$$

• Simplify: $\frac{-4}{6} = -\frac{2}{3}$

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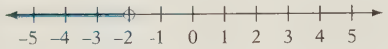
Example 3 Solve and graph the solution set of $-7x > 14$.

Solution

$$-7x > 14$$

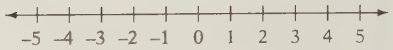
$$\frac{-7x}{-7} < \frac{14}{-7}$$

$$x < -2$$



You Try It 3 Solve and graph the solution set of $-3x > -9$.

Your solution



Example 4 Solve: $-\frac{5}{8}x \le \frac{5}{12}$

Solution

$$-\frac{5}{8}x \le \frac{5}{12}$$

$$-\frac{8}{5}\left(-\frac{5}{8}x\right) \ge -\frac{8}{5}\left(\frac{5}{12}\right)$$

$$x \ge -\frac{2}{3}$$

You Try It 4 Solve: $-\frac{3}{4}x \ge 18$

Your solution

Solutions on p. S22

Objective C To solve application problems

30 CT

Example 5

A student must have at least 450 points out of 500 points on five tests to receive an A in a course. One student's results on the first four tests were 94, 87, 77, and 95. What scores on the last test will enable this student to receive an A in the course?

Strategy

To find the scores, write and solve an inequality using N to represent the possible scores on the last test.

Solution

Total number of points on the 5 tests	is greater than or equal to	450

$$94 + 87 + 77 + 95 + N \ge 450$$

$$353 + N \ge 450$$

$$353 - 353 + N \ge 450 - 353$$

$$N \ge 97$$

The student's score on the last test must be equal to or greater than 97.

You Try It 5

An appliance dealer will make a profit on the sale of a television set if the cost of the new set is less than 70% of the selling price. What selling prices will enable the dealer to make a profit on a television set that costs the dealer \$314?

Your strategy

Your solution

Solution on p. S22

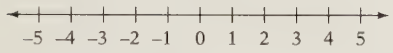
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9.2 Exercises

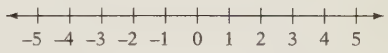
Objective A

Solve and graph the solution set.

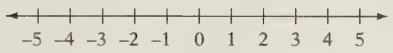
1. $x + 1 < 3$



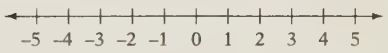
2. $y + 2 < 2$



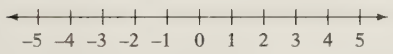
3. $x - 5 > -2$



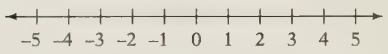
4. $x - 3 > -2$



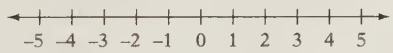
5. $7 \le n + 4$



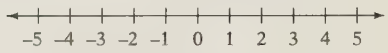
6. $3 \le 5 + x$



7. $x - 6 \le -10$



8. $y - 8 \le -11$



Solve.

9. $y - 3 \ge -12$

10. $x + 8 \ge -14$

11. $3x - 5 < 2x + 7$

12. $5x + 4 < 4x - 10$

13. $8x - 7 \ge 7x - 2$

14. $3n - 9 \ge 2n - 8$

15. $2x + 4 < x - 7$

16. $9x + 7 < 8x - 7$

17. $4x - 8 \le 2 + 3x$

18. $5b - 9 < 3 + 4b$

19. $6x + 4 \ge 5x - 2$

20. $7x - 3 \ge 6x - 2$

21. $2x - 12 > x - 10$

22. $3x + 9 > 2x + 7$

23. $d + \frac{1}{2} < \frac{1}{3}$

24. $x - \frac{3}{8} < \frac{5}{6}$

25. $x + \frac{5}{8} \ge -\frac{2}{3}$

26. $y + \frac{5}{12} \ge -\frac{3}{4}$

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27. $x - \frac{3}{8} < \frac{1}{4}$

28. $y + \frac{5}{9} \leq \frac{5}{6}$

29. $2x - \frac{1}{2} < x + \frac{3}{4}$

30. $6x - \frac{1}{3} \leq 5x - \frac{1}{2}$

31. $3x + \frac{5}{8} > 2x + \frac{5}{6}$

32. $4b - \frac{7}{12} \geq 3b - \frac{9}{16}$

33. $3.8x < 2.8x - 3.8$

34. $1.2x < 0.2x - 7.3$

35. $x + 5.8 \leq 4.6$

36. $n - 3.82 \leq 3.95$

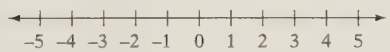
37. $x - 3.5 < 2.1$

38. $x - 0.23 \leq 0.47$

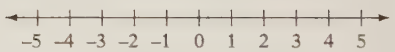
Objective B

Solve and graph the solution set.

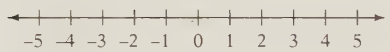
39. $3x < 12$



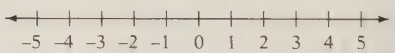
40. $8x \leq -24$



41. $15 \leq 5y$



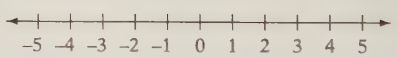
42. $-48 < 24x$



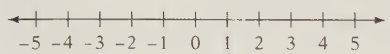
43. $16x \leq 16$



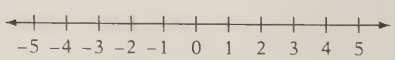
44. $3x > 0$



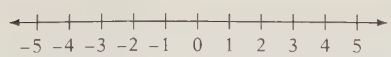
45. $-8x > 8$



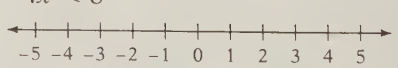
46. $-2n \leq -8$



47. $-6b > 24$



48. $-4x < 8$



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49. $-5y \geq 0$ **50.** $-3z < 0$ **51.** $7x > 2$ **52.** $6x \leq -1$

53. $2x \leq -5$ **54.** $\frac{5}{6}n < 15$ **55.** $\frac{3}{4}x < 12$ **56.** $\frac{2}{3}y \geq 4$

57. $10 \leq \frac{5}{8}x$ **58.** $4 \geq \frac{2}{3}x$ **59.** $-\frac{3}{7}x \leq 6$ **60.** $-\frac{2}{11}b \geq -6$

61. $-\frac{4}{7}x \geq -12$ **62.** $\frac{2}{3}n < \frac{1}{2}$ **63.** $-\frac{3}{5}x < 0$ **64.** $-\frac{2}{3}x \geq 0$

65. $-\frac{3}{8}x \geq \frac{9}{14}$ **66.** $-\frac{3}{5}x < -\frac{6}{7}$ **67.** $-\frac{4}{5}x < -\frac{8}{15}$ **68.** $-\frac{3}{4}y \geq -\frac{5}{8}$

69. $-\frac{8}{9}x \geq -\frac{16}{27}$ **70.** $1.5x \leq 6.30$ **71.** $2.3x \leq 5.29$ **72.** $-3.5d > 7.35$

73. $-0.24x > 0.768$ **74.** $4.25m > -34$ **75.** $-3.9x \geq -19.5$

Objective C Application Problems

76. Three-fifths of a number is greater than two-thirds. Find the smallest integer that satisfies this inequality.

77. To be eligible for a basketball tournament, a basketball team must win at least 60% of its remaining games. If the team has 17 games remaining, how many games must the team win to qualify for the tournament?

78. To avoid a tax penalty, at least 90% of a self-employed person's total annual income tax liability must be paid by April 15. What amount of income tax must a person with an annual income tax liability of $3500 pay?

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79. A service organization will receive a bonus of $200 for collecting more than 1850 lb of aluminum cans during its four collection drives. On the first three drives, the organization collected 505 lb, 493 lb, and 412 lb. How many pounds of cans must the organization collect on the fourth drive to receive the bonus?

80. Computer software engineers are fond of saying that software takes at least twice as long to develop as they think it will. Applying that saying, how many hours will it take to develop a software product that an engineer thinks can be finished in 50 h?

81. A government agency recommends a minimum daily allowance of vitamin C of 60 mg. How many additional milligrams of vitamin C does a person who drank a glass of orange juice with 10 mg of vitamin C need in order to satisfy the recommended daily allowance?

82. To pass a course with a B grade, a student must have an average of 80 points on five tests. The student's grades on the first four tests were 75, 83, 86, and 78. What scores can the student receive on the fifth test to earn a B grade?

83. A professor scores all tests with a maximum of 100 points. To earn an A grade in this course, a student must have an average of 92 on four tests. A student's grades on the first three tests were 89, 86, and 90. Can this student earn an A grade?

84. A health official recommends a maximum cholesterol level of 200 units. How many units must a patient with a cholesterol level of 275 units reduce her cholesterol level to satisfy the recommended maximum level?

APPLYING THE CONCEPTS

Given that $a > b$ and that a and b are real numbers, determine for which real numbers c the statement is true. Use set-builder notation to write the answer.

85. $ac > bc$

86. $ac < bc$

87. $a + c > b + c$

88. $a + c < b + c$

89. $\frac{a}{c} > \frac{b}{c}$

90. $\frac{a}{c} < \frac{b}{c}$

91. In your own words, state the Addition Property of Inequalities.

92. In your own words, state the Multiplication Property of Inequalities.

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9.3 General Inequalities

Objective A To solve general inequalities

Solving an inequality frequently requires application of both the Addition and the Multiplication Properties of Inequalities.

Solve: $4y - 3 \geq 6y + 5$

$$4y - 3 \geq 6y + 5$$

$$4y - 6y - 3 \geq 6y - 6y + 5$$ Subtract $6y$ from each side of the inequality.

$$-2y - 3 \geq 5$$ Simplify.

$$-2y - 3 + 3 \geq 5 + 3$$ Add 3 to each side of the inequality.

$$-2y \geq 8$$ Simplify.

$$\frac{-2y}{-2} \leq \frac{8}{-2}$$ Divide each side of the inequality by -2. Because -2 is a negative number, the inequality symbol must be reversed.

$$y \leq -4$$

When an inequality contains parentheses, one of the steps in solving the inequality requires the use of the Distributive Property.

Solve: $-2(x - 7) > 3 - 4(2x - 3)$

$$-2(x - 7) > 3 - 4(2x - 3)$$

$$-2x + 14 > 3 - 8x + 12$$ Use the Distributive Property to remove parentheses.

$$-2x + 14 > -8x + 15$$ Simplify.

$$-2x + 8x + 14 > -8x + 8x + 15$$ Add $8x$ to each side of the inequality.

$$6x + 14 > 15$$ Simplify.

$$6x + 14 - 14 > 15 - 14$$ Subtract 14 from each side of the inequality.

$$6x > 1$$ Simplify.

$$\frac{6x}{6} > \frac{1}{6}$$ Divide each side of the inequality by 6.

$$x > \frac{1}{6}$$

Example 1 Solve: $7x - 3 \leq 3x + 17$

Solution

$$7x - 3 \leq 3x + 17$$
$$7x - 3x - 3 \leq 3x - 3x + 17$$
$$4x - 3 \leq 17$$
$$4x - 3 + 3 \leq 17 + 3$$
$$4x \leq 20$$
$$\frac{4x}{4} \leq \frac{20}{4}$$
$$x \leq 5$$

You Try It 1 Solve: $5 - 4x > 9 - 8x$

Your solution

Solution on p. S22

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Example 2

Solve:

$3(3 - 2x) \geq -5x - 2(3 - x)$

Solution

$$\begin{aligned} 3(3 - 2x) &\geq -5x - 2(3 - x) \\ 9 - 6x &\geq -5x - 6 + 2x \\ 9 - 6x &\geq -3x - 6 \\ 9 - 6x + 3x &\geq -3x + 3x - 6 \\ 9 - 3x &\geq -6 \\ 9 - 9 - 3x &\geq -6 - 9 \\ -3x &\geq -15 \\ \frac{-3x}{-3} &\leq \frac{-15}{-3} \end{aligned}$$

You Try It 2

Solve:

$8 - 4(3x + 5) \leq 6(x - 8)$

Your solution

Solution on p. S22

Objective B To solve application problems

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Example 3

A rectangle is 10 ft wide and $(2x + 4)$ ft long. Express as an integer the maximum length of the rectangle when the area is less than 200 ft^2. (The area of a rectangle is equal to its length times its width.)

Strategy

To find the maximum length:

Replace the variables in the area formula by the given values and solve for x.

Replace the variable in the expression $2x + 4$ with the value found for x.

Solution

Length times width	is less than	200 ft^2

$$\begin{aligned} 10(2x + 4) &< 200 \\ 20x + 40 &< 200 \\ 20x + 40 - 40 &< 200 - 40 \\ 20x &< 160 \\ \frac{20x}{20} &< \frac{160}{20} \\ x &< 8 \end{aligned}$$

The length is $(2x + 4)$ ft. Because $x < 8$, $2x + 4 < 2(8) + 4 = 20$. Therefore, the length is less than 20 ft. The maximum length is 19 ft.

You Try It 3

Company A rents cars for \$8 a day and \$.10 for every mile driven. Company B rents cars for \$10 a day and \$.08 per mile driven. You want to rent a car for one week. What is the maximum number of miles you can drive a Company A car if it is to cost you less than a Company B car?

Your strategy

Your solution

Solution on p. S22

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9.3 Exercises

Objective A

Solve.

1. $4x - 8 < 2x$
2. $7x - 4 < 3x$
3. $2x - 8 > 4x$
4. $3y + 2 > 7y$
5. $8 - 3x \leq 5x$
6. $10 - 3x \leq 7x$
7. $3x + 2 > 5x - 8$
8. $2n - 9 \geq 5n + 4$
9. $5x - 2 < 3x - 2$
10. $8x - 9 > 3x - 9$
11. $0.1(180 + x) > x$
12. $x > 0.2(50 + x)$
13. $2(2y - 5) \leq 3(5 - 2y)$
14. $2(5x - 8) \leq 7(x - 3)$
15. $5(2 - x) > 3(2x - 5)$
16. $4(3d - 1) > 3(2 - 5d)$
17. $4 - 3(3 - n) \leq 3(2 - 5n)$
18. $15 - 5(3 - 2x) \leq 4(x - 3)$
19. $2x - 3(x - 4) \geq 4 - 2(x - 7)$
20. $4 + 2(3 - 2y) \leq 4(3y - 5) - 6y$

Objective B Application Problems

21. The sales agent for a jewelry company is offered a flat monthly salary of \$3200 or a salary of \$1000 plus an 11% commission on the selling price of each item sold by the agent. If the agent chooses the \$3200, what dollar amount does the agent expect to sell in one month?

22. A baseball player is offered an annual salary of \$200,000 or a base salary of \$100,000 plus a bonus of \$1000 for each hit over 100 hits. How many hits must the baseball player make to earn more than \$200,000?

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23. A computer bulletin board service charges a flat fee of \$10 per month or a fee of \$4 per month plus \$.10 for each minute the service is used. How many minutes must a person use this service to exceed \$10?

24. A site licensing fee for a computer program is \$1500. Paying this fee allows the company to use the program at any computer terminal within the company. Alternatively, the company can choose to pay \$200 for each individual computer it has. How many individual computers must a company have for the site license to be more economical for the company?

25. For a product to be labeled orange juice, a state agency requires that at least 80% of the drink be real orange juice. How many ounces of artificial flavors can be added to 32 oz of real orange juice and have it still be legal to label the drink orange juice?

26. Grade A hamburger cannot contain more than 20% fat. How much fat can a butcher mix with 300 lb of lean meat to meet the 20% requirement?

27. A shuttle service taking skiers to a ski area charges \$8 per person each way. Four skiers are debating whether to take the shuttle bus or rent a car for \$45 plus \$.25 per mile. Assuming that the skiers will share the cost of the car and that they want the least expensive method of transportation, find how far away the ski area is if they choose the shuttle service.

APPLYING THE CONCEPTS

28. Determine whether the statement is always true, sometimes true, or never true, given that a, b, and c are real numbers.

a. If $a > b$, then $-a > -b$.

b. If $a < b$, then $ac < bc$.

c. If $a > b$, then $a + c > b + c$.

d. If $a \neq 0$, $b \neq 0$, and $a > b$, then $\frac{1}{a} > \frac{1}{b}$.

Use the roster method to list the set of positive integers that are solutions of the inequality.

29. $7 - 2b \leq 15 - 5b$

30. $-6(2 - d) \geq 4d - 9$

Use the roster method to list the set of integers that are common to the solution sets of the two inequalities.

31. $5x - 12 \leq x + 8$
$3x - 4 \geq 2 + x$

32. $3(x + 2) > 9x - 2$
$4(x + 5) > 3(x + 6)$

33. Determine the solution set of $2 - 3(x + 4) < 5 - 3x$.

34. Determine the solution set of $3x + 2(x - 1) > 5(x + 1)$.

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9.4 Graphing Linear Inequalities

Objective A To graph an inequality in two variables

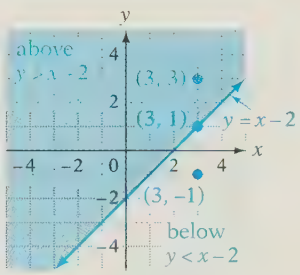
26 CT

POINT OF INTEREST

Linear inequalities play an important role in applied mathematics. They are used in a branch of mathematics called *linear programming*, which was developed during World War II to solve problems in supplying the Air Force with the machine parts necessary to keep planes flying. Today, its applications have been broadened to many other disciplines.

The graph of the linear equation $y = x - 2$ separates a plane into three sets:

the set of points on the line
the set of points above the line
the set of points below the line



The point (3, 1) is a solution of $y = x - 2$.

$$\begin{array}{r|l} y = x - 2 \\ \hline 1 & 3 - 2 \\ 1 = 1 \end{array}$$

The point (3, 3) is a solution of $y > x - 2$.

$$\begin{array}{r|l} y > x - 2 \\ \hline 3 & 3 - 2 \\ 3 > 1 \end{array}$$

Any point above the line is a solution of $y > x - 2$.

The point (3, −1) is a solution of $y < x - 2$.

$$\begin{array}{r|l} y < x - 2 \\ \hline -1 & 3 - 2 \\ -1 < 1 \end{array}$$

Any point below the line is a solution of $y < x - 2$.

The solution set of $y = x - 2$ is all points on the line. The solution set of $y > x - 2$ is all points above the line. The solution set of $y < x - 2$ is all points below the line. The solution set of an inequality in two variables is a **half-plane.**

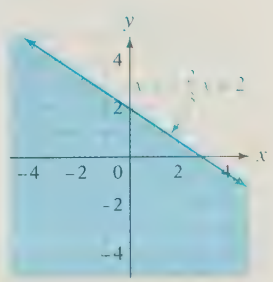
The following illustrates the procedure for graphing a linear inequality.

➡ Graph the solution set of $2x + 3y \le 6$.

Solve the inequality for y.

$$\begin{aligned} 2x + 3y &\le 6 \\ 2x - 2x + 3y &\le -2x + 6 && \text{Subtract } 2x \text{ from each side.} \\ 3y &\le -2x + 6 && \text{Simplify.} \\ \frac{3y}{3} &\le \frac{-2x + 6}{3} && \text{Divide each side by 3.} \\ y &\le -\frac{2}{3}x + 2 && \text{Simplify.} \end{aligned}$$

Change the inequality to an equality and graph $y = -\frac{2}{3}x + 2$. If the inequality is **≥ or ≤,** the line is in the solution set and is shown by a **solid line.** If the inequality is **> or <**, the line is not a part of the solution set and is shown by a **dotted line.**



If the inequality is **> or ≥,** shade the **upper half-plane.** If the inequality is **< or ≤,** shade the **lower half-plane.**

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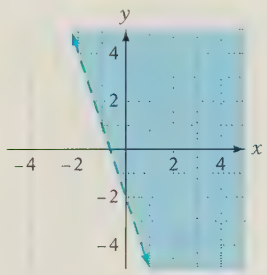
Example 1

Graph the solution set of $3x + y > -2$.

Solution

$$\begin{aligned} 3x + y &> -2 \\ 3x - 3x + y &> -3x - 2 \\ y &> -3x - 2 \end{aligned}$$

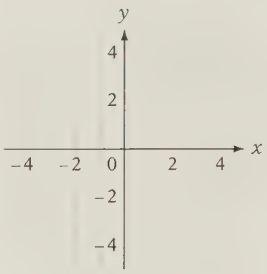
Graph $y = -3x - 2$ as a dotted line. Shade the upper half-plane.



You Try It 1

Graph the solution set of $x - 3y < 2$.

Your solution



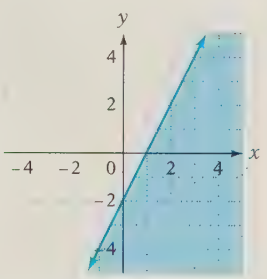
Example 2

Graph the solution set of $2x - y \geq 2$.

Solution

$$\begin{aligned} 2x - y &\geq 2 \\ 2x - 2x - y &\geq -2x + 2 \\ -y &\geq -2x + 2 \\ -1(-y) &\leq -1(-2x + 2) \\ y &\leq 2x - 2 \end{aligned}$$

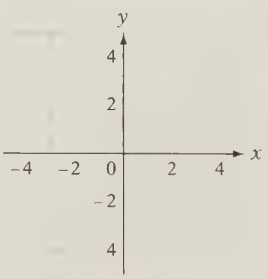
Graph $y = 2x - 2$ as a solid line. Shade the lower half-plane.



You Try It 2

Graph the solution set of $2x - 4y \leq 8$.

Your solution

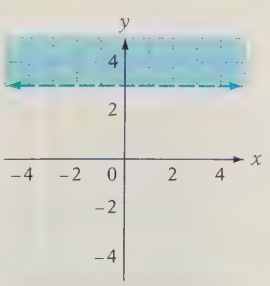


Example 3

Graph the solution set of $y > 3$.

Solution

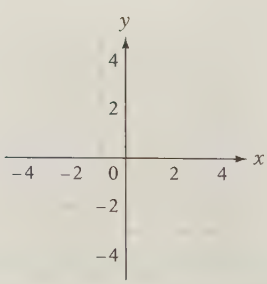
Graph $y = 3$ as a dotted line. Shade the upper half-plane.



You Try It 3

Graph the solution set of $x < 3$.

Your solution



Solutions on pp. S22–S23

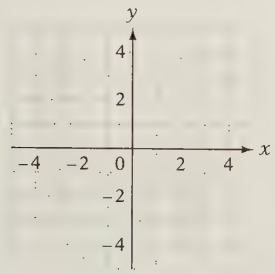
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9.4 Exercises

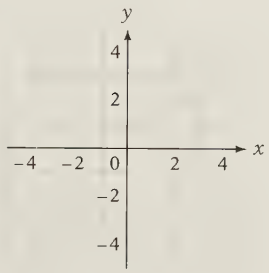
Objective A

Graph the solution set.

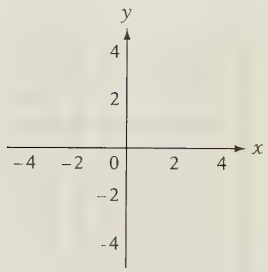
1. $x + y > 4$



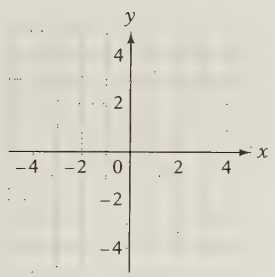
2. $x - y > -3$



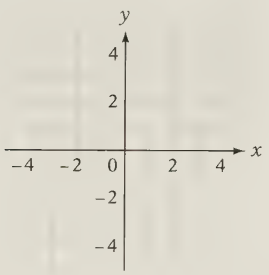
3. $2x - y < -3$



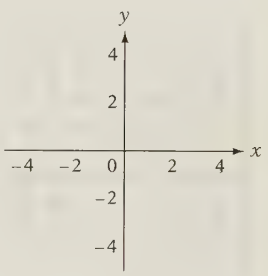
4. $3x - y < 9$



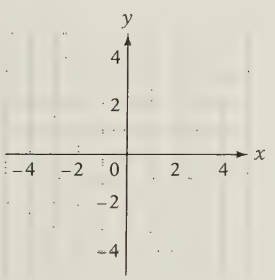
5. $2x + y \geq 4$



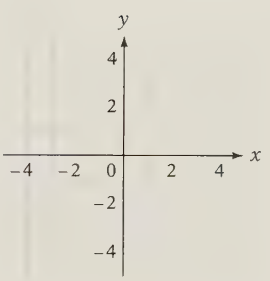
6. $3x + y \geq 6$



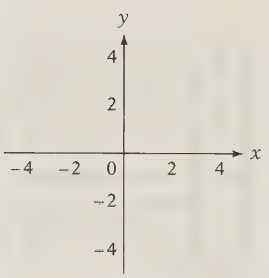
7. $y \leq -2$



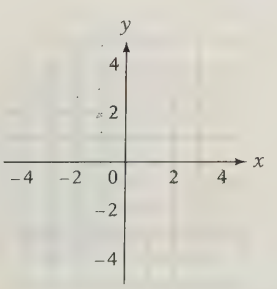
8. $y > 3$



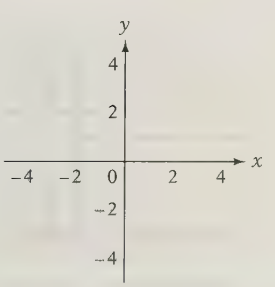
9. $3x - 2y < 8$



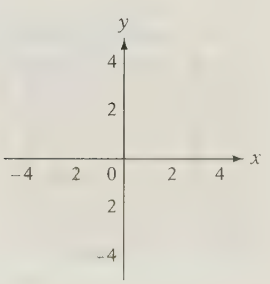
10. $5x + 4y > 4$



11. $-3x - 4y \geq 4$

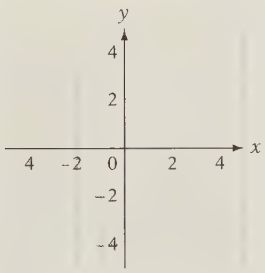


12. $-5x - 2y \geq 8$

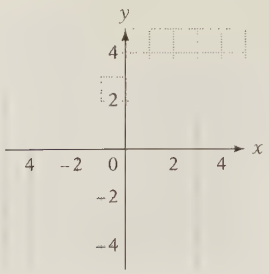


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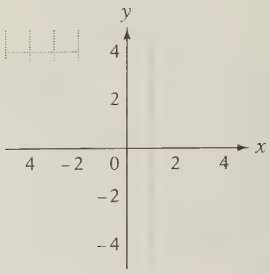
13. $6x + 5y \leq -10$



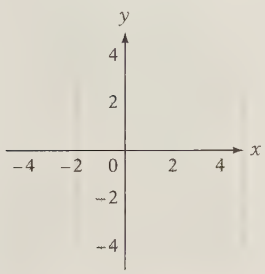
14. $2x + 2y \leq -4$



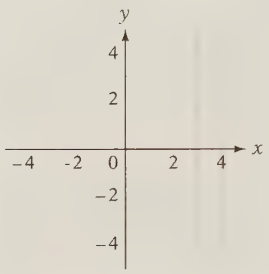
15. $-4x + 3y < -12$



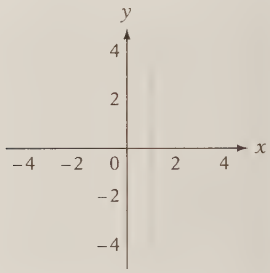
16. $-4x + 5y < 15$



17. $-2x + 3y \leq 6$



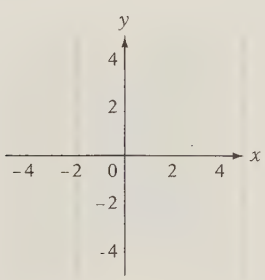
18. $3x - 4y > 12$



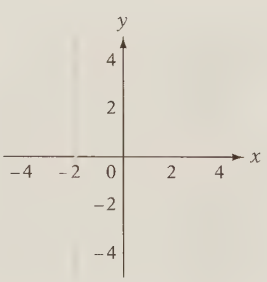
APPLYING THE CONCEPTS

Graph the solution set.

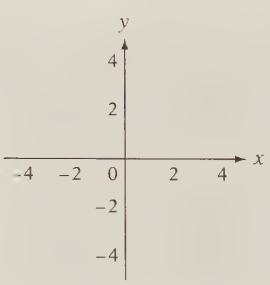
19. $\frac{x}{4} + \frac{y}{2} > 1$



20. $2x - 3(y + 1) > y - (4 - x)$

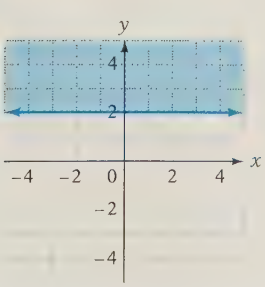


21. $4y - 2(x + 1) \geq 3(y - 1) + 3$

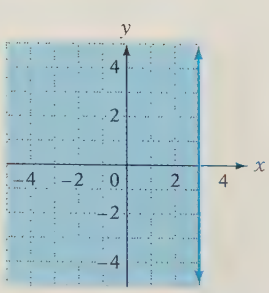


Write the inequality given its graph.

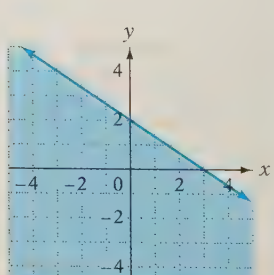
22.



23.



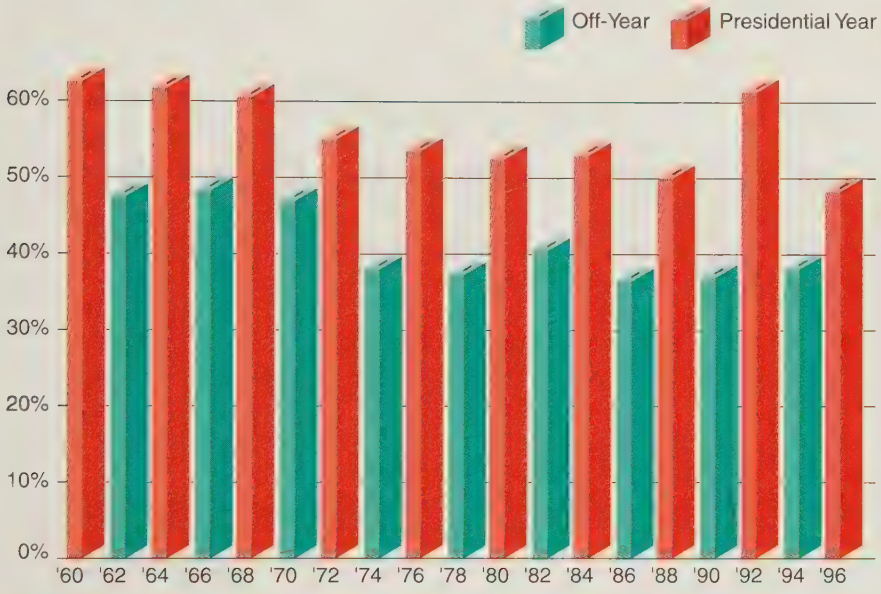
24.



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Focus on Problem Solving

Graphing Data Graphs are very useful in displaying data. Highs and lows can be shown and trends observed from the graph. By showing the trends, predictions can be made for a future time by assuming that these trends will continue.



The bar graph at the left depicts U.S. voter turnout during presidential election years and during off-year elections.

The heights of the bars indicate the percents of eligible voters who cast ballots.

1. During which presidential election years was voter turnout greater than 60%?
2. During which off-year elections was voter turnout less than 40%?
3. During which presidential election years was voter turnout less than 50%?
4. Write a paragraph describing any pattern you see in the graph.
5. Using any patterns that you may have observed, make a prediction about voter turnout in 1998 and 2000.

The table at the right shows the number of mobile phone users (in millions) for the years 1988 to 1996.

Year	*Number*	*Year*	*Number*
'88	2.1	'93	16.0
'89	3.5	'94	24.1
'90	5.3	'95	33.8
'91	7.6	'96	44.0
'92	11.0		

6. Make a bar graph of the number of mobile phone users.
7. Note the trend that this bar graph indicates. Do you expect the number of mobile phone users to be closer to 48 million, 55 million, or 80 million in 1997?

The federal budget deficit (in billions) since 1980 is shown below. The deficits for the years 1997, 1998, and 1999 are estimates made by the Office of Management and Budget.

Year	*Deficit*	*Year*	*Deficit*	*Year*	*Deficit*	*Year*	*Deficit*	*Year*	*Deficit*
'80	73.8	'84	185.3	'88	155.2	'92	290.4	'96	107.3
'81	78.9	'85	212.3	'89	152.5	'93	255.0	'97	67.0
'82	127.9	'86	221.2	'90	221.2	'94	203.1	'98	90.0
'83	207.8	'87	149.8	'91	269.4	'95	163.9	'99	90.0

8. Make a bar graph showing the federal budget deficits.
9. During which consecutive three-year period was the deficit the greatest?
10. Write a paragraph describing any pattern you see in the graph.

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Projects and Group Activities

Mean and Standard Deviation

An automotive engineer tests the miles-per-gallon ratings of 15 cars and records the results as follows:

25 22 21 27 25 35 29 31 25 26 21 39 34 32 28

The **mean** of the data is the sum of the measurements divided by the number of measurements. The symbol for the mean is $\bar{x}$.

$$\text{Mean} = \bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$$

To find the mean for the data above, add the numbers and then divide by 15.

$$\bar{x} = \frac{25 + 22 + 21 + 27 + 25 + 35 + 29 + 31 + 25 + 26 + 21 + 39 + 34 + 32 + 28}{15}$$

$$= \frac{420}{15} = 28$$

The mean number of miles per gallon for the 15 cars tested was 28 mi/gal.

The mean is one of the most frequently computed averages. It is the one that is commonly used to calculate a student's performance in a class.

The scores for a history student on 5 tests were 78, 82, 91, 87, and 93. What was the mean score for this student?

To find the mean, add the numbers. Then divide by 5.

$$\bar{x} = \frac{78 + 82 + 91 + 87 + 93}{5}$$

$$= \frac{431}{5} = 86.2$$

The mean score for the history student was 86.2.

Consider two students, each of whom has taken 5 exams.

Scores for Student A

84	86	83	85	87

$$\bar{x} = \frac{84 + 86 + 83 + 85 + 87}{5} = \frac{425}{5} = 85$$

The mean for Student A is 85.

Scores for Student B

90	75	94	68	98

$$\bar{x} = \frac{90 + 75 + 94 + 68 + 98}{5} = \frac{425}{5} = 85$$

The mean for Student B is 85.

For each of these students, the mean (average) for the 5 exams is 85. However, Student A has a more consistent record of scores than Student B. One way to measure the consistency, or "clustering" near the mean, of data is to use the **standard deviation.**

To calculate the standard deviation:

Step 1. Sum the squares of the differences between each value of the data and the mean.

Step 2. Divide the result in step 1 by the number of items in the set of data.

Step 3. Take the square root of the result in step 2.

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The calculation for Student A is shown at the right.

Step 1:

x	$x - \bar{x}$	$(x - \bar{x})^2$
84	$84 - 85$	$(-1)^2 = 1$
86	$86 - 85$	$1^2 = 1$
83	$83 - 85$	$(-2)^2 = 4$
85	$85 - 85$	$0^2 = 0$
87	$87 - 85$	$2^2 = 4$
		Total $= 10$

Step 2: $\frac{10}{5} = 2$

Step 3: $\sigma = \sqrt{2} \approx 1.414$

The symbol for standard deviation is the lower case Greek letter *sigma*, σ.

The standard deviation for Student A's scores is approximately 1.414.

Following a similar procedure for Student B shows that the standard deviation for Student B's scores is approximately 11.524. Because the standard deviation of Student B's scores is greater than that of Student A's ($11.524 > 1.414$), Student B's scores are not as consistent as those of Student A.



1. The weights in ounces of 6 newborn infants were recorded by a hospital. The weights were 96, 105, 84, 90, 102, and 99. Find the standard deviation of the weights.

2. The numbers of rooms occupied in a hotel on 6 consecutive days were 234, 321, 222, 246, 312, and 396. Find the standard deviation for the number of rooms occupied.

3. Seven coins were tossed 100 times. The numbers of heads recorded for each coin were 56, 63, 49, 50, 48, 53, and 52. Find the standard deviation of the number of heads.

4. The temperatures for 11 consecutive days at a desert resort were 95°, 98°, 98°, 104°, 97°, 100°, 96°, 97°, 108°, 93°, and 104°. For the same days, temperatures in Antarctica were 27°, 28°, 28°, 30°, 28°, 27°, 30°, 25°, 24°, 26°, and 21°. Which location has the greater standard deviation of temperatures?

5. The scores for 5 college basketball games were 56, 68, 60, 72, and 64. The scores for 5 professional basketball games were 106, 118, 110, 122, and 114. Which scores have the greater standard deviation?

6. The weights in pounds of the 5-man front line of a college football team are 210, 245, 220, 230, and 225. Find the standard deviation of the weights.

7. One student received test scores of 85, 92, 86, and 89. A second student received scores of 90, 97, 91, and 94 (exactly 5 points more on each test). Are the means of the two students the same? If not, what is the relationship between the means of the two students? Are the standard deviations of the scores of the two students the same? If not, what is the relationship between the standard deviations of the scores of the two students?

8. Grade-point average (GPA) is a *weighted* mean. It is called a weighted mean because a grade in a 5-unit course has more influence on your GPA than a grade in a 2-unit course. GPA is calculated by multiplying the numerical equivalent of each grade by the number of units, adding those products, and then dividing by the total number of units. Calculate your GPA for the last quarter or semester.

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9. If you average 40 mph for 1 h and then 50 mph for 1 h, is your average speed $\frac{40 + 50}{2} = 45$ mph? Why or why not?

10. A company is negotiating with its employees the terms of a raise in salary. One proposal would add \$500 a year to each employee's salary. The second proposal would give each employee a 4% raise. Explain how each of these proposals would affect the current mean and standard deviation of salaries for the company.

Chapter Summary

Key Words

A *set* is a collection of objects. The objects of a set are called the *elements* of the set.

The *roster method* of writing a set encloses a list of the elements in braces.

The *empty set,* or *null set,* written $\varnothing$ or $\{\ \}$, is the set that contains no elements.

The *union* of two sets, written $A \cup B$, is the set that contains all the elements of A and all the elements of B (any elements that are in both sets A and B are listed only once).

The *intersection* of two sets, written $A \cap B$, is the set that contains the elements that are common to both A and B.

An *inequality* is an expression that contains the symbol $<$, $>$, $\leq$, or $\geq$.

The *solution set of an inequality* is a set of numbers, each element of which, when substituted for the variable, results in a true inequality. The solution set of an inequality can be graphed on the number line.

The solution set of an inequality in two variables is a *half-plane*.

Essential Rules

Addition Property of Inequalities

The same term can be added to each side of an inequality without changing the solution set of the inequality.

If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

The Addition Property of Inequalities also holds true for an inequality containing the symbol $\geq$ or $\leq$.

Multiplication Property of Inequalities

Each side of an inequality can be multiplied by the same **positive number** without changing the solution set of the inequality.

If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.

If each side of an inequality is multiplied by the same **negative number** and the inequality symbol is reversed, then the solution set of the inequality is not changed.

If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

The Multiplication Property of Inequalities also holds true for an inequality containing the symbol $\geq$ or $\leq$.

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Chapter Review

1. Solve: $2x - 3 > x + 15$

2. Find $A \cap B$, given $A = \{0, 2, 4, 6, 8\}$ and $B = \{-2, -4\}$.

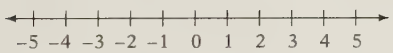
3. Use set-builder notation to write the set of odd integers greater than -8.

4. Find $A \cup B$, given $A = \{6, 8, 10\}$ and $B = \{2, 4, 6\}$.

5. Use the roster method to write the set of odd positive integers less than 8.

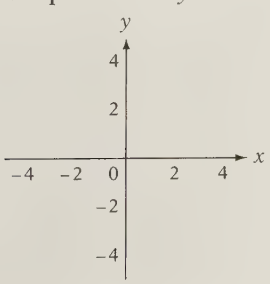
6. Solve: $12 - 4(x - 1) \leq 5(x - 4)$

7. Graph: $x > 3$

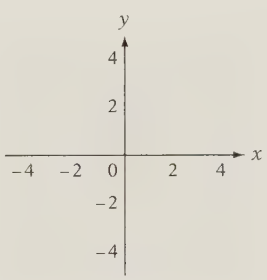


8. Solve: $3x + 4 \geq -8$

9. Graph: $3x + 2y \leq 12$

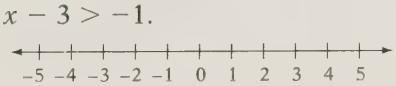


10. Graph: $5x + 2y < 6$



11. Use set-builder notation to write the set of real numbers greater than 3.

12. Solve and graph the solution set of $x - 3 > -1$.



13. Find $A \cap B$, given $A = \{1, 5, 9, 13\}$ and $B = \{1, 3, 5, 7, 9\}$.

14. Graph: $\{x | x < 2\} \cup \{x | x > 5\}$



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15. Graph: $\{x|x > -1\} \cap \{x|x \leq 2\}$



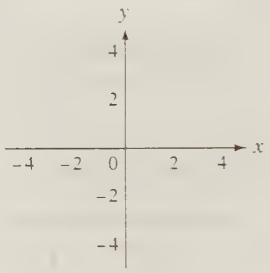
16. Solve: $-15x \leq 45$

17. Solve: $6x - 9 < 4x + 3(x + 3)$

18. Solve: $5 - 4(x + 9) > 11(12x - 9)$

19. Solve: $-\frac{3}{4}x > \frac{2}{3}$

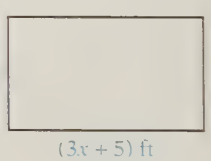
20. Graph: $2x - 3y < 9$



21. Solve: $7x - 2(x + 3) \geq x + 10$

22. Florist A charges a \$3 delivery fee plus \$21 per bouquet delivered. Florist B charges a \$15 delivery fee plus \$18 per bouquet delivered. A church wants to supply each resident of a small nursing home with a bouquet for Grandparent's Day. Find the number of residents of the nursing home if Florist B is more economical than Florist A.

23. The width of a rectangular garden is 12 ft. The length of the garden is $(3x + 5)$ ft. Express as an integer the minimum length of the garden when the area is greater than 276 ft^2. (The area of a rectangle is equal to its length times its width.)



24. Six less than a number is greater than twenty-five. Find the smallest integer that will satisfy the inequality.

25. A student's grades on five sociology tests were 68, 82, 90, 73, 95. What is the lowest score the student can receive on the next test and still be able to attain a minimum of 480 points?

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Chapter Test

1. Graph: $\{x|x < 5\} \cap \{x|x > 0\}$



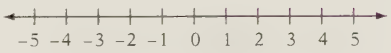
2. Use set-builder notation to write the set of the positive integers less than 50.

3. Use the roster method to write the set of the even positive integers between 3 and 9.

4. Solve: $3(2x - 5) \geq 8x - 9$

5. Solve: $x + \frac{1}{2} > \frac{5}{8}$

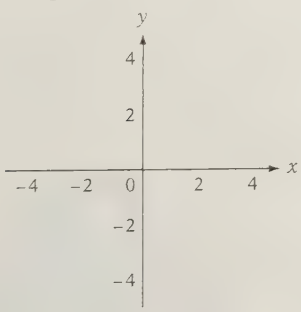
6. Graph: $x > -2$



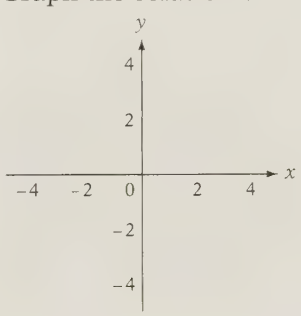
7. Solve: $5 - 3x > 8$

8. Use set-builder notation to write the set of the real numbers greater than -23.

9. Graph the solution set of $3x + y > 4$.

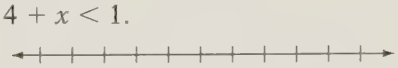


10. Graph the solution set of $4x - 5y \geq 15$.



11. Find $A \cap B$, given $A = \{6, 8, 10, 12\}$ and $B = \{12, 14, 16\}$.

12. Solve and graph the solution set of $4 + x < 1$.

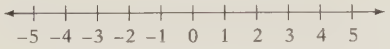


13. Solve: $-\frac{3}{8}x \leq 5$

14. Solve: $6x - 3(2 - 3x) < 4(2x - 7)$

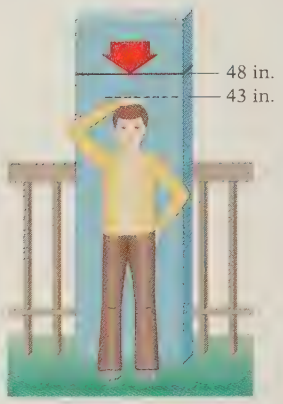
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15. Solve and graph the solution set of $\frac{2}{3}x \geq 2$.

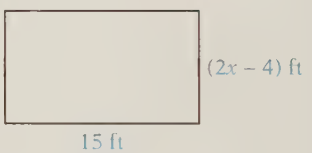


16. Solve: $2x - 7 \leq 6x + 9$

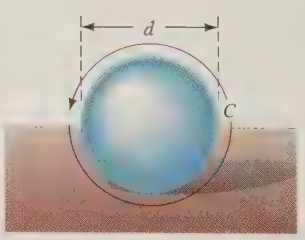
17. To ride a certain roller coaster at an amusement park, a person must be at least 48 in. tall. How many inches must a child who is 43 in. tall grow to be eligible to ride the roller coaster?



18. A rectangle is 15 ft long and $(2x - 4)$ ft wide. Express an integer as the maximum width of the rectangle if the area is less than 180 ft^2. (The area of a rectangle is equal to its length times its width.)



19. A ball bearing for a rotary engine must have a circumference between 0.1220 in. and 0.1240 in. What are the allowable diameters for the bearings to the nearest ten-thousandth of an inch? Recall that $C = \pi d$.



20. A stockbroker receives a monthly salary that is the greater of \$2500 or \$1000 plus 2% of the total value of all stock transactions the broker processes during the month. What dollar amounts of transactions did the broker process in a month for which the broker's salary was \$2500?

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Cumulative Review

1. Simplify: $2[5a - 3(2 - 5a) - 8]$

2. Solve: $\frac{5}{8} - 4x = \frac{1}{8}$

3. Solve: $2x - 3[x - 2(x - 3)] = 2$

4. Simplify: $(-3a)(-2a^3b^2)^2$

5. Simplify: $\frac{27a^3b^2}{(-3ab^2)^3}$

6. Simplify: $(16x^2 - 12x - 2) \div (4x - 1)$

7. Given $f(x) = x^2 - 4x - 5$, find $f(-1)$.

8. Factor: $27a^2x^2 - 3a^2$

9. Simplify: $\frac{x^2 - 2x}{x^2 - 2x - 8} \div \frac{x^3 - 5x^2 + 6x}{x^2 - 7x + 12}$

10. Simplify: $\frac{4a}{2a - 3} - \frac{2a}{a + 3}$

11. Solve: $\frac{5y}{6} - \frac{5}{9} = \frac{y}{3} - \frac{5}{6}$

12. Solve $R = \frac{C - S}{t}$ for C.

13. Find the slope of the line that passes through the points $(2, -3)$ and $(-1, 4)$.

14. Find the equation of the line that passes through the point $(1, -3)$ and has slope $-\frac{3}{2}$.

15. Solve by substitution.

$$x = 3y + 1$$
$$2x + 5y = 13$$

16. Solve by the addition method.

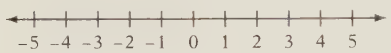
$$9x - 2y = 17$$
$$5x + 3y = -7$$

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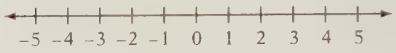
17. Find $A \cup B$, given $A = \{0, 1, 2\}$ and $B = \{-10, -2\}$.

18. Use set-builder notation to write the set of the real numbers less than 48.

19. Graph: $\{x|x > 1\} \cup \{x|x < -1\}$



20. Graph the solution set of $\frac{3}{8}x > -\frac{3}{4}$.



21. Solve: $-\frac{4}{5}x > 12$

22. Solve: $15 - 3(5x - 7) < 2(7 - 2x)$

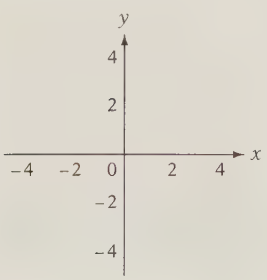
23. Three-fifths of a number is less than negative fifteen. What integers satisfy this inequality? Write the answer in set builder notation.

24. Company A rents cars for \$6 a day and \$.25 for every mile driven. Company B rents cars for \$15 a day and \$.10 per mile. You want to rent a car for 6 days. What is the maximum number of miles you can drive a Company A car if it is to cost you less than a Company B car?

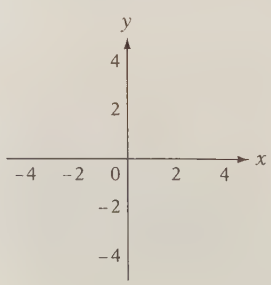
25. In a lake, 100 fish are caught, tagged, and then released. Later, 150 fish are caught. Three of the 150 fish are found to have tags. Estimate the number of fish in the lake.

26. The first angle of a triangle is 30 degrees more than the second angle. The third angle is 10 degrees more than twice the second angle. Find the measure of each angle.

27. Graph: $y = 2x - 1$



28. Graph the solution set of $6x - 3y \geq 6$.



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CHAPTER

10 Radical Expressions



Drafters prepare detailed drawings from sketches and specifications made by architects, designers, engineers, and scientists. Usually drafters specialize in a particular field, such as architectural, civil, mechanical, electrical, or aeronautical drafting. The study of radical expressions is important for drafters as working with right triangles requires radicals. See Section 4 of this chapter.

Objectives

Section 10.1

To simplify numerical radical expressions

To simplify variable radical expressions

Section 10.2

To add and subtract radical expressions

Section 10.3

To multiply radical expressions

To divide radical expressions

Section 10.4

To solve an equation containing a radical expression

To solve application problems

A Table of Square Roots

The practice of finding the square root of a number has existed for at least two thousand years. Because the process of finding a square root before the invention of the calculator was tedious and time-consuming, it was convenient to have tables of square roots.

The table shown at the left is part of an old Babylonian clay tablet that was written around 350 B.C. It is an incomplete table of square roots written in a style called *cuneiform*.



The number base of the Babylonians was 60 instead of 10, as we use today. The symbol was used for 1, and 10 was written as  . Some examples of numbers written in this system are given below.

 = 9  = 40

Translations of the first two lines of the table are given below. The number given in parentheses is the equivalent base-10 number that would be used today. You might try to translate the third line. The answer is given at the bottom of this page.

$40 \times 60 + 1$ (= 2401), which is the square of 49

$41 \times 60 + 40$ (= 2500), which is the square of 50

Answer: $43 \times 60 + 21$ (= 2601), which is the square of 51

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10.1 Introduction to Radical Expressions

Objective A To simplify numerical radical expressions

A **square root** of a positive number x is a number whose square is x.

A square root of 16 is 4 because $4^2 = 16$.
A square root of 16 is -4 because $(-4)^2 = 16$.

POINT OF INTEREST

The radical symbol was first used in 1525 but was written as $\surd$. Some historians suggest that the radical symbol also developed into the symbols for "less than" and "greater than." Because typesetters of that time did not want to make additional symbols, the radical was rotated to the position $>$ and used as a "greater than" symbol and rotated to $<$ and used for the "less than" symbol. Other evidence, however, suggests that the "less than" and "greater than" symbols were developed independently of the radical symbol.

Every positive number has two square roots, one a positive and one a negative number. The symbol $\sqrt{\ }$, called a **radical sign,** is used to indicate the positive or **principal square root** of a number. For example, $\sqrt{16} = 4$ and $\sqrt{25} = 5$. The number under the radical sign is called the **radicand.**

When the negative square root of a number is to be found, a negative sign is placed in front of the radical. For example, $-\sqrt{16} = -4$ and $-\sqrt{25} = -5$.

The square of an integer is a **perfect square.** 49, 81, and 144 are examples of perfect squares.

$7^2 = 49$
$9^2 = 81$
$12^2 = 144$

An integer that is a perfect square can be written as the product of prime factors, each of which has an even exponent when expressed in exponential form.

$49 = 7 \cdot 7 = 7^2$
$81 = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4$
$144 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^4 3^2$

➡ Simplify $\sqrt{625}$.

TAKE NOTE

To find the square root of a perfect square written in exponential form, remove the radical sign and multiply the exponent by $\frac{1}{2}$.

$\sqrt{625} = \sqrt{5^4}$ • Write the prime factorization of the radicand in exponential form.

$= 5^2$ • Remove the radical sign and multiply the exponent by $\frac{1}{2}$.

$= 25$ • Simplify.

If a number is not a perfect square, its square root can only be approximated. For example, 2 and 7 are not perfect squares. The square roots of these numbers are **irrational numbers.** Their decimal representations never terminate or repeat.

$\sqrt{2} \approx 1.4142135\cdots$ $\sqrt{7} \approx 2.6457513\cdots$

A radical expression is in simplest form when the radicand contains no factor greater than 1 that is a perfect square. The Product Property of Square Roots is used to simplify radical expressions.

The Product Property of Square Roots

If a and b are positive real numbers, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

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➡ Simplify: $\sqrt{96}$

$\sqrt{96} = \sqrt{2^5 \cdot 3}$ • Write the prime factorization of the radicand in exponential form.

$= \sqrt{2^4(2 \cdot 3)}$ • Write the radicand as a product of a perfect square and factors that do not contain a perfect square.

$= \sqrt{2^4}\sqrt{2 \cdot 3}$ • Use the Product Property of Square Roots.

$= 2^2\sqrt{6}$ • Simplify.

$= 4\sqrt{6}$

➡ Simplify: $\sqrt{360}$

$\sqrt{360} = \sqrt{2^3 \cdot 3^2 \cdot 5}$ • Write the prime factorization of the radicand in exponential form.

$= \sqrt{(2^2 \cdot 3^2)(2 \cdot 5)}$ • Write the radicand as a product of perfect squares and factors that do not contain a perfect square.

$= \sqrt{2^2 \cdot 3^2}\sqrt{2 \cdot 5}$ • Use the Product Property of Square Roots.

$= (2 \cdot 3)\sqrt{10}$ • Simplify.

$= 6\sqrt{10}$

From the last example, note that $\sqrt{360} = 6\sqrt{10}$. The two expressions are different representations of the same number. Using a calculator, we find that $\sqrt{360} \approx 18.973666$ and $6\sqrt{10} \approx 6(3.1622777) = 18.9736662$.

➡ Simplify: $\sqrt{-16}$

Because the square of any real number is positive, there is no real number whose square is -16. $\sqrt{-16}$ is not a real number.

Example 1 Simplify: $3\sqrt{90}$

Solution

$3\sqrt{90} = 3\sqrt{2 \cdot 3^2 \cdot 5}$

$= 3\sqrt{3^2(2 \cdot 5)}$

$= 3\sqrt{3^2}\sqrt{2 \cdot 5}$

$= 3 \cdot 3\sqrt{10} = 9\sqrt{10}$

You Try It 1 Simplify: $-5\sqrt{32}$

Your solution

Example 2 Simplify: $\sqrt{252}$

Solution

$\sqrt{252} = \sqrt{2^2 \cdot 3^2 \cdot 7}$

$= \sqrt{2^2 \cdot 3^2}\sqrt{7}$

$= 2 \cdot 3\sqrt{7}$

$= 6\sqrt{7}$

You Try It 2 Simplify: $\sqrt{216}$

Your solution

Solutions on p. S23

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Objective B To simplify variable radical expressions

Variable expressions that contain radicals do not always represent real numbers. For example, if $a = -4$, then

$$\sqrt{a^3} = \sqrt{(-4)^3} = \sqrt{-64}$$

and $\sqrt{-64}$ is not a real number.

Now consider the expression $\sqrt{x^2}$. Evaluate this expression for $x = -2$ and $x = 2$.

$$\sqrt{x^2}$$
$$\sqrt{(-2)^2} = \sqrt{4} = 2 = |-2|$$

$$\sqrt{x^2}$$
$$\sqrt{2^2} = \sqrt{4} = 2 = |2|$$

This suggests the following:

For any real number a, $\sqrt{a^2} = |a|$. If $a \geq 0$, then $\sqrt{a^2} = a$.

In order to avoid variable expressions that do not represent real numbers, and so that absolute-value signs are not needed for certain expressions, the variables in this chapter will represent *positive* numbers unless otherwise stated.

A variable or a product of variables written in exponential form is a perfect square when each exponent is an even number.

To find the square root of a perfect square, remove the radical sign and multiply each exponent by $\frac{1}{2}$.

➡ Simplify: $\sqrt{a^6}$

$\sqrt{a^6} = a^3$ • Remove the radical sign and multiply the exponent by $\frac{1}{2}$.

A variable radical expression is in simplest form when the radicand contains no factor greater than 1 that is a perfect square.

➡ Simplify: $\sqrt{x^7}$

$\sqrt{x^7} = \sqrt{x^6 \cdot x}$ • Write x^7 as the product of a perfect square and x.

$= \sqrt{x^6}\sqrt{x}$ • Use the Product Property of Square Roots.

$= x^3\sqrt{x}$ • Simplify the perfect square.

➡ Simplify: $3x\sqrt{8x^3y^{13}}$

$3x\sqrt{8x^3y^{13}} = 3x\sqrt{2^3x^3y^{13}}$ • Write the prime factorization of the coefficient of the radicand in exponential form.

$= 3x\sqrt{2^2x^2y^{12}(2xy)}$ • Write the radicand as a product of perfect squares and factors that do not contain a perfect square.

$= 3x\sqrt{2^2x^2y^{12}}\sqrt{2xy}$ • Use the Product Property of Square Roots.

$= 3x \cdot 2xy^6\sqrt{2xy}$ • Simplify.

$= 6x^2y^6\sqrt{2xy}$

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➡ Simplify: $\sqrt{25(x+2)^2}$

$$\sqrt{25(x+2)^2} = \sqrt{5^2(x+2)^2}$$
$$= 5(x+2)$$
$$= 5x + 10$$

• Write the prime factorization of 25 in exponential form.

Example 3
Simplify: $\sqrt{b^{15}}$

Solution
$\sqrt{b^{15}} = \sqrt{b^{14} \cdot b} = \sqrt{b^{14}} \cdot \sqrt{b} = b^7\sqrt{b}$

You Try It 3
Simplify: $\sqrt{y^{19}}$

Your solution

Example 4
Simplify: $\sqrt{24x^5}$

Solution
$$\sqrt{24x^5} = \sqrt{2^3 \cdot 3 \cdot x^5} = \sqrt{2^2x^4(2 \cdot 3x)}$$
$$= \sqrt{2^2 \cdot x^4}\sqrt{2 \cdot 3x}$$
$$= 2x^2\sqrt{6x}$$

You Try It 4
Simplify: $\sqrt{45b^7}$

Your solution

Example 5
Simplify: $2a\sqrt{18a^3b^{10}}$

Solution
$$2a\sqrt{18a^3b^{10}} = 2a\sqrt{2 \cdot 3^2 \cdot a^3b^{10}}$$
$$= 2a\sqrt{3^2a^2b^{10}(2a)}$$
$$= 2a\sqrt{3^2a^2b^{10}}\sqrt{2a}$$
$$= 2a \cdot 3ab^5\sqrt{2a}$$
$$= 6a^2b^5\sqrt{2a}$$

You Try It 5
Simplify: $3a\sqrt{28a^9b^{18}}$

Your solution

Example 6
Simplify: $\sqrt{16(x+5)^2}$

Solution
$$\sqrt{16(x+5)^2} = \sqrt{2^4(x+5)^2} = 2^2(x+5)$$
$$= 4(x+5) = 4x + 20$$

You Try It 6
Simplify: $\sqrt{25(a+3)^2}$

Your solution

Example 7
Simplify: $\sqrt{x^2 + 10x + 25}$

Solution
$\sqrt{x^2 + 10x + 25} = \sqrt{(x+5)^2} = x + 5$

You Try It 7
Simplify: $\sqrt{x^2 + 14x + 49}$

Your solution

Solutions on p. S23

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10.1 Exercises

Objective A

Simplify.

1. $\sqrt{16}$ 2. $\sqrt{64}$ 3. $\sqrt{49}$ 4. $\sqrt{144}$ 5. $\sqrt{32}$ 6. $\sqrt{50}$

7. $\sqrt{8}$ 8. $\sqrt{12}$ 9. $6\sqrt{18}$ 10. $-3\sqrt{48}$ 11. $5\sqrt{40}$ 12. $2\sqrt{28}$

13. $\sqrt{15}$ 14. $\sqrt{21}$ 15. $\sqrt{29}$ 16. $\sqrt{13}$ 17. $-9\sqrt{72}$ 18. $11\sqrt{80}$

19. $\sqrt{45}$ 20. $\sqrt{225}$ 21. $\sqrt{0}$ 22. $\sqrt{210}$ 23. $6\sqrt{128}$ 24. $9\sqrt{288}$

Find the decimal approximation rounded to the nearest thousandth.

25. $\sqrt{240}$ 26. $\sqrt{300}$ 27. $\sqrt{288}$ 28. $\sqrt{600}$ 29. $\sqrt{256}$ 30. $\sqrt{324}$

Objective B

Simplify.

31. $\sqrt{x^6}$ 32. $\sqrt{x^{12}}$ 33. $\sqrt{y^{15}}$ 34. $\sqrt{y^{11}}$

35. $\sqrt{a^{20}}$ 36. $\sqrt{a^{16}}$ 37. $\sqrt{x^4y^4}$ 38. $\sqrt{x^{12}y^8}$

39. $\sqrt{4x^4}$ 40. $\sqrt{25y^8}$ 41. $\sqrt{24x^2}$ 42. $\sqrt{x^3y^{15}}$

43. $\sqrt{60x^5}$ 44. $\sqrt{72y^7}$ 45. $\sqrt{49a^4b^8}$ 46. $\sqrt{144x^2y^8}$

47. $\sqrt{18x^5y^7}$ 48. $\sqrt{32a^5b^{15}}$ 49. $\sqrt{40x^{11}y^7}$ 50. $\sqrt{72x^9y^3}$

51. $\sqrt{80a^9b^{10}}$ 52. $\sqrt{96a^5b^7}$ 53. $2\sqrt{16a^2b^3}$ 54. $5\sqrt{25a^4b^7}$

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55. $x\sqrt{x^4y^2}$ **56.** $y\sqrt{x^5y^6}$ **57.** $4\sqrt{20a^4b^7}$ **58.** $5\sqrt{12a^3b^4}$

59. $3x\sqrt{12x^2y^7}$ **60.** $4y\sqrt{18x^5y^4}$ **61.** $2x^2\sqrt{8x^5y^3}$ **62.** $3y^2\sqrt{27x^4y^3}$

63. $\sqrt{25(a+4)^2}$ **64.** $\sqrt{81(x+y)^4}$ **65.** $\sqrt{4(x+2)^4}$ **66.** $\sqrt{9(x+2)^8}$

67. $\sqrt{x^2+4x+4}$ **68.** $\sqrt{b^2+8b+16}$ **69.** $\sqrt{y^2+2y+1}$ **70.** $\sqrt{a^2+6a+9}$

APPLYING THE CONCEPTS

71. If a and b are positive real numbers, does $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$? If not, give an example in which the expressions are not equal.

72. **a.** Find the two-digit perfect square that has exactly nine factors.
b. Find two whole numbers such that their difference is 10, the smaller number is a perfect square, and the larger number is two less than a perfect square.

73. Describe in your own words how to simplify a radical expression.

74. Explain why $2\sqrt{2}$ is in simplest form and $\sqrt{8}$ is not in simplest form.

75. Use the roster method to list the whole numbers between $\sqrt{8}$ and $\sqrt{90}$.

76. Simplify. Assume that no radicand is negative.
a. $\sqrt{x^2y^3+x^3y^2}$ **b.** $\sqrt{4a^5b^4-4a^4b^5}$ **c.** $\sqrt{(x^2-y^2)(x-y)}$

77. You are to grade the solution to the problem "Write $\sqrt{72}$ in simplest form." Is the solution correct? If not, what error was made? What is the correct solution?

$$\sqrt{72}=\sqrt{4}\sqrt{18}$$
$$=2\sqrt{18}$$

78. Simplify.
a. $\sqrt{\sqrt{16}}$ **b.** $\sqrt{\sqrt{81}}$

79. Approximate to the nearest ten-thousandth.
a. $\sqrt{\sqrt{77}}$ **b.** $\sqrt{\sqrt{17}}$

80. Given $f(x)=\sqrt{2x-1}$, find each of the following. Write your answer in simplest form.
a. $f(1)$ **b.** $f(5)$ **c.** $f(14)$

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10.2 Addition and Subtraction of Radical Expressions

Objective A *To add and subtract radical expressions*

The Distributive Property is used to simplify the sum or difference of radical expressions with like radicands.

$5\sqrt{2} + 3\sqrt{2} = (5 + 3)\sqrt{2} = 8\sqrt{2}$

$6\sqrt{2x} - 4\sqrt{2x} = (6 - 4)\sqrt{2x} = 2\sqrt{2x}$

Radical expressions that are in simplest form and have unlike radicands cannot be simplified by the Distributive Property.

$2\sqrt{3} + 4\sqrt{2}$ cannot be simplified by the Distributive Property.

➡ Simplify: $4\sqrt{8} - 10\sqrt{2}$

$4\sqrt{8} - 10\sqrt{2} = 4\sqrt{2^3} - 10\sqrt{2}$ • Write the prime factorization of the radicands.

$= 4\sqrt{2^2 \cdot 2} - 10\sqrt{2}$

$= 4\sqrt{2^2}\sqrt{2} - 10\sqrt{2}$ • Use the Product Property of Radicals.

$= 4 \cdot 2\sqrt{2} - 10\sqrt{2}$

$= 8\sqrt{2} - 10\sqrt{2}$

$= (8 - 10)\sqrt{2}$ • Simplify the expression by using the Distributive Property.

$= -2\sqrt{2}$

➡ Simplify: $8\sqrt{18x} - 2\sqrt{32x}$

$8\sqrt{18x} - 2\sqrt{32x} = 8\sqrt{2 \cdot 3^2 x} - 2\sqrt{2^5 x}$ • Write the prime factorization of the coefficients of the radicands.

$= 8\sqrt{3^2 \cdot 2x} - 2\sqrt{2^4 \cdot 2x}$

$= 8\sqrt{3^2}\sqrt{2x} - 2\sqrt{2^4}\sqrt{2x}$ • Use the Product Property of Radicals.

$= 8 \cdot 3\sqrt{2x} - 2 \cdot 2^2\sqrt{2x}$

$= 24\sqrt{2x} - 8\sqrt{2x}$

$= (24 - 8)\sqrt{2x}$ • Simplify the expression by using the Distributive Property.

$= 16\sqrt{2x}$

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Example 1
Simplify: $5\sqrt{2} - 3\sqrt{2} + 12\sqrt{2}$

Solution
$5\sqrt{2} - 3\sqrt{2} + 12\sqrt{2} = 14\sqrt{2}$

You Try It 1
Simplify: $9\sqrt{3} + 3\sqrt{3} - 18\sqrt{3}$

Your solution

Example 2
Simplify: $3\sqrt{12} - 5\sqrt{27}$

Solution
$$\begin{aligned} 3\sqrt{12} - 5\sqrt{27} &= 3\sqrt{2^2 \cdot 3} - 5\sqrt{3^3} \\ &= 3\sqrt{2^2}\sqrt{3} - 5\sqrt{3^2}\sqrt{3} \\ &= 3 \cdot 2\sqrt{3} - 5 \cdot 3\sqrt{3} \\ &= 6\sqrt{3} - 15\sqrt{3} \\ &= -9\sqrt{3} \end{aligned}$$

You Try It 2
Simplify: $2\sqrt{50} - 5\sqrt{32}$

Your solution

Example 3
Simplify: $3\sqrt{12x^3} - 2x\sqrt{3x}$

Solution
$$\begin{aligned} &3\sqrt{12x^3} - 2x\sqrt{3x} \\ &= 3\sqrt{2^2 \cdot 3 \cdot x^3} - 2x\sqrt{3x} \\ &= 3\sqrt{2^2 \cdot x^2}\sqrt{3x} - 2x\sqrt{3x} \\ &= 3 \cdot 2 \cdot x\sqrt{3x} - 2x\sqrt{3x} \\ &= 6x\sqrt{3x} - 2x\sqrt{3x} \\ &= 4x\sqrt{3x} \end{aligned}$$

You Try It 3
Simplify: $y\sqrt{28y} + 7\sqrt{63y^3}$

Your solution

Example 4
Simplify: $2x\sqrt{8y} - 3\sqrt{2x^2y} + 2\sqrt{32x^2y}$

Solution
$$\begin{aligned} &2x\sqrt{8y} - 3\sqrt{2x^2y} + 2\sqrt{32x^2y} \\ &= 2x\sqrt{2^3y} - 3\sqrt{2x^2y} + 2\sqrt{2^5x^2y} \\ &= 2x\sqrt{2^2}\sqrt{2y} - 3\sqrt{x^2}\sqrt{2y} + 2\sqrt{2^4x^2}\sqrt{2y} \\ &= 2x \cdot 2\sqrt{2y} - 3 \cdot x\sqrt{2y} + 2 \cdot 2^2 \cdot x\sqrt{2y} \\ &= 4x\sqrt{2y} - 3x\sqrt{2y} + 8x\sqrt{2y} \\ &= 9x\sqrt{2y} \end{aligned}$$

You Try It 4
Simplify: $2\sqrt{27a^5} - 4a\sqrt{12a^3} + a^2\sqrt{75a}$

Your solution

Solutions on p. S23

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10.2 Exercises

Objective A

Simplify.

1. $2\sqrt{2} + \sqrt{2}$
2. $3\sqrt{5} + 8\sqrt{5}$
3. $-3\sqrt{7} + 2\sqrt{7}$
4. $4\sqrt{5} - 10\sqrt{5}$
5. $-3\sqrt{11} - 8\sqrt{11}$
6. $-3\sqrt{3} - 5\sqrt{3}$
7. $2\sqrt{x} + 8\sqrt{x}$
8. $3\sqrt{y} + 2\sqrt{y}$
9. $8\sqrt{y} - 10\sqrt{y}$
10. $-5\sqrt{2a} + 2\sqrt{2a}$
11. $-2\sqrt{3b} - 9\sqrt{3b}$
12. $-7\sqrt{5a} - 5\sqrt{5a}$
13. $3x\sqrt{2} - x\sqrt{2}$
14. $2y\sqrt{3} - 9y\sqrt{3}$
15. $2a\sqrt{3a} - 5a\sqrt{3a}$
16. $-5b\sqrt{3x} - 2b\sqrt{3x}$
17. $3\sqrt{xy} - 8\sqrt{xy}$
18. $-4\sqrt{xy} + 6\sqrt{xy}$
19. $\sqrt{45} + \sqrt{125}$
20. $\sqrt{32} - \sqrt{98}$
21. $2\sqrt{2} + 3\sqrt{8}$
22. $4\sqrt{128} - 3\sqrt{32}$
23. $5\sqrt{18} - 2\sqrt{75}$
24. $5\sqrt{75} - 2\sqrt{18}$
25. $5\sqrt{4x} - 3\sqrt{9x}$
26. $-3\sqrt{25y} + 8\sqrt{49y}$
27. $3\sqrt{3x^2} - 5\sqrt{27x^2}$
28. $-2\sqrt{8y^2} + 5\sqrt{32y^2}$
29. $2x\sqrt{xy^2} - 3y\sqrt{x^2y}$
30. $4a\sqrt{b^2a} - 3b\sqrt{a^2b}$
31. $3x\sqrt{12x} - 5\sqrt{27x^3}$
32. $2a\sqrt{50a} + 7\sqrt{32a^3}$
33. $4y\sqrt{8y^3} - 7\sqrt{18y^5}$
34. $2a\sqrt{8ab^2} - 2b\sqrt{2a^3}$
35. $b^2\sqrt{a^5b} + 3a^2\sqrt{ab^5}$
36. $y^2\sqrt{x^5y} + x\sqrt{x^3y^5}$

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37. $4\sqrt{2} - 5\sqrt{2} + 8\sqrt{2}$

38. $3\sqrt{3} + 8\sqrt{3} - 16\sqrt{3}$

39. $5\sqrt{x} - 8\sqrt{x} + 9\sqrt{x}$

40. $\sqrt{x} - 7\sqrt{x} + 6\sqrt{x}$

41. $8\sqrt{2} - 3\sqrt{y} - 8\sqrt{2}$

42. $8\sqrt{3} - 5\sqrt{2} - 5\sqrt{3}$

43. $8\sqrt{8} - 4\sqrt{32} - 9\sqrt{50}$

44. $2\sqrt{12} - 4\sqrt{27} + \sqrt{75}$

45. $-2\sqrt{3} + 5\sqrt{27} - 4\sqrt{45}$

46. $-2\sqrt{8} - 3\sqrt{27} + 3\sqrt{50}$

47. $4\sqrt{75} + 3\sqrt{48} - \sqrt{99}$

48. $2\sqrt{75} - 5\sqrt{20} + 2\sqrt{45}$

49. $\sqrt{25x} - \sqrt{9x} + \sqrt{16x}$

50. $\sqrt{4x} - \sqrt{100x} - \sqrt{49x}$

51. $3\sqrt{3x} + \sqrt{27x} - 8\sqrt{75x}$

52. $5\sqrt{5x} + 2\sqrt{45x} - 3\sqrt{80x}$

53. $2a\sqrt{75b} - a\sqrt{20b} + 4a\sqrt{45b}$

54. $2b\sqrt{75a} - 5b\sqrt{27a} + 2b\sqrt{20a}$

55. $x\sqrt{3y^2} - 2y\sqrt{12x^2} + xy\sqrt{3}$

56. $a\sqrt{27b^2} + 3b\sqrt{147a^2} - ab\sqrt{3}$

APPLYING THE CONCEPTS

57. Given $G(x) = \sqrt{x + 5} + \sqrt{5x + 3}$, write $G(3)$ in simplest form.

58. Is the equation $\sqrt{a^2 + b^2} = \sqrt{a} + \sqrt{b}$ true for all real numbers a and b?

59. Use complete sentences to explain the steps in simplifying $4\sqrt{2a^3b} + 5\sqrt{5a^3b}$.

60. For each problem, write "ok" if the answer is correct. If the answer is incorrect, write the correct answer.

a. $3\sqrt{ab} + 5\sqrt{ab} = 8\sqrt{2ab}$

b. $7\sqrt{x^3} - 3x\sqrt{x} - x\sqrt{16x} = 0$

c. $5 - 2\sqrt{y} = 3\sqrt{y}$

Simplify.

61. $2\sqrt{8x + 4y} - 5\sqrt{18x + 9y}$

62. $6\sqrt{16x - 16} + \sqrt{25x - 25}$

63. $3\sqrt{a^3 + a^2} + 5\sqrt{4a^3 + 4a^2}$

64. $3\sqrt{x^3y^2 + x^2y^3} + xy\sqrt{4x + 4y}$

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10.3 Multiplication and Division of Radical Expressions

Objective A To multiply radical expressions

The Product Property of Square Roots is used to multiply variable radical expressions.

$$\sqrt{2x}\sqrt{3y} = \sqrt{2x \cdot 3y} = \sqrt{6xy}$$

➡ Simplify: $\sqrt{2x^2}\sqrt{32x^5}$

$$\begin{aligned}\sqrt{2x^2}\sqrt{32x^5} &= \sqrt{2x^2 \cdot 32x^5} \\ &= \sqrt{64x^7} \\ &= \sqrt{2^6x^7} \\ &= \sqrt{2^6x^6}\sqrt{x} \\ &= 2^3x^3\sqrt{x} \\ &= 8x^3\sqrt{x}\end{aligned}$$

- Use the Product Property of Square Roots.
- Multiply the radicands.
- Simplify.

➡ Simplify: $\sqrt{2x}(x + \sqrt{2x})$

$$\begin{aligned}\sqrt{2x}(x + \sqrt{2x}) &= \sqrt{2x}(x) + \sqrt{2x}\sqrt{2x} \\ &= x\sqrt{2x} + \sqrt{4x^2} \\ &= x\sqrt{2x} + \sqrt{2^2x^2} \\ &= x\sqrt{2x} + 2x\end{aligned}$$

- Use the Distributive Property to remove parentheses.
- Simplify.

➡ Simplify: $(\sqrt{2} - 3x)(\sqrt{2} + x)$

$$\begin{aligned}(\sqrt{2} - 3x)(\sqrt{2} + x) &= \sqrt{2 \cdot 2} + x\sqrt{2} - 3x\sqrt{2} - 3x^2 \\ &= \sqrt{2^2} + (x - 3x)\sqrt{2} - 3x^2 \\ &= 2 - 2x\sqrt{2} - 3x^2\end{aligned}$$

- Use the FOIL method to remove parentheses.

The expressions $a + b$ and $a - b$, which are the sum and difference of two terms, are called **conjugates** of each other. You will recall that $(a + b)(a - b) = a^2 - b^2$.

➡ Simplify: $(2 + \sqrt{7})(2 - \sqrt{7})$

$$\begin{aligned}(2 + \sqrt{7})(2 - \sqrt{7}) &= 2^2 - (\sqrt{7})^2 \\ &= 4 - 7 \\ &= -3\end{aligned}$$

- $(2 + \sqrt{7})(2 - \sqrt{7})$ is the product of conjugates.

TAKE NOTE

For $x > 0$, $(\sqrt{x})^2 = x$.

➡ Simplify: $(3 + \sqrt{y})(3 - \sqrt{y})$

$$\begin{aligned}(3 + \sqrt{y})(3 - \sqrt{y}) &= 3^2 - (\sqrt{y})^2 \\ &= 9 - y\end{aligned}$$

- $(3 + \sqrt{y})(3 - \sqrt{y})$ is the product of conjugates.

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Example 1
Simplify: $\sqrt{3x^4}\sqrt{2x^2y}\sqrt{6xy^2}$

Solution
$$\begin{aligned}\sqrt{3x^4}\sqrt{2x^2y}\sqrt{6xy^2} &= \sqrt{36x^7y^3}\\ &= \sqrt{2^2 3^2 x^7 y^3}\\ &= \sqrt{2^2 3^2 x^6 y^2}\sqrt{xy}\\ &= 2 \cdot 3x^3y\sqrt{xy}\\ &= 6x^3y\sqrt{xy}\end{aligned}$$

You Try It 1
Simplify: $\sqrt{5a}\sqrt{15a^3b^4}\sqrt{3b^5}$

Your solution

Example 2
Simplify: $\sqrt{3ab}(\sqrt{3a} + \sqrt{9b})$

Solution
$$\begin{aligned}&\sqrt{3ab}(\sqrt{3a} + \sqrt{9b})\\ &= \sqrt{3^2a^2b} + \sqrt{3^3ab^2}\\ &= \sqrt{3^2a^2}\sqrt{b} + \sqrt{3^2b^2}\sqrt{3a}\\ &= 3a\sqrt{b} + 3b\sqrt{3a}\end{aligned}$$

You Try It 2
Simplify: $\sqrt{5x}(\sqrt{5x} - \sqrt{25y})$

Your solution

Example 3
Simplify: $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$

Solution
$$\begin{aligned}(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) &= (\sqrt{a})^2 - (\sqrt{b})^2\\ &= a - b\end{aligned}$$

You Try It 3
Simplify: $(2\sqrt{x} + 7)(2\sqrt{x} - 7)$

Your solution

Example 4
Simplify: $(2\sqrt{x} - \sqrt{y})(5\sqrt{x} - 2\sqrt{y})$

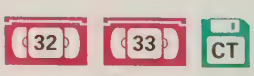
Solution
$$\begin{aligned}&(2\sqrt{x} - \sqrt{y})(5\sqrt{x} - 2\sqrt{y})\\ &= 10(\sqrt{x})^2 - 4\sqrt{xy} - 5\sqrt{xy} + 2(\sqrt{y})^2\\ &= 10x - 9\sqrt{xy} + 2y\end{aligned}$$

You Try It 4
Simplify: $(3\sqrt{x} - \sqrt{y})(5\sqrt{x} - 2\sqrt{y})$

Your solution

Solutions on p. S23

Objective B To divide radical expressions



The Quotient Property of Square Roots

If a and b are positive real numbers, then

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad \text{and} \quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

The square root of a quotient is equal to the quotient of the square roots.

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POINT OF INTEREST

A radical expression that occurs in Einstein's Theory of Relativity is

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

where v is the velocity of an object and c is the speed of light.

➡ Simplify: $\sqrt{\frac{4x^2}{z^6}}$

$$\sqrt{\frac{4x^2}{z^6}} = \frac{\sqrt{4x^2}}{\sqrt{z^6}}$$
$$= \frac{\sqrt{2^2x^2}}{\sqrt{z^6}} = \frac{2x}{z^3}$$

- Rewrite the radical expression as the quotient of the square roots.
- Simplify.

➡ Simplify: $\sqrt{\frac{24x^3y^7}{3x^7y^2}}$

$$\sqrt{\frac{24x^3y^7}{3x^7y^2}} = \sqrt{\frac{8y^5}{x^4}}$$
$$= \frac{\sqrt{8y^5}}{\sqrt{x^4}}$$
$$= \frac{\sqrt{2^3y^5}}{\sqrt{x^4}}$$
$$= \frac{\sqrt{2^2y^4}\sqrt{2y}}{\sqrt{x^4}}$$
$$= \frac{2y^2\sqrt{2y}}{x^2}$$

- Simplify the radicand.
- Rewrite the radical expression as the quotient of the square roots.
- Simplify.

➡ Simplify: $\frac{\sqrt{4x^2y}}{\sqrt{xy}}$

$$\frac{\sqrt{4x^2y}}{\sqrt{xy}} = \sqrt{\frac{4x^2y}{xy}}$$
$$= \sqrt{4x}$$
$$= \sqrt{2^2}\sqrt{x}$$
$$= 2\sqrt{x}$$

- Use the Quotient Property of Square Roots.
- Simplify the radicand.
- Simplify the radical expression.

A radical expression is not considered to be in simplest form if a radical remains in the denominator. The procedure used to remove a radical from the denominator is called **rationalizing the denominator.**

➡ Simplify: $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \boxed{\frac{\sqrt{3}}{\sqrt{3}}}$$
$$= \frac{2\sqrt{3}}{(\sqrt{3})^2}$$
$$= \frac{2\sqrt{3}}{3}$$

- Multiply the expression by $\frac{\sqrt{3}}{\sqrt{3}}$, which equals 1.
- The radicand in the denominator is a perfect square.
- Simplify.

The radical expression is in simplest form, because no radical remains in the denominator and the radical in the numerator contains no perfect-square factors other than 1.

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When the denominator contains a binomial radical expression, simplify the radical expression by multiplying the numerator and denominator by the conjugate of the denominator.

Simplify: $\dfrac{\sqrt{2y}}{\sqrt{y}+3}$

$$\frac{\sqrt{2y}}{\sqrt{y}+3}=\frac{\sqrt{2y}}{\sqrt{y}+3}\cdot\frac{\sqrt{y}-3}{\sqrt{y}-3}$$

- Multiply the numerator and denominator by $\sqrt{y}-3$, the conjugate of $\sqrt{y}+3$.

$$=\frac{\sqrt{2y^2}-3\sqrt{2y}}{(\sqrt{y})^2-3^2}$$

- Simplify.

$$=\frac{y\sqrt{2}-3\sqrt{2y}}{y-9}$$

Example 5

Simplify: $\dfrac{\sqrt{4x^2y^5}}{\sqrt{3x^4y}}$

Solution

$$\frac{\sqrt{4x^2y^5}}{\sqrt{3x^4y}}=\sqrt{\frac{2^2x^2y^5}{3x^4y}}=\sqrt{\frac{2^2y^4}{3x^2}}=\frac{\sqrt{2^2y^4}}{\sqrt{3x^2}}=\frac{2y^2}{x\sqrt{3}}$$

$$=\frac{2y^2}{x\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{2y^2\sqrt{3}}{3x}$$

You Try It 5

Simplify: $\dfrac{\sqrt{15x^6y^7}}{\sqrt{3x^7y^9}}$

Your solution

Example 6

Simplify: $\dfrac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}$

Solution

The denominator contains a binomial expression. Multiply the numerator and denominator by the conjugate of the denominator.

$$\frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}=\frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}\cdot\frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}$$

$$=\frac{\sqrt{4}-\sqrt{12}}{2-6}=\frac{2-2\sqrt{3}}{-4}=\frac{2(1-\sqrt{3})}{-4}$$

$$=\frac{1-\sqrt{3}}{-2}=\frac{1-\sqrt{3}}{2}$$

You Try It 6

Simplify: $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{6}}$

Your solution

Example 7

Simplify: $\dfrac{3-\sqrt{5}}{2+3\sqrt{5}}$

Solution

$$\frac{3-\sqrt{5}}{2+3\sqrt{5}}=\frac{3-\sqrt{5}}{2+3\sqrt{5}}\cdot\frac{2-3\sqrt{5}}{2-3\sqrt{5}}$$

$$=\frac{6-9\sqrt{5}-2\sqrt{5}+3(\sqrt{5})^2}{4-9\cdot 5}$$

$$=\frac{6-11\sqrt{5}+15}{4-45}$$

$$=\frac{21-11\sqrt{5}}{-41}=\frac{21-11\sqrt{5}}{41}$$

You Try It 7

Simplify: $\dfrac{5+\sqrt{y}}{1-2\sqrt{y}}$

Your solution

Solutions on p. S24

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10.3 Exercises

Objective A

Simplify.

1. $\sqrt{5} \cdot \sqrt{5}$
2. $\sqrt{11} \cdot \sqrt{11}$
3. $\sqrt{3} \cdot \sqrt{12}$
4. $\sqrt{2} \cdot \sqrt{8}$
5. $\sqrt{x} \cdot \sqrt{x}$
6. $\sqrt{y} \cdot \sqrt{y}$
7. $\sqrt{xy^3} \cdot \sqrt{x^5y}$
8. $\sqrt{a^3b^5} \cdot \sqrt{ab^5}$
9. $\sqrt{3a^2b^5} \cdot \sqrt{6ab^7}$
10. $\sqrt{5x^3y} \cdot \sqrt{10x^2y}$
11. $\sqrt{6a^3b^2} \cdot \sqrt{24a^5b}$
12. $\sqrt{8ab^5} \cdot \sqrt{12a^7b}$
13. $\sqrt{2}(\sqrt{2} - \sqrt{3})$
14. $3(\sqrt{12} - \sqrt{3})$
15. $\sqrt{x}(\sqrt{x} - \sqrt{y})$
16. $\sqrt{b}(\sqrt{a} - \sqrt{b})$
17. $\sqrt{5}(\sqrt{10} - \sqrt{x})$
18. $\sqrt{6}(\sqrt{y} - \sqrt{18})$
19. $\sqrt{8}(\sqrt{2} - \sqrt{5})$
20. $\sqrt{10}(\sqrt{20} - \sqrt{a})$
21. $(\sqrt{x} - 3)^2$
22. $(2\sqrt{a} - y)^2$
23. $\sqrt{3a}(\sqrt{3a} - \sqrt{3b})$
24. $\sqrt{5x}(\sqrt{10x} - \sqrt{x})$
25. $\sqrt{2ac} \cdot \sqrt{5ab} \cdot \sqrt{10cb}$
26. $\sqrt{3xy} \cdot \sqrt{6x^3y} \cdot \sqrt{2y^2}$
27. $(\sqrt{5} + 3)(2\sqrt{5} - 4)$
28. $(2 - 3\sqrt{7})(5 + 2\sqrt{7})$
29. $(4 + \sqrt{8})(3 + \sqrt{2})$
30. $(6 - \sqrt{27})(2 + \sqrt{3})$
31. $(2\sqrt{x} + 4)(3\sqrt{x} - 1)$
32. $(5 + \sqrt{y})(6 - 3\sqrt{y})$
33. $(3\sqrt{x} - 2y)(5\sqrt{x} - 4y)$
34. $(5\sqrt{x} + 2\sqrt{y})(3\sqrt{x} - \sqrt{y})$
35. $(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y})$

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Objective B

Simplify.

36. $\frac{\sqrt{32}}{\sqrt{2}}$ **37.** $\frac{\sqrt{45}}{\sqrt{5}}$ **38.** $\frac{\sqrt{98}}{\sqrt{2}}$ **39.** $\frac{\sqrt{48}}{\sqrt{3}}$ **40.** $\frac{\sqrt{27a}}{\sqrt{3a}}$

41. $\frac{\sqrt{72x^5}}{\sqrt{2x}}$ **42.** $\frac{\sqrt{15x^3y}}{\sqrt{3xy}}$ **43.** $\frac{\sqrt{40x^5y^2}}{\sqrt{5xy}}$ **44.** $\frac{\sqrt{2a^5b^4}}{\sqrt{98ab^4}}$ **45.** $\frac{\sqrt{48x^5y^2}}{\sqrt{3x^3y}}$

46. $\frac{\sqrt{9xy^2}}{\sqrt{27x}}$ **47.** $\frac{\sqrt{4x^2y}}{\sqrt{3xy^3}}$ **48.** $\frac{\sqrt{16x^3y^2}}{\sqrt{8x^3y}}$ **49.** $\frac{\sqrt{2}}{\sqrt{8}+4}$

50. $\frac{1}{\sqrt{2}-3}$ **51.** $\frac{5}{\sqrt{7}-3}$ **52.** $\frac{3}{5+\sqrt{5}}$ **53.** $\frac{\sqrt{3}}{5-\sqrt{27}}$

54. $\frac{7}{\sqrt{2}-7}$ **55.** $\frac{3-\sqrt{6}}{5-2\sqrt{6}}$ **56.** $\frac{6-2\sqrt{3}}{4+3\sqrt{3}}$ **57.** $\frac{-6}{4+\sqrt{2}}$

58. $\frac{\sqrt{2}+2\sqrt{6}}{2\sqrt{2}-3\sqrt{6}}$ **59.** $\frac{2\sqrt{3}-\sqrt{6}}{5\sqrt{3}+2\sqrt{6}}$ **60.** $\frac{3+\sqrt{x}}{2-\sqrt{x}}$ **61.** $\frac{-\sqrt{15}}{3-\sqrt{12}}$

62. $\frac{\sqrt{a}-4}{2\sqrt{a}+2}$ **63.** $\frac{\sqrt{xy}}{\sqrt{x}-\sqrt{y}}$ **64.** $\frac{\sqrt{x}}{\sqrt{x}-\sqrt{y}}$ **65.** $\frac{-12}{\sqrt{6}-3}$

APPLYING THE CONCEPTS

66. In your own words, describe the process of rationalizing the denominator.

67. Show that $(1+\sqrt{6})$ and $(1-\sqrt{6})$ are solutions of the equation $x^2-2x-5=0$.

68. Answer true or false. If the answer is false, write the correct answer.

a. $(\sqrt{y})^4 = y^2$ **b.** $(2\sqrt{x})^3 = 8x\sqrt{x}$ **c.** $(\sqrt{x}+1)^2 = x+1$ **d.** $\frac{1}{2-\sqrt{3}} = 2+\sqrt{3}$

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10.4 Solving Equations Containing Radical Expressions

Objective A To solve an equation containing a radical expression



An equation that contains a variable expression in a radicand is a **radical equation.**

$$\sqrt{x} = 4 \qquad \sqrt{x+2} = \sqrt{x-7}$$

Radical equations

The following property of equality states that if two numbers are equal, the squares of the numbers are equal. This property is used to solve radical equations.

Property of Squaring Both Sides of an Equation

If a and b are real numbers and $a = b$, then $a^2 = b^2$.

The first step when solving a radical equation is to isolate a radical in the equation.

➡ Solve: $\sqrt{x-2} - 7 = 0$

$$\begin{aligned} \sqrt{x-2} - 7 &= 0 \\ \sqrt{x-2} &= 7 \\ (\sqrt{x-2})^2 &= 7^2 \\ x - 2 &= 49 \\ x &= 51 \end{aligned}$$

- Rewrite the equation with the radical on one side of the equation and the constant on the other side.
- Square both sides of the equation.
- Solve the resulting equation.

Check: $\sqrt{x-2} - 7 = 0$

$\sqrt{51-2} - 7$	0
$\sqrt{49} - 7$	0
$\sqrt{7^2} - 7$	0
$7 - 7$	0
$0 = 0$	A true equation

The solution is 51.

When both sides of an equation are squared, the resulting equation may have a solution that is not a solution of the original equation. Checking a proposed solution of a radical equation, as we did in the previous example, is a necessary step.

➡ Solve: $\sqrt{2x-5} + 3 = 0$

$$\begin{aligned} \sqrt{2x-5} + 3 &= 0 \\ \sqrt{2x-5} &= -3 \\ (\sqrt{2x-5})^2 &= (-3)^2 \\ 2x - 5 &= 9 \\ 2x &= 14 \\ x &= 7 \end{aligned}$$

- Rewrite the equation with the radical on one side of the equation and the constant on the other side.
- Square each side of the equation.
- Solve for x.

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TAKE NOTE

Any time each side of an equation is squared, you must check the proposed solution of the equation.

Here is the check for the equation on the previous page.

Check: $\sqrt{2x - 5} + 3 = 0$

$$\begin{array}{r|l} \sqrt{2 \cdot 7 - 5} + 3 & 0 \\ \sqrt{14 - 5} + 3 & 0 \\ \sqrt{9} + 3 & 0 \\ 3 + 3 & 0 \\ 6 \neq 0 & \end{array}$$

7 does not check as a solution. The equation has no solution.

Example 1

Solve: $\sqrt{3x} + 2 = 5$

Solution

$$\begin{aligned} \sqrt{3x} + 2 &= 5 \\ \sqrt{3x} &= 3 \\ (\sqrt{3x})^2 &= 3^2 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

Check: $\sqrt{3x} + 2 = 5$

$$\begin{array}{r|l} \sqrt{3 \cdot 3} + 2 & 5 \\ \sqrt{3^2} + 2 & 5 \\ 3 + 2 & 5 \\ 5 = 5 & \end{array}$$

The solution is 3.

You Try It 1

Solve: $\sqrt{4x} + 3 = 7$

Your solution

Example 2

Solve: $\sqrt{x} - \sqrt{x - 5} = 1$

Solution

$$\begin{aligned} \sqrt{x} - \sqrt{x - 5} &= 1 \\ \sqrt{x} &= 1 + \sqrt{x - 5} \\ (\sqrt{x})^2 &= (1 + \sqrt{x - 5})^2 \\ x &= 1 + 2\sqrt{x - 5} + (x - 5) \\ 4 &= 2\sqrt{x - 5} \\ 2 &= \sqrt{x - 5} \\ 2^2 &= (\sqrt{x - 5})^2 \\ 4 &= x - 5 \\ 9 &= x \end{aligned}$$

Check:

$$\begin{array}{r|l} \sqrt{x} - \sqrt{x - 5} = 1 & \\ \sqrt{9} - \sqrt{9 - 5} & 1 \\ \sqrt{9} - \sqrt{4} & 1 \\ 3 - 2 & 1 \\ 1 = 1 & \end{array}$$

The solution is 9.

You Try It 2

Solve: $\sqrt{x} + \sqrt{x + 9} = 9$

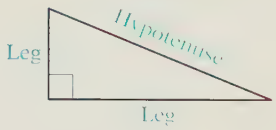
Your solution

Solutions on p. S24

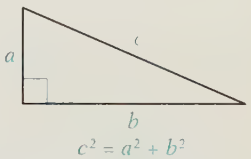
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Objective B To solve application problems

A right triangle contains one 90° angle. The side opposite the 90° angle is called the **hypotenuse.** The other two sides are called **legs.**



Pythagoras, a Greek mathematician who lived around 550 B.C., is given credit for the Pythagorean Theorem. It states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. Actually, this theorem was known to the Babylonians around 1200 B.C.



POINT OF INTEREST

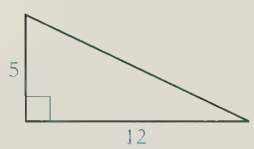
The first known proof of this theorem occurs in a Chinese text, *Arithmetic Classic,* which was first written around 600 B.C. (but there are no existing copies) and revised over a period of 500 years. The earliest known copy of this text dates from approximately 100 B.C.

Pythagorean Theorem

If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $c^2 = a^2 + b^2$.

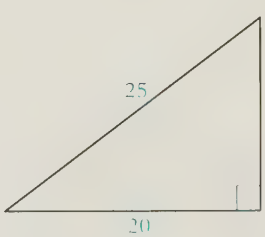
Using this theorem, we can find the hypotenuse of a right triangle when we know the two legs. Use the formula

$$\begin{aligned}\text{Hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{leg})^2}\\ c &= \sqrt{a^2 + b^2}\\ &= \sqrt{(5)^2 + (12)^2}\\ &= \sqrt{25 + 144}\\ &= \sqrt{169}\\ &= 13\end{aligned}$$



The leg of a right triangle can be found when one leg and the hypotenuse are known. Use the formula

$$\begin{aligned}\text{Leg} &= \sqrt{(\text{hypotenuse})^2 - (\text{leg})^2}\\ a &= \sqrt{c^2 - b^2}\\ &= \sqrt{(25)^2 - (20)^2}\\ &= \sqrt{625 - 400}\\ &= \sqrt{225}\\ &= 15\end{aligned}$$



Example 3 and You Try It 3 on the following page illustrate the use of the Pythagorean Theorem. Example 4 and You Try It 4 illustrate other applications of radical equations.

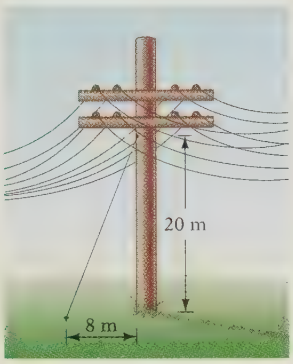
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Example 3

A guy wire is attached to a point 20 m above the ground on a telephone pole. The wire is anchored to the ground at a point 8 m from the base of the pole. Find the length of the guy wire. Round to the nearest tenth.

Strategy

To find the length of the guy wire, use the Pythagorean Theorem. One leg is 20 m. The other leg is 8 m. The guy wire is the hypotenuse. Solve the Pythagorean Theorem for the hypotenuse.



Solution

$$c = \sqrt{a^2 + b^2}$$
$$= \sqrt{(20)^2 + (8)^2}$$
$$= \sqrt{400 + 64} = \sqrt{464} \approx 21.5$$

The guy wire has a length of 21.5 m.

You Try It 3

A ladder 8 ft long is resting against a building. How high on the building will the ladder reach when the bottom of the ladder is 3 ft from the building? Round to the nearest hundredth.

Your strategy

Your solution

Example 4

How far would a submarine periscope have to be above the water to locate a ship 4 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water. Round to the nearest hundredth.

Strategy

To find the height above the water, replace d in the equation with the given value and solve for h.

Solution

$$\sqrt{1.5h} = d$$
$$\sqrt{1.5h} = 4$$
$$(\sqrt{1.5h})^2 = 4^2$$
$$1.5h = 16$$
$$h = \frac{16}{1.5} \approx 10.67$$

The periscope must be 10.67 ft above the water.

You Try It 4

Find the length of a pendulum that makes one swing in 2.5 s. The equation for the time for one swing is $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Use 3.14 for π. Round to the nearest hundredth.

Your strategy

Your solution

Solutions on p. S24

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10.4 Exercises

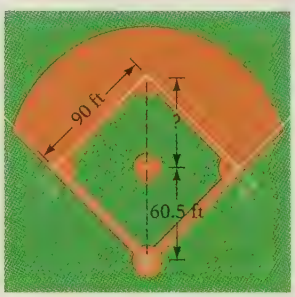
Objective A

Solve and check.

1. $\sqrt{x} = 5$
2. $\sqrt{y} = 7$
3. $\sqrt{a} = 12$
4. $\sqrt{a} = 9$
5. $\sqrt{5x} = 5$
6. $\sqrt{4x} + 5 = 2$
7. $\sqrt{3x} + 9 = 4$
8. $\sqrt{3x - 2} = 4$
9. $\sqrt{5x + 6} = 1$
10. $\sqrt{2x + 1} = 7$
11. $\sqrt{5x + 4} = 3$
12. $0 = 2 - \sqrt{3 - x}$
13. $0 = 5 - \sqrt{10 + x}$
14. $\sqrt{5x + 2} = 0$
15. $\sqrt{3x - 7} = 0$
16. $\sqrt{3x} - 6 = -4$
17. $\sqrt{5x} + 8 = 23$
18. $0 = 3 - \sqrt{3x - 9}$
19. $\sqrt{x + 2} = \sqrt{x + 1}$
20. $\sqrt{3x - 2} = \sqrt{5 - 2x}$

Objective B Application Problems

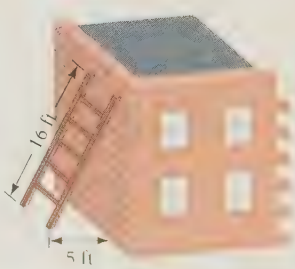
21. The infield of a baseball diamond is a square. The distance between successive bases is 90 ft. The pitcher's mound is on the diagonal between home plate and second base at a distance of 60.5 ft from home plate. (See the figure to the right.) Is the pitcher's mound more or less than halfway between home plate and second base?



22. The infield of a softball diamond is a square. The distance between successive bases is 60 ft. The pitcher's mound is on the diagonal between home plate and second base at a distance of 46 ft from home plate. Is the pitcher's mound more or less than halfway between home plate and second base?

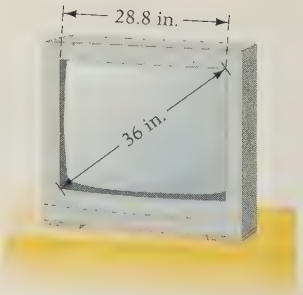
23. How far would a submarine periscope have to be above the water to locate a ship 5 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water. Round to the nearest hundredth.

24. A 16-foot ladder is leaning against a building. How high on the building will the ladder reach when the bottom of the ladder is 5 ft from the building? (See the figure to the right.) Round to the nearest tenth.



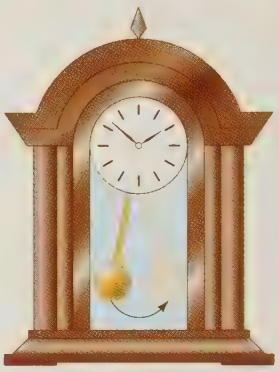
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25. The measure of a big-screen television is given by the length of a diagonal across the screen. A 36-inch television has a width of 28.8 in. Find the height of the screen to the nearest tenth of an inch.



26. The measure of a television screen is given by the length of a diagonal across the screen. A 33-inch big-screen television has a width of 26.4 in. Find the height of the screen to the nearest tenth of an inch.

27. The speed of a child riding a merry-go-round at a carnival is given by the equation $v = \sqrt{12r}$, where v is the speed in feet per second and r is the distance in feet from the center of the merry-go-round to the rider. If a child is moving at 15 ft/s, how far is the child from the center of the merry-go-round?



28. Find the length of a pendulum that makes one swing in 1.5 s. The equation for the time of one swing of a pendulum is $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Round to the nearest hundredth.

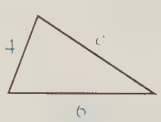
APPLYING THE CONCEPTS

29. In the coordinate plane, a triangle is formed by drawing lines between the points (0, 0) and (5, 0), (5, 0) and (5, 12), and (5, 12) and (0, 0). Find the perimeter of the triangle.

30. The hypotenuse of a right triangle is $5\sqrt{2}$ cm, and one leg is $4\sqrt{2}$ cm.
a. Find the perimeter of the triangle.
b. Find the area of the triangle.

31. If a and b are real numbers and $a^2 = b^2$, does $a = b$? Explain your answer.

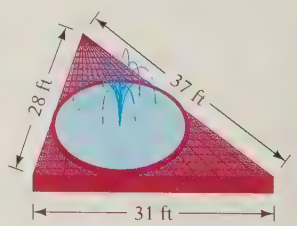
32. Can the Pythagorean Theorem be used to find the length of side c of the triangle at the right? If so, determine c. If not, explain why the theorem cannot be used.



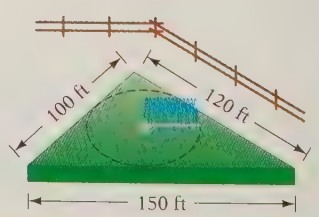
33. A circular fountain is being designed for a triangular plaza in a cultural center. The fountain is placed so that each side of the triangle touches the fountain as shown in the diagram at the right. Find the area of the fountain. The formula for the radius of the circle is given by

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

where $s = \frac{1}{2}(a + b + c)$ and a, b, and c are the lengths of the sides of the triangle. Round to the nearest hundredth.



34. A farmer owns a triangular piece of land that measures 100 ft by 120 ft by 150 ft, as shown at the right. Find the maximum area that can be irrigated with a circular irrigation system in which the irrigation does not go outside the triangular shape. Use the formulas in Exercise 33. Round to the nearest hundredth.



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Focus on Problem Solving

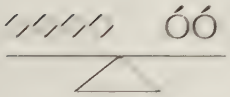
Deductive Reasoning

Deductive reasoning uses a rule or statement of fact to reach a conclusion. We established the rule in the section "Ratio and Proportion," Objective C, in the Rational Expressions chapter, that if two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar. Thus any time we establish this fact about two triangles, we know that the triangles are similar. Below are two examples of deductive reasoning.



Given that ΔΔΔ = ◇◇◇◇ and ◇◇◇◇ = ÓÓ, then ΔΔΔΔΔΔ is equivalent to how many Ós?

Because 3 Δs = 4 ◇s and 4 ◇s = 2 Ós, 3 Δs = 2 Ós.



6 Δs is twice 3 Δs. We need to find twice 2 Ós, which is 4 Ós.

Therefore, ΔΔΔΔΔΔ = ÓÓÓÓ.

Lomax, Parish, Thorpe, and Wong are neighbors. Each drives a different type of vehicle: a compact car, a sedan, a sports car, or a station wagon. From the following statements, determine which type of vehicle each of the neighbors drives.

1. Although the vehicle owned by Lomax has more mileage on it than does either the sedan or the sports car, it does not have the highest mileage of all four cars. (Use X1 in the chart below to eliminate possibilities due to this statement.)
2. Wong and the owner of the sports car live on one side of the street, and Thorpe and the owner of the compact car live on the other side of the street. (Use X2 to eliminate possibilities due to this statement.)
3. Thorpe owns the vehicle with the most mileage on it. (Use X3 to eliminate possibilities due to this statement.)

TAKE NOTE

To use the chart to solve this problem, write an X in a box to indicate that a possibility has been eliminated. Write a ✓ to show that a match has been found. When a row or column has 3 X's, a ✓ is written in the remaining open box in that row or column of the chart.

	Compact	*Sedan*	*Sports Car*	*Wagon*
Lomax	✓	X1	X1	X2
Parish	X2	X2	✓	X2
Thorpe	X2	X3	X2	✓
Wong	X2		X2	

Lomax drives the compact car, Parish drives the sports car, Thorpe drives the station wagon, and Wong drives the sedan.

4. Given that ‡‡ = ••••• and ••••• = ΛΛ, then ‡‡‡‡‡ = how many Λs?

5. Given that □□□□□□ = ÓÓÓÓ and ÓÓÓÓ = ÎÎ, then □□□ = how many Îs?

6. Given that ▫▫▫▫ = ΩΩΩ and ΩΩΩ = ΔΔ, then ΔΔΔΔ = how many ▫s?

7. Given that ŸŸŸŸŸ = §§ and §§ = ÂÂÂ, then ÂÂÂÂÂÂ = how many Ÿs?

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8. Anna, Kay, Megan, and Nicole decide to travel together during spring break, but they need to find a destination where each of them will be able to participate in her favorite sport (golf, horseback riding, sailing, or tennis). From the following statements, determine the favorite sport of each student.
   a. Anna and the student whose favorite sport is sailing both like to swim, whereas Nicole and the student whose favorite sport is tennis would prefer to scuba dive.
   b. Megan and the student whose favorite sport is sailing are roommates. Nicole and the student whose favorite sport is golf each live in a single.

9. Chang, Nick, Pablo, and Saul each take a different form of transportation (bus, car, subway, or taxi) from the office to the airport. From the following statements, determine which form of transportation each takes.
   a. Chang spent more on transportation than the fellow who took the bus but less than the fellow who took the taxi.
   b. Pablo, who did not travel by bus and who spent the least on transportation, arrived at the airport after Nick but before the fellow who took the subway.
   c. Saul spent less on transportation than either Chang or Nick.

Projects and Group Activities

Measurements as Approximations

From arithmetic, you know the rules for rounding decimals.

If the digit to the right of the given place value is less than 5, drop that digit and all digits to the right.

6.31 rounded to the nearest tenth is 6.3.

If the digit to the right of the given place value is greater than or equal to 5, increase the given place value by 1 and drop all digits to its right.

6.28 rounded to the nearest tenth is 6.3.

Given the rules for rounding numbers, what range of values can the number 6.3 represent? The smallest possible value of 6.3 is 6.25; any number smaller than that would not have been rounded up to 6.3. What about the largest possible value of 6.3? The number 6.34 would be rounded down to 6.3. So would the numbers 6.349, 6.3499, 6.34999, and so on. Therefore, we cannot name the largest possible value of 6.3. We can say that the number must be less than 6.35. Any number less than 6.35 would be rounded down to 6.3. The exact value of 6.3 is greater than or equal to 6.25 and less than 6.35.

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The dimensions of a rectangle are given as 4.3 cm by 3.2 cm. Using the smallest and the largest possible values of the length and of the width, we can represent the possible values of the area, A, of the rectangle as follows:

$$4.25(3.15) \le A < 4.35(3.25)$$

$$13.3875 \le A < 14.1375$$

The area is greater than or equal to 13.3875 cm^2 and less than 14.1375 cm^2.

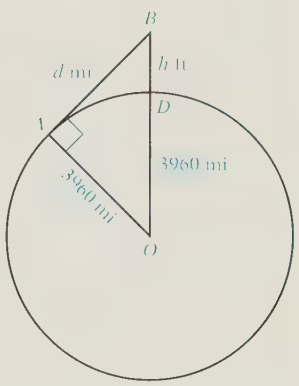
1. The measurements of the three sides of a triangle are given as 8.37 m, 5.42 m, and 9.61 m. Find the possible lengths of the perimeter of the triangle.
2. The length of a side of a square is given as 4.7 cm. Find the possible values for the area of the square.
3. What are the possible values for the area of a rectangle whose dimensions are 6.5 cm by 7.8 cm?
4. The length of a box is 40 cm, the width is 25 cm, and the height is 8 cm. What are the possible values of the volume of the box?

Distance to the Horizon

In Section 4 of this chapter, we used the formula $d = \sqrt{1.5h}$ to calculate the approximate distance d (in miles) that a person could see who uses a periscope h feet above the water. That formula is derived by using the Pythagorean Theorem.

Consider the diagram (not to scale) at the right, which shows the earth as a sphere and the periscope extending h feet above the surface. From geometry, because AB is tangent to the circle and OA is a radius, triangle AOB is a right triangle. Therefore,

$$(OA)^2 + (AB)^2 = (OB)^2$$



Substituting into this formula, we have

$$3960^2 + d^2 = \left(3960 + \frac{h}{5280}\right)^2$$

Because h is in feet, $\frac{h}{5280}$ is in miles.

$$3960^2 + d^2 = 3960^2 + \frac{2 \cdot 3960}{5280}h + \left(\frac{h}{5280}\right)^2$$

$$d^2 = \frac{3}{2}h + \left(\frac{h}{5280}\right)^2$$

$$d = \sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2}$$

At this point, an assumption is made that $\sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2} \approx \sqrt{1.5h}$, where we have written $\frac{3}{2}$ as 1.5. Thus $d \approx \sqrt{1.5h}$ is used to approximate the distance that can be seen using a periscope h feet above the water.

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1. Write a paragraph that justifies the assumption that

$$\sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2} \approx \sqrt{1.5h}$$

(*Suggestion:* Evaluate each expression for various values of h. Because h is the height of a periscope above water, it is unlikely that $h > 25$ ft.)

2. The distance d is the distance from the top of the periscope to A. The distance along the surface of the water is given by arc AD. This distance, D, can be approximated by the equation

$$D \approx \sqrt{1.5h} + 0.306186\left(\sqrt{\frac{h}{5280}}\right)^3$$

Using this formula, calculate D when $h = 10$.

Chapter Summary

Key Words

A *square root* of a positive number x is a number whose square is x.

The *principal square root* of a number is the positive square root.

The symbol $\sqrt{\ }$ is called a *radical sign* and is used to indicate the principal square root of a number. The *radicand* is the number under the radical sign.

The square of an integer is a *perfect square.*

If a whole number is not a perfect square, its square root can only be approximated. Such numbers are *irrational numbers.* Their decimal representations never terminate or repeat.

Conjugates are binomial expressions that differ only in the sign of a term. (The expressions $a + b$ and $a - b$ are conjugates.)

Rationalizing the denominator is the procedure used to remove a radical from the denominator of a fraction.

A *radical equation* is an equation that contains a variable expression in a radicand.

Essential Rules

The Product Property of Square Roots

If a and b are positive real numbers, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

The Quotient Property of Square Roots

If a and b are positive real numbers, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$.

Property of Squaring Both Sides of an Equation

If a and b are real numbers and $a = b$, then $a^2 = b^2$.

Pythagorean Theorem

If a and b are legs of a right triangle and c is the hypotenuse, then $c^2 = a^2 + b^2$.

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Chapter Review

1. Simplify: $\sqrt{3}(\sqrt{12} - \sqrt{3})$

2. Simplify: $3\sqrt{18a^5b}$

3. Simplify: $2\sqrt{36}$

4. Simplify: $\sqrt{6a}(\sqrt{3a} + \sqrt{2a})$

5. Simplify: $\dfrac{2x}{\sqrt{3} - \sqrt{5}}$

6. Simplify: $-3\sqrt{120}$

7. Solve: $\sqrt{5x} = 10$

8. Simplify: $5\sqrt{48}$

9. Simplify: $\dfrac{\sqrt{98x^7y^9}}{\sqrt{2x^3y}}$

10. Solve: $3 - \sqrt{7x} = 5$

11. Simplify: $6a\sqrt{80b} - \sqrt{180a^2b} + 5a\sqrt{b}$

12. Simplify: $4\sqrt{250}$

13. Simplify: $2x\sqrt{60x^3y^3} + 3x^2y\sqrt{15xy}$

14. Simplify: $(4\sqrt{y} - \sqrt{5})(2\sqrt{y} + 3\sqrt{5})$

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15. Simplify: $3\sqrt{12x} + 5\sqrt{48x}$

16. Solve: $\sqrt{2x - 3} + 4 = 0$

17. Simplify: $\frac{8}{\sqrt{x} - 3}$

18. Simplify: $4y\sqrt{243x^{17}y^9}$

19. Simplify: $y\sqrt{24y^6}$

20. Solve: $\sqrt{5x + 1} = \sqrt{20x - 8}$

21. Simplify:
$2x^2\sqrt{18x^2y^5} + 6y\sqrt{2x^6y^3} - 9xy^2\sqrt{8x^4y}$

22. Simplify: $\frac{16}{\sqrt{a}}$

23. The weight of an object is related to the distance the object is above the surface of the earth. An equation for this relationship is $d = 4000\sqrt{\frac{W_0}{W_d}} - 4000$, where W_0 is an object's weight on the surface of the earth and W_d is the object's weight at a distance of d miles above the earth's surface. If a space explorer weighs 36 lb at a distance of 4000 mi above the surface of the earth, how much does the explorer weigh on the surface of the earth?

24. A tsunami is a great sea wave produced by underwater earthquakes or volcanic eruption. The velocity of a tsunami as it approaches land depends on the depth of the water and can be approximated by the equation $v = 3\sqrt{d}$, where d is the depth of the water in feet and v is the velocity of the tsunami in feet per second. Find the depth of the water if the velocity is 30 ft/s.

25. A bicycle will overturn if it rounds a corner too sharply or too fast. An equation for the maximum velocity at which a cyclist can turn a corner without tipping over is $v = 4\sqrt{r}$, where v is the velocity of the bicycle in miles per hour and r is the radius of the corner in feet. What is the radius of the sharpest corner that a cyclist can safely turn if riding at 20 mph?

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Chapter Test

1. Simplify: $\sqrt{121x^8y^2}$

2. Simplify: $\sqrt{3x^2y}\sqrt{6xy^2}\sqrt{2x}$

3. Simplify: $5\sqrt{8} - 3\sqrt{50}$

4. Simplify: $\sqrt{45}$

5. Simplify: $\dfrac{\sqrt{162}}{\sqrt{2}}$

6. Solve: $\sqrt{9x} + 3 = 18$

7. Simplify: $\sqrt{32a^5b^{11}}$

8. Simplify: $\dfrac{\sqrt{98a^6b^4}}{\sqrt{2a^3b^2}}$

9. Simplify: $\dfrac{2}{\sqrt{3} - 1}$

10. Simplify: $\sqrt{8x^3y}\sqrt{10xy^4}$

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11. Solve: $\sqrt{2x - 4} = \sqrt{3x - 5}$

12. Simplify: $3\sqrt{8y} - 2\sqrt{72x} + 5\sqrt{18y}$

13. Simplify: $\sqrt{72x^7y^2}$

14. Simplify: $(\sqrt{y} - 3)(\sqrt{y} + 5)$

15. Simplify: $2x\sqrt{3xy^3} - 2y\sqrt{12x^3y} - 3xy\sqrt{xy}$

16. Simplify: $\dfrac{2 - \sqrt{5}}{6 + \sqrt{5}}$

17. Simplify: $\sqrt{a}(\sqrt{a} - \sqrt{b})$

18. Simplify: $\sqrt{75}$

19. Find the length of a pendulum that makes one swing in 3 s. The equation for the time of one swing of a pendulum is $T = 2\pi\sqrt{\dfrac{L}{32}}$, where T is the time in seconds and L is the length in feet. Round to the nearest hundredth.

20. The square root of the sum of two consecutive odd integers is equal to 10. Find the larger integer.

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Cumulative Review

1. Simplify:
$\left(\frac{2}{3}\right)^2 \cdot \left(\frac{3}{4} - \frac{3}{2}\right) + \left(\frac{1}{2}\right)^2$

2. Simplify:
$-3[x - 2(3 - 2x) - 5x] + 2x$

3. Solve:
$2x - 4[3x - 2(1 - 3x)] = 2(3 - 4x)$

4. Simplify: $(-3x^2y)(-2x^3y^4)$

5. Simplify: $\frac{12b^4 - 6b^2 + 2}{-6b^2}$

6. Given $f(x) = \frac{2x}{x - 3}$, find $f(-3)$.

7. Factor: $2a^3 - 16a^2 + 30a$

8. Simplify: $\frac{3x^3 - 6x^2}{4x^2 + 4x} \cdot \frac{3x - 9}{9x^3 - 45x^2 + 54x}$

9. Simplify: $\frac{x + 2}{x - 4} - \frac{6}{(x - 4)(x - 3)}$

10. Solve: $\frac{x}{2x - 5} - 2 = \frac{3x}{2x - 5}$

11. Find the equation of the line that contains the point $(-2, -3)$ and has slope $\frac{1}{2}$.

12. Solve by substitution:
$4x - 3y = 1$
$2x + y = 3$

13. Solve by the addition method:
$5x + 4y = 7$
$3x - 2y = 13$

14. Solve: $3(x - 7) \geq 5x - 12$

15. Simplify: $\sqrt{108}$

16. Simplify: $3\sqrt{32} - 2\sqrt{128}$

17. Simplify: $2a\sqrt{2ab^3} + b\sqrt{8a^3b} - 5ab\sqrt{ab}$

18. Simplify: $\sqrt{2a^9b}\sqrt{98ab^3}\sqrt{2a}$

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19. Simplify: $\sqrt{3}(\sqrt{6} - \sqrt{x^2})$

20. Simplify: $\frac{\sqrt{320}}{\sqrt{5}}$

21. Simplify: $\frac{3}{2 - \sqrt{5}}$

22. Solve: $\sqrt{3x - 2} - 4 = 0$

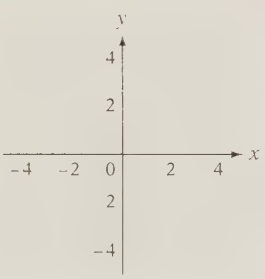
23. The selling price for a book is \$29.40. The markup rate used by the bookstore is 20%. Find the cost of the book. Use the formula $S = C + rC$, where S is the selling price, C is the cost, and r is the markup rate.

24. How many ounces of pure water must be added to 40 oz of a 12% salt solution to make a salt solution that is 5% salt?

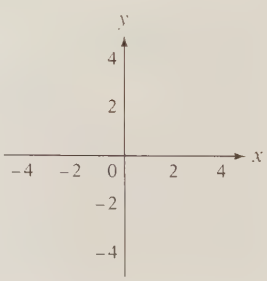
25. The sum of two numbers is twenty-one. The product of the two numbers is one hundred four. Find the two numbers.

26. A small water pipe takes twice as long to fill a tank as does a larger water pipe. With both pipes open, it takes 16 h to fill the tank. Find the time it would take the small pipe working alone to fill the tank.

27. Solve by graphing: $3x - 2y = 8$
$4x + 5y = 3$



28. Graph the solution set of $3x + y \leq 2$.



29. The square root of the sum of two consecutive integers is equal to 9. Find the smaller integer.

30. A stone is dropped from a building and hits the ground 5 s later. How high is the building? The equation for the distance an object falls in T seconds is $T = \sqrt{\frac{d}{16}}$, where d is the distance in feet.

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CHAPTER

11 Quadratic Equations



A forensic scientist assists police in the investigation of crimes. Chemistry, biology, and mathematics are critical tools for the forensic scientist when analyzing evidence such as matching the DNA of a suspect to the DNA found at a crime scene.

Objectives

Section 11.1

To solve a quadratic equation by factoring

To solve a quadratic equation by taking square roots

Section 11.2

To solve a quadratic equation by completing the square

Section 11.3

To solve a quadratic equation by using the quadratic formula

Section 11.4

To graph a quadratic equation of the form $y = ax^2 + bx + c$

Section 11.5

To solve application problems



Algebraic Symbolism

The way in which an algebraic expression or equation is written has gone through several stages of development. First there was the *rhetoric*, which was in vogue until the late 13th century. In this method, expressions were written out in sentences. The word *res* was used to represent an unknown.

Rhetoric: From the additive *res* in the additive *res* results in a square *res*. From the three in an additive *res* comes three additive *res* and from the subtractive four in the additive *res* comes subtractive four *res*. From three in subtractive four comes subtractive twelve.

Modern: $(x + 3)(x - 4) = x^2 - x - 12$

The second stage was *syncoptic*, which was a shorthand in which abbreviations were used for words.

Syncoptic: a 6 in b quad $-$ c plano 4 in b + b cub

Modern: $6ab^2 - 4cb + b^3$

The current modern stage, called the *symbolic* stage, began with the use of exponents rather than words to symbolize exponential expressions. This occurred near the beginning of the 17th century with the publication of the book *La Geometrie* by René Descartes. Modern notation is still evolving as mathematicians continue to search for convenient methods to symbolize concepts.

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11.1 Solving Quadratic Equations by Factoring or by Taking Square Roots

Objective A To solve a quadratic equation by factoring



An equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants and $a \neq 0$, is a **quadratic equation.**

$4x^2 - 3x + 1 = 0, a = 4, b = -3, c = 1$
$3x^2 - 4 = 0, a = 3, b = 0, c = -4$
$\frac{x^2}{2} - 2x + 4 = 0, a = \frac{1}{2}, b = -2, c = 4$

A quadratic equation is also called a **second-degree equation.**

A quadratic equation is in **standard form** when the polynomial is in descending order and equal to zero.

Recall that the Principle of Zero Products states that if the product of two factors is zero, then at least one of the factors must be zero.

If $a \cdot b = 0$,
then $a = 0$ or $b = 0$.

The Principle of Zero Products can be used in solving quadratic equations.

➡ Solve by factoring: $2x^2 - x = 1$

$$2x^2 - x = 1$$
$$2x^2 - x - 1 = 0$$
$$(2x + 1)(x - 1) = 0$$
$$2x + 1 = 0 \qquad x - 1 = 0$$
$$2x = -1 \qquad x = 1$$
$$x = -\frac{1}{2}$$

- Write the equation in standard form.
- Factor.
- Use the Principle of Zero Products to set each factor equal to zero.
- Rewrite each equation in the form *variable = constant.*

TAKE NOTE

You should always check your solutions by substituting the proposed solutions back into the *original* equation.

Check:

$$\begin{array}{r|l} 2\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) & 1 \\ 2 \cdot \frac{1}{4} + \frac{1}{2} & 1 \\ \frac{1}{2} + \frac{1}{2} & 1 \\ 1 = 1 & \end{array} \qquad \begin{array}{r|l} 2(1)^2 - 1 & 1 \\ 2 \cdot 1 - 1 & 1 \\ 2 - 1 & 1 \\ 1 = 1 & \end{array}$$

The solutions are $-\frac{1}{2}$ and 1.

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➡ Solve by factoring: $3x^2 - 4x + 8 = (4x + 1)(x - 2)$

$3x^2 - 4x + 8 = (4x + 1)(x - 2)$

$3x^2 - 4x + 8 = 4x^2 - 7x - 2$ • Multiply the factors on the right side of the equation.

$0 = x^2 - 3x - 10$ • Write the equation in standard form.

$0 = (x - 5)(x + 2)$ • Factor.

$x - 5 = 0 \qquad x + 2 = 0$ • Use the Principle of Zero Products to set each factor equal to zero.

$x = 5 \qquad x = -2$ • Rewrite each equation in the form *variable = constant*.

Check:

$3x^2 - 4x + 8 = (4x + 1)(x - 2)$	
$3(5)^2 - 4(5) + 8$	$(4[5] + 1)(5 - 2)$
$3(25) - 20 + 8$	$(20 + 1)(3)$
$75 - 20 + 8$	$(21)(3)$
$63 = 63$	

$3x^2 - 4x + 8 = (4x + 1)(x - 2)$	
$3(-2)^2 - 4(-2) + 8$	$(4[-2] + 1)(-2 - 2)$
$3(4) + 8 + 8$	$(-8 + 1)(-4)$
$12 + 8 + 8$	$(-7)(-4)$
$28 = 28$	

The solutions are 5 and -2.

➡ Solve by factoring: $x^2 - 10x + 25 = 0$

$x^2 - 10x + 25 = 0$

$(x - 5)(x - 5) = 0$ • Factor.

$x - 5 = 0 \qquad x - 5 = 0$ • Use the Principle of Zero Products.

$x = 5 \qquad x = 5$ • Solve each equation for x.

The solution is 5.

In this last example, 5 is called a **double root** of the quadratic equation.

Example 1

Solve by factoring: $\frac{z^2}{2} - \frac{z}{4} - \frac{1}{4} = 0$

Solution

$\frac{z^2}{2} - \frac{z}{4} - \frac{1}{4} = 0$

$4\left(\frac{z^2}{2} - \frac{z}{4} - \frac{1}{4}\right) = 4(0)$ • Multiply each side by 4.

$2z^2 - z - 1 = 0$

$(2z + 1)(z - 1) = 0$

$2z + 1 = 0 \qquad z - 1 = 0$

$2z = -1 \qquad z = 1$

$z = -\frac{1}{2}$

The solutions are $-\frac{1}{2}$ and 1.

You Try It 1

Solve by factoring: $\frac{3y^2}{2} + y - \frac{1}{2} = 0$

Your solution

Solution on p. S25

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Objective B To solve a quadratic equation by taking square roots

Consider a quadratic equation of the form $x^2 = a$. This equation can be solved by factoring.

$$\begin{aligned} x^2 &= 25 \\ x^2 - 25 &= 0 \\ (x - 5)(x + 5) &= 0 \\ x = 5 \quad & x = -5 \end{aligned}$$

The solutions are 5 and -5. The solutions are plus or minus the same number, which is frequently written by using $\pm$; for example, "the solutions are ± 5." Because ± 5 can be written as $\pm\sqrt{25}$, an alternative method of solving this equation is suggested.

The Square Root Property of an Equality

If $x^2 = a$, then $x = \pm\sqrt{a}$.

Solve by taking square roots: $x^2 = 25$

$$\begin{aligned} x^2 &= 25 \\ \sqrt{x^2} &= \sqrt{25} \\ x &= \pm\sqrt{25} = \pm 5 \end{aligned}$$

- Take the square root of each side of the equation. Then simplify.

The solutions are 5 and -5.

Solve by taking square roots: $3x^2 = 36$

$$\begin{aligned} 3x^2 &= 36 \\ x^2 &= 12 \\ \sqrt{x^2} &= \sqrt{12} \\ x &= \pm\sqrt{12} = \pm 2\sqrt{3} \end{aligned}$$

- Solve for x^2.
- Take the square root of each side.
- Simplify.

The solutions are $2\sqrt{3}$ and $-2\sqrt{3}$.

Solve by taking square roots: $49y^2 - 25 = 0$

$$\begin{aligned} 49y^2 - 25 &= 0 \\ 49y^2 &= 25 \\ y^2 &= \frac{25}{49} \\ \sqrt{y^2} &= \sqrt{\frac{25}{49}} \\ y &= \pm\frac{5}{7} \end{aligned}$$

- Solve for y^2.
- Take the square root of each side.
- Simplify.

The solutions are $\frac{5}{7}$ and $-\frac{5}{7}$.

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An equation that contains the square of a binomial can be solved by taking square roots.

➡ Solve by taking square roots: $2(x - 1)^2 - 36 = 0$

$2(x - 1)^2 - 36 = 0$

$2(x - 1)^2 = 36$ • Solve for $(x - 1)^2$.

$(x - 1)^2 = 18$

$\sqrt{(x - 1)^2} = \sqrt{18}$ • Take the square root of each side of the equation.

$x - 1 = \pm\sqrt{18}$

$x - 1 = \pm 3\sqrt{2}$ • Simplify.

$x - 1 = 3\sqrt{2}$ $\qquad$ $x - 1 = -3\sqrt{2}$ • Solve for x.

$x = 1 + 3\sqrt{2}$ $\qquad$ $x = 1 - 3\sqrt{2}$

The solutions are $1 + 3\sqrt{2}$ and $1 - 3\sqrt{2}$.

Example 2

Solve by taking square roots:
$x^2 + 16 = 0$

Solution

$x^2 + 16 = 0$

$x^2 = -16$

$\sqrt{x^2} = \sqrt{-16}$

$\sqrt{-16}$ is not a real number.

The equation has no real number solution.

You Try It 2

Solve by taking square roots:
$x^2 + 81 = 0$

Your solution

Example 3

Solve by taking square roots:
$5(y - 4)^2 = 25$

Solution

$5(y - 4)^2 = 25$

$(y - 4)^2 = 5$

$\sqrt{(y - 4)^2} = \sqrt{5}$

$y - 4 = \pm\sqrt{5}$

$y = 4 \pm \sqrt{5}$

The solutions are $4 + \sqrt{5}$ and $4 - \sqrt{5}$.

You Try It 3

Solve by taking square roots:
$7(z + 2)^2 = 21$

Your solution

Solutions on p. S25

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11.1 Exercises

Objective A

Solve by factoring.

1. $x^2 + 2x - 15 = 0$ **2.** $t^2 + 3t - 10 = 0$ **3.** $z^2 - 4z + 3 = 0$ **4.** $s^2 - 5s + 4 = 0$

5. $p^2 + 3p + 2 = 0$ **6.** $v^2 + 6v + 5 = 0$ **7.** $x^2 - 6x + 9 = 0$ **8.** $y^2 - 8y + 16 = 0$

9. $12y^2 + 8y = 0$ **10.** $6x^2 - 9x = 0$ **11.** $r^2 - 10 = 3r$ **12.** $t^2 - 12 = 4t$

13. $3v^2 - 5v + 2 = 0$ **14.** $2p^2 - 3p - 2 = 0$ **15.** $3s^2 + 8s = 3$

16. $3x^2 + 5x = 12$ **17.** $\frac{3}{4}z^2 - z = -\frac{1}{3}$ **18.** $\frac{r^2}{2} = 1 - \frac{r}{12}$

19. $4t^2 = 4t + 3$ **20.** $5y^2 + 11y = 12$ **21.** $4v^2 - 4v + 1 = 0$

22. $9s^2 - 6s + 1 = 0$ **23.** $x^2 - 9 = 0$ **24.** $t^2 - 16 = 0$

25. $4y^2 - 1 = 0$ **26.** $9z^2 - 4 = 0$ **27.** $x + 15 = x(x - 1)$

28. $p + 18 = p(p - 2)$ **29.** $r^2 - r - 2 = (2r - 1)(r - 3)$ **30.** $s^2 + 5s - 4 = (2s + 1)(s - 4)$

Objective B

Solve by taking square roots.

31. $x^2 = 36$ **32.** $y^2 = 49$ **33.** $v^2 - 1 = 0$

34. $z^2 - 64 = 0$ **35.** $4x^2 - 49 = 0$ **36.** $9w^2 - 64 = 0$

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37. $9y^2 = 4$

38. $4z^2 = 25$

39. $16v^2 - 9 = 0$

40. $25x^2 - 64 = 0$

41. $y^2 + 81 = 0$

42. $z + 49 = 0$

43. $w^2 - 24 = 0$

44. $v^2 - 48 = 0$

45. $(x - 1)^2 = 36$

46. $(y + 2)^2 = 49$

47. $2(x + 5)^2 = 8$

48. $4(z - 3)^2 = 100$

49. $9(x - 1)^2 - 16 = 0$

50. $4(y + 3)^2 - 81 = 0$

51. $49(v + 1)^2 - 25 = 0$

52. $81(y - 2)^2 - 64 = 0$

53. $(x - 4)^2 - 20 = 0$

54. $(y + 5)^2 - 50 = 0$

55. $(x + 1)^2 + 36 = 0$

56. $2\left(z - \frac{1}{2}\right)^2 = 12$

57. $3\left(v + \frac{3}{4}\right)^2 = 36$

APPLYING THE CONCEPTS

Solve for x.

58. $(x^2 - 1)^2 = 9$

59. $(x^2 + 3)^2 = 25$

60. $(6x^2 - 5)^2 = 1$

61. $ax^2 - bx = 0, a > 0$ and $b > 0$

62. $ax^2 - b = 0, a > 0$ and $b > 0$

63. $x^2 = x$

64. The value P of an initial investment of A dollars after two years is given by $P = A(1 + r)^2$, where r is the annual percentage rate earned by the investment. If an initial investment of \$1500 grew to a value of \$1782.15 in two years, what was the annual percentage rate?

65. An initial investment of \$5000 grew to a value of \$5832 in two years. Use the formula in Exercise 64 to find the annual percentage rate.

66. The kinetic energy of a moving body is given by $E = \frac{1}{2}mv^2$, where E is the kinetic energy, m is the mass, and v is the velocity. What is the velocity of a moving body whose mass is 5 kg and whose kinetic energy is 250 newton-meters?

67. On a certain type of street surface, the equation $d = 0.0074v^2$ can be used to approximate the distance d a car traveling v miles per hour will slide when its brakes are applied. After applying the brakes, the owner of a car involved in an accident skidded 40 ft. Did the traffic officer investigating the accident issue the car owner a ticket for speeding if the speed limit is 65 mph?

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11.2 Solving Quadratic Equations by Completing the Square

Objective A To solve a quadratic equation by completing the square



Recall that a perfect-square trinomial is the square of a binomial.

Perfect-Square Trinomial		Square of a Binomial
$x^2 + 6x + 9$	$=$	$(x + 3)^2$
$x^2 - 10x + 25$	$=$	$(x - 5)^2$
$x^2 + 8x + 16$	$=$	$(x + 4)^2$

For each perfect-square trinomial, the square of $\frac{1}{2}$ of the coefficient of x equals the constant term.

$x^2 + 6x + 9$, $\left(\frac{1}{2} \cdot 6\right)^2 = 9$

$x^2 - 10x + 25$, $\left[\frac{1}{2}(-10)\right]^2 = 25$

$x^2 + 8x + 16$, $\left(\frac{1}{2} \cdot 8\right)^2 = 16$

Adding to a binomial the constant term that makes it a perfect-square trinomial is called **completing the square.**

➡ Complete the square of $x^2 - 8x$. Write the resulting perfect-square trinomial as the square of a binomial.

$\left[\frac{1}{2}(-8)\right]^2 = 16$ • Find the constant term.

$x^2 - 8x + 16$ • Complete the square of $x^2 - 8x$ by adding the constant term.

$x^2 - 8x + 16 = (x - 4)^2$ • Write the resulting perfect-square trinomial as the square of a binomial.

➡ Complete the square of $y^2 + 5y$. Write the resulting perfect-square trinomial as the square of a binomial.

$\left(\frac{1}{2} \cdot 5\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$ • Find the constant term.

$y^2 + 5y + \frac{25}{4}$ • Complete the square of $y^2 + 5y$ by adding the constant term.

$y^2 + 5y + \frac{25}{4} = \left(y + \frac{5}{2}\right)^2$ • Write the resulting perfect-square trinomial as the square of a binomial.

A quadratic equation that cannot be solved by factoring can be solved by completing the square. Add to each side of the equation the term that completes the square. Rewrite the quadratic equation in the form $(x + a)^2 = b$. Take the square root of each side of the equation and then solve for x.

POINT OF INTEREST

Early mathematicians solved quadratic equations by literally *completing the square.* For these mathematicians, all equations had geometric interpretations. They found that a quadratic equation could be solved by making certain figures into squares. See the fourth "Projects and Group Activities" at the end of this chapter for an idea of how this was done.

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➡ Solve by completing the square: $x^2 + 8x - 2 = 0$

$$x^2 + 8x - 2 = 0$$
$$x^2 + 8x = 2$$ • Add 2 to each side of the equation.

$$x^2 + 8x + \left(\frac{1}{2} \cdot 8\right)^2 = 2 + \left(\frac{1}{2} \cdot 8\right)^2$$ • Complete the square of $x^2 + 8x$. Add $\left(\frac{1}{2} \cdot 8\right)^2$ to each side of the equation.

$$x^2 + 8x + 16 = 2 + 16$$ • Simplify.

$$(x + 4)^2 = 18$$ • Factor the perfect-square trinomial.

$$\sqrt{(x + 4)^2} = \sqrt{18}$$ • Take the square root of each side of the equation.

$$x + 4 = \pm\sqrt{18} = \pm 3\sqrt{2}$$ • Solve for x.

$$x + 4 = -3\sqrt{2} \qquad x + 4 = 3\sqrt{2}$$
$$x = -4 - 3\sqrt{2} \qquad x = -4 + 3\sqrt{2}$$

Check:

$x^2 + 8x - 2 = 0$	
$(-4 - 3\sqrt{2})^2 + 8(-4 - 3\sqrt{2}) - 2$	0
$16 + 24\sqrt{2} + 18 - 32 - 24\sqrt{2} - 2$	0
$0 = 0$	

$x^2 + 8x - 2 = 0$	
$(-4 + 3\sqrt{2})^2 + 8(-4 + 3\sqrt{2}) - 2$	0
$16 - 24\sqrt{2} + 18 - 32 + 24\sqrt{2} - 2$	0
$0 = 0$	

The solutions are $-4 - 3\sqrt{2}$ and $-4 + 3\sqrt{2}$.

If the coefficient of the second-degree term is not 1, a step in completing the square is to multiply each side of the equation by the reciprocal of that coefficient.

➡ Solve by completing the square: $2x^2 - 3x + 1 = 0$

$$2x^2 - 3x + 1 = 0$$
$$2x^2 - 3x = -1$$ • Subtract 1 from each side of the equation.

$$\frac{1}{2}(2x^2 - 3x) = \frac{1}{2} \cdot (-1)$$ • To complete the square, the coefficient of x^2 must be 1. Multiply each side of the equation by $\frac{1}{2}$.

$$x^2 - \frac{3}{2}x = -\frac{1}{2}$$

$$x^2 - \frac{3}{2}x + \left[\frac{1}{2}\left(-\frac{3}{2}\right)\right]^2 = -\frac{1}{2} + \left[\frac{1}{2}\left(-\frac{3}{2}\right)\right]^2$$ • Complete the square. Add $\left[\frac{1}{2}\left(-\frac{3}{2}\right)\right]^2$ to each side of the equation.

$$x^2 - \frac{3}{2}x + \frac{9}{16} = -\frac{1}{2} + \frac{9}{16}$$ • Simplify.

$$\left(x - \frac{3}{4}\right)^2 = \frac{1}{16}$$ • Factor the perfect-square trinomial.

$$\sqrt{\left(x - \frac{3}{4}\right)^2} = \sqrt{\frac{1}{16}}$$ • Take the square root of each side of the equation.

$$x - \frac{3}{4} = \pm\frac{1}{4}$$ • Solve for x.

$$x - \frac{3}{4} = -\frac{1}{4} \qquad x - \frac{3}{4} = \frac{1}{4}$$
$$x = \frac{1}{2} \qquad x = 1$$

The solutions are $\frac{1}{2}$ and 1.

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Example 1

Solve by completing the square:
$2x^2 - 4x - 1 = 0$

Solution

$$2x^2 - 4x - 1 = 0$$
$$2x^2 - 4x = 1$$
$$\frac{1}{2}(2x^2 - 4x) = \frac{1}{2} \cdot 1$$
$$x^2 - 2x = \frac{1}{2}$$

Complete the square.

$$x^2 - 2x + 1 = \frac{1}{2} + 1$$
$$(x - 1)^2 = \frac{3}{2}$$
$$\sqrt{(x - 1)^2} = \sqrt{\frac{3}{2}}$$
$$x - 1 = \pm\frac{\sqrt{6}}{2}$$

$$x - 1 = \frac{\sqrt{6}}{2} \qquad x - 1 = -\frac{\sqrt{6}}{2}$$
$$x = 1 + \frac{\sqrt{6}}{2} \qquad x = 1 - \frac{\sqrt{6}}{2}$$
$$= \frac{2 + \sqrt{6}}{2} \qquad = \frac{2 - \sqrt{6}}{2}$$

Check:

$$\begin{array}{r|l} 2x^2 - 4x - 1 = 0 & \\ \hline 2\left(\frac{2 + \sqrt{6}}{2}\right)^2 - 4\left(\frac{2 + \sqrt{6}}{2}\right) - 1 & 0 \\ 2\left(\frac{4 + 4\sqrt{6} + 6}{4}\right) - 2(2 + \sqrt{6}) - 1 & 0 \\ 2 + 2\sqrt{6} + 3 - 4 - 2\sqrt{6} - 1 & 0 \\ 0 = 0 & \end{array}$$

$$\begin{array}{r|l} 2x^2 - 4x - 1 = 0 & \\ \hline 2\left(\frac{2 - \sqrt{6}}{2}\right)^2 - 4\left(\frac{2 - \sqrt{6}}{2}\right) - 1 & 0 \\ 2\left(\frac{4 - 4\sqrt{6} + 6}{4}\right) - 2(2 - \sqrt{6}) - 1 & 0 \\ 2 - 2\sqrt{6} + 3 - 4 + 2\sqrt{6} - 1 & 0 \\ 0 = 0 & \end{array}$$

The solutions are $\frac{2 + \sqrt{6}}{2}$ and $\frac{2 - \sqrt{6}}{2}$.

You Try It 1

Solve by completing the square:
$3x^2 - 6x - 2 = 0$

Your solution

Solution on p. S25

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Example 2

Solve by completing the square:
$x^2 + 4x + 5 = 0$

Solution

$$x^2 + 4x + 5 = 0$$
$$x^2 + 4x = -5$$

Complete the square.

$$x^2 + 4x + 4 = -5 + 4$$
$$(x + 2)^2 = -1$$
$$\sqrt{(x + 2)^2} = \sqrt{-1}$$

$\sqrt{-1}$ is not a real number.

The quadratic equation has no real number solution.

You Try It 2

Solve by completing the square:
$x^2 + 6x + 12 = 0$

Your solution

Example 3

Solve $\frac{x^2}{4} + \frac{3x}{2} + 1 = 0$ by completing the square. Approximate the solutions to the nearest thousandth.

Solution

$$\frac{x^2}{4} + \frac{3x}{2} + 1 = 0$$
$$4\left(\frac{x^2}{4} + \frac{3x}{2} + 1\right) = 4(0)$$
$$x^2 + 6x + 4 = 0$$
$$x^2 + 6x = -4$$

Complete the square.

$$x^2 + 6x + 9 = -4 + 9$$
$$(x + 3)^2 = 5$$
$$\sqrt{(x + 3)^2} = \sqrt{5}$$
$$x + 3 = \pm\sqrt{5}$$

$x + 3 = \sqrt{5}$

$$x = -3 + \sqrt{5}$$
$$\approx -3 + 2.236$$
$$\approx -0.764$$

$x + 3 = -\sqrt{5}$

$$x = -3 - \sqrt{5}$$
$$\approx -3 - 2.236$$
$$\approx -5.236$$

The solutions are approximately -0.764 and -5.236.

You Try It 3

Solve $\frac{x^2}{8} + x + 1 = 0$ by completing the square. Approximate the solutions to the nearest thousandth.

Your solution

Solutions on p. S25

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11.2 Exercises

Objective A

Solve by completing the square.

1. $x^2 + 2x - 3 = 0$ **2.** $y^2 + 4y - 5 = 0$ **3.** $z^2 - 6z - 16 = 0$ **4.** $w^2 + 8w - 9 = 0$

5. $x^2 = 4x - 4$ **6.** $z^2 = 8z - 16$ **7.** $v^2 - 6v + 13 = 0$ **8.** $x^2 + 4x + 13 = 0$

9. $y^2 + 5y + 4 = 0$ **10.** $v^2 - 5v - 6 = 0$ **11.** $w^2 + 7w = 8$ **12.** $y^2 + 5y = -4$

13. $v^2 + 4v + 1 = 0$ **14.** $y^2 - 2y - 5 = 0$ **15.** $x^2 + 6x = 5$

16. $w^2 - 8w = 3$ **17.** $\frac{z^2}{2} = z + \frac{1}{2}$ **18.** $\frac{y^2}{10} = y - 2$

19. $p^2 + 3p = 1$ **20.** $r^2 + 5r = 2$ **21.** $t^2 - 3t = -2$

22. $z^2 - 5z = -3$ **23.** $v^2 + v - 3 = 0$ **24.** $x^2 - x = 1$

25. $y^2 = 7 - 10y$ **26.** $v^2 = 14 + 16v$ **27.** $r^2 - 3r = 5$

28. $s^2 + 3s = -1$ **29.** $t^2 - t = 4$ **30.** $y^2 + y - 4 = 0$

31. $x^2 - 3x + 5 = 0$ **32.** $z^2 + 5z + 7 = 0$ **33.** $2t^2 - 3t + 1 = 0$

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34. $2x^2 - 7x + 3 = 0$ **35.** $2r^2 + 5r = 3$ **36.** $2y^2 - 3y = 9$ **37.** $2s^2 = 7s - 6$

38. $2x^2 = 3x + 20$ **39.** $2v^2 = v + 1$ **40.** $2z^2 = z + 3$ **41.** $3r^2 + 5r = 2$

42. $3t^2 - 8t = 3$ **43.** $3y^2 + 8y + 4 = 0$ **44.** $3z^2 - 10z - 8 = 0$ **45.** $4x^2 + 4x - 3 = 0$

46. $4v^2 + 4v - 15 = 0$ **47.** $6s^2 + 7s = 3$ **48.** $6z^2 = z + 2$ **49.** $6p^2 = 5p + 4$

50. $6t^2 = t - 2$ **51.** $4v^2 - 4v - 1 = 0$ **52.** $2s^2 - 4s - 1 = 0$

Solve by completing the square. Approximate the solutions to the nearest thousandth.

53. $y^2 + 3y = 5$ **54.** $w^2 + 5w = 2$ **55.** $2z^2 - 3z = 7$

56. $2x^2 + 3x = 11$ **57.** $4x^2 + 6x - 1 = 0$ **58.** $4x^2 + 2x - 3 = 0$

APPLYING THE CONCEPTS

59. Explain why the equation $(x - 2)^2 = -4$ does not have a real-number solution.

Solve.

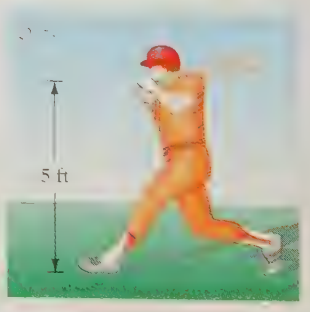
60. $\frac{x^2}{6} - \frac{x}{3} = 1$ **61.** $\sqrt{x + 2} = x - 4$ **62.** $\sqrt{3x + 4} - x = 2$

63. $\frac{x}{3} + \frac{3}{x} = \frac{8}{3}$ **64.** $\frac{x + 1}{2} + \frac{3}{x - 1} = 4$ **65.** $\frac{x - 2}{3} + \frac{2}{x + 2} = 4$

66. $4\sqrt{x + 1} - x = 4$ **67.** $\sqrt{2x^2 + 7} = x + 2$ **68.** $3\sqrt{x - 1} + 3 = x$

69. A basketball player shoots at a basket 25 ft away. The height of the ball above the ground at time t is given by $h = -16t^2 + 32t + 6.5$. How many seconds after the ball is released does it hit the basket? *Hint:* When it hits the basket, $h = 10$ ft.

70. A ball player hits a ball. The height of the ball above the ground can be approximated by the equation $h = -16t^2 + 76t + 5$. When will the ball hit the ground? *Hint:* The ball strikes the ground when $h = 0$ ft.



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11.3 Solving Quadratic Equations by Using the Quadratic Formula

Objective A To solve a quadratic equation by using the quadratic formula

Any quadratic equation can be solved by completing the square. Applying this method to the standard form of a quadratic equation produces a formula that can be used to solve any quadratic equation.

Solve $ax^2 + bx + c = 0$ by completing the square.

$$ax^2 + bx + c = 0$$

Add the opposite of the constant term to each side of the equation.

$$ax^2 + bx + c + (-c) = 0 + (-c)$$
$$ax^2 + bx = -c$$

Multiply each side of the equation by the reciprocal of a, the coefficient of x^2.

$$\frac{1}{a}(ax^2 + bx) = \frac{1}{a}(-c)$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Complete the square by adding $\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2$ to each side of the equation.

$$x^2 + \frac{b}{a}x + \left(\frac{1}{2}\cdot\frac{b}{a}\right)^2 = \left(\frac{1}{2}\cdot\frac{b}{a}\right)^2 - \frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

Simplify the right side of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \left(\frac{c}{a}\cdot\frac{4a}{4a}\right)$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

Factor the perfect-square trinomial on the left side of the equation.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Take the square root of each side of the equation.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Solve for x.

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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➡ Solve by using the quadratic formula: $2x^2 = 4x - 1$.

First write the equation in standard form. Then use the quadratic formula.

$$2x^2 = 4x - 1$$

$$2x^2 - 4x + 1 = 0$$

- Subtract $4x$ and add 1 to each side of the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The quadratic formula.

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \cdot 2 \cdot 1)}}{2 \cdot 2}$$

- $a = 2$, $b = -4$, $c = 1$. Replace a, b, and c by their values.

$$= \frac{4 \pm \sqrt{16 - 8}}{4} = \frac{4 \pm \sqrt{8}}{4}$$

- Simplify.

$$= \frac{4 \pm 2\sqrt{2}}{4} = \frac{2 \pm \sqrt{2}}{2}$$

The solutions are $\frac{2 + \sqrt{2}}{2}$ and $\frac{2 - \sqrt{2}}{2}$.

Example 1

Solve by using the quadratic formula:
$2x^2 - 3x + 1 = 0$

Solution

$2x^2 - 3x + 1 = 0$ $\quad a = 2, b = -3, c = 1$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(1)}}{2 \cdot 2}$$

$$= \frac{3 \pm \sqrt{9 - 8}}{4} = \frac{3 \pm \sqrt{1}}{4} = \frac{3 \pm 1}{4}$$

$$x = \frac{3 + 1}{4} \qquad x = \frac{3 - 1}{4}$$

$$= \frac{4}{4} = 1 \qquad = \frac{2}{4} = \frac{1}{2}$$

The solutions are 1 and $\frac{1}{2}$.

You Try It 1

Solve by using the quadratic formula:
$3x^2 + 4x - 4 = 0$

Your solution

Example 2

Solve by using the quadratic formula:
$\frac{x^2}{2} = 2x - \frac{5}{4}$

Solution

$$\frac{x^2}{2} = 2x - \frac{5}{4}$$

$$4\left(\frac{x^2}{2}\right) = 4\left(2x - \frac{5}{4}\right)$$

$$2x^2 = 8x - 5$$

$2x^2 - 8x + 5 = 0$ $\quad a = 2, b = -8, c = 5$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(5)}}{2 \cdot 2}$$

$$= \frac{8 \pm \sqrt{64 - 40}}{4} = \frac{8 \pm \sqrt{24}}{4}$$

$$= \frac{8 \pm 2\sqrt{6}}{4} = \frac{4 \pm \sqrt{6}}{2}$$

The solutions are $\frac{4 + \sqrt{6}}{2}$ and $\frac{4 - \sqrt{6}}{2}$.

You Try It 2

Solve by using the quadratic formula:
$\frac{x^2}{4} + \frac{x}{2} = \frac{1}{4}$

Your solution

Solutions on p. S26

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11.3 Exercises

Objective A

Solve by using the quadratic formula.

1. $x^2 - 4x - 5 = 0$
2. $y^2 + 3y + 2 = 0$
3. $z^2 - 2z - 15 = 0$
4. $v^2 + 5v + 4 = 0$
5. $y^2 = 2y + 3$
6. $w^2 = 3w + 18$
7. $r^2 = 5 - 4r$
8. $z^2 = 3 - 2z$
9. $2y^2 - y - 1 = 0$
10. $2t^2 - 5t + 3 = 0$
11. $w^2 + 3w + 5 = 0$
12. $x^2 - 2x + 6 = 0$
13. $p^2 - p = 0$
14. $2v^2 + v = 0$
15. $4t^2 - 9 = 0$
16. $4s^2 - 25 = 0$
17. $4y^2 + 4y = 15$
18. $6y^2 + 5y - 4 = 0$
19. $2x^2 + x + 1 = 0$
20. $3r^2 - r + 2 = 0$
21. $\frac{1}{2}t^2 - t = \frac{5}{2}$
22. $y^2 - 4y = 6$
23. $\frac{1}{3}t^2 + 2t - \frac{1}{3} = 0$
24. $z^2 + 4z + 1 = 0$
25. $w^2 = 4w + 9$
26. $y^2 = 8y + 3$
27. $9y^2 + 6y - 1 = 0$
28. $9s^2 - 6s - 2 = 0$
29. $4p^2 + 4p + 1 = 0$
30. $9z^2 + 12z + 4 = 0$
31. $\frac{x^2}{2} = x - \frac{5}{4}$
32. $r^2 = \frac{5}{3}r - 2$
33. $4p^2 + 16p = -11$
34. $4y^2 - 12y = -1$
35. $4x^2 = 4x + 11$

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36. $4s^2 + 12s = 3$ **37.** $9v^2 = -30v - 23$ **38.** $9t^2 = 30t + 17$

Solve by using the quadratic formula. Approximate the solutions to the nearest thousandth.

39. $x^2 - 2x - 21 = 0$ **40.** $y^2 + 4y - 11 = 0$ **41.** $s^2 - 6s - 13 = 0$

42. $w^2 + 8w - 15 = 0$ **43.** $2p^2 - 7p - 10 = 0$ **44.** $3t^2 - 8t - 1 = 0$

45. $4z^2 + 8z - 1 = 0$ **46.** $4x^2 + 7x + 1 = 0$ **47.** $5v^2 - v - 5 = 0$

APPLYING THE CONCEPTS

48. Factoring, completing the square, and using the quadratic formula are three methods of solving quadratic equations. Describe each method, and cite the advantages and disadvantages of using each.

49. Explain why the equation $0x^2 + 3x + 4 = 0$ cannot be solved by the quadratic formula.

50. Solve $x^2 + ax + b = 0$ for x.

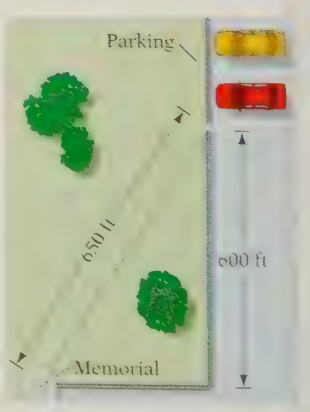
51. True or False?
a. The equations $x = \sqrt{12 - x}$ and $x^2 = 12 - x$ have the same solutions.
b. If $\sqrt{a} + \sqrt{b} = c$, then $a + b = c^2$.
c. $\sqrt{9} = \pm 3$
d. $\sqrt{x^2} = |x|$

Solve.

52. $\sqrt{x + 3} = x - 3$ **53.** $\sqrt{x + 4} = x + 4$ **54.** $\sqrt{x + 1} = x - 1$

55. $\sqrt{x^2 + 2x + 1} = x - 1$ **56.** $\frac{x}{4} + \frac{3}{x} = \frac{5}{2}$ **57.** $\frac{x + 1}{5} - \frac{4}{x - 1} = 2$

58. An L-shaped sidewalk from the parking lot to a memorial is shown in the figure at the right. The distance directly across the grass to the memorial is 650 ft. The distance to the corner is 600 ft. Find the distance from the corner to the memorial.



59. A commuter plane leaves an airport traveling due south at 400 mph. Another plane leaving at the same time travels due east at 300 mph. Find the distance between the two planes after 2 h.

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11.4 Graphing Quadratic Equations in Two Variables

Objective A To graph a quadratic equation of the form $y = ax^2 + bx + c$

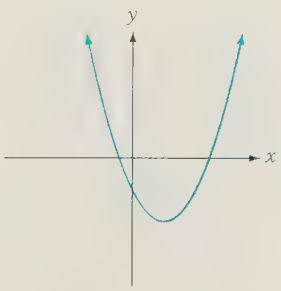
37 CT

An equation of the form $y = ax^2 + bx + c$, $a \neq 0$, is a **quadratic equation in two variables.** Examples of quadratic equations in two variables are shown at the right.

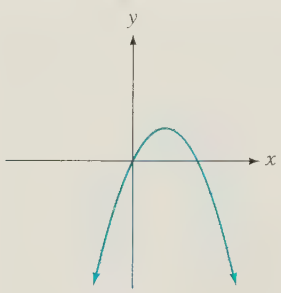
$y = 3x^2 - x + 1$
$y = -x^2 - 3$
$y = 2x^2 - 5x$

For these equations, y is a function of x, and we can write $f(x) = ax^2 + bx + c$. This represents a **quadratic function.**

The graph of a quadratic equation in two variables is a **parabola.** The graph is cup shaped and opens either up or down. The graphs of two parabolas are shown below.



Parabola that opens up



Parabola that opens down

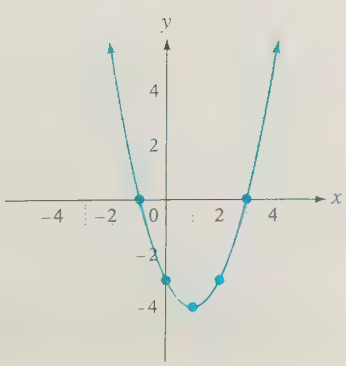
➡ Graph $y = x^2 - 2x - 3$.

x	y
0	−3
1	−4
−1	0
2	−3
3	0

• Find several solutions of the equation. Because the graph is not a straight line, several solutions must be found in order to determine the cup shape. Display the ordered-pair solutions in a table.

POINT OF INTEREST

One of the "Projects and Group Activities" at the end of this chapter shows how to graph a quadratic equation by using a graphing calculator. You may want to verify the graphs you draw in this section against those drawn by a graphing calculator.



• Graph the ordered-pair solutions on a rectangular coordinate system. Draw a parabola through the points.

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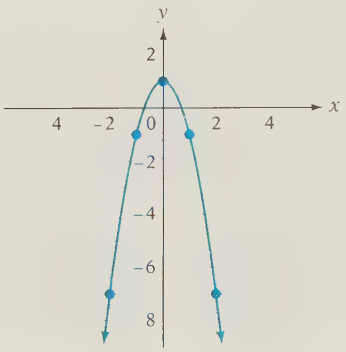
POINT OF INTEREST

Mirrors in some telescopes are ground into the shape of a parabola. The mirror at the Palomar Mountain Observatory is 2 ft thick at the ends and weighs 14.75 tons. The mirror has been ground to a true paraboloid (the three-dimensional version of a parabola) to within 0.0000015 in. A possible equation of the mirror is $y = 2640x^2$.

➡ Graph $y = -2x^2 + 1$.

x	y
0	1
1	−1
−1	−1
2	−7
−2	−7

Find enough solutions of the equation to determine the cup shape. Display the ordered-pair solutions in a table.



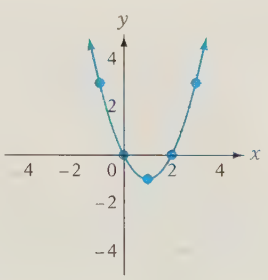
Graph the ordered-pair solutions on a rectangular coordinate system. Draw a parabola through the points.

Note in the example on page 493 that the coefficient of x^2 is **positive** and the graph **opens up.** In the example above, the coefficient of x^2 is **negative** and the graph **opens down.**

Example 1 Graph $y = x^2 - 2x$.

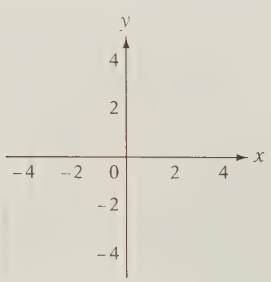
Solution

x	y
0	0
1	−1
−1	3
2	0
3	3



You Try It 1 Graph $y = x^2 + 2$.

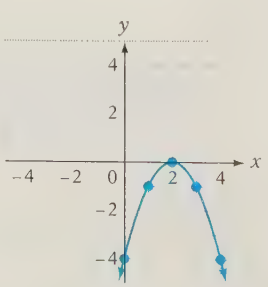
Your solution



Example 2 Graph $y = -x^2 + 4x - 4$.

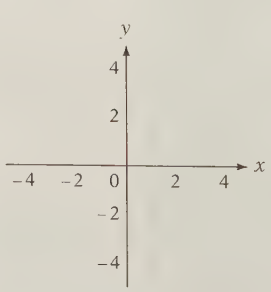
Solution

x	y
0	−4
1	−1
2	0
3	−1
4	−4



You Try It 2 Graph $y = -x^2 - 2x - 1$.

Your solution



Solutions on p. S26

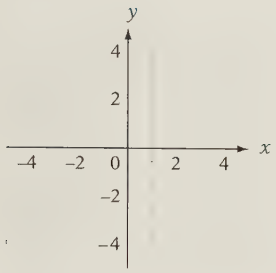
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11.4 Exercises

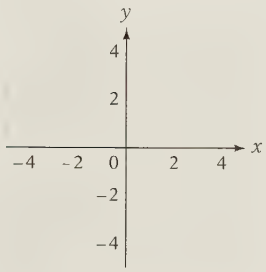
Objective A

Graph.

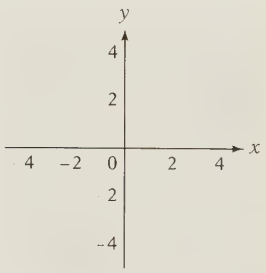
1. $y = x^2$



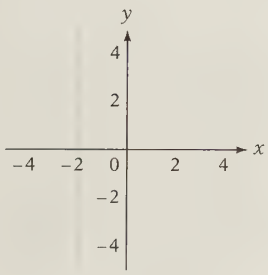
2. $y = -x^2$



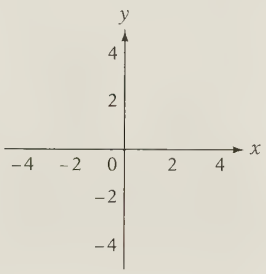
3. $y = -x^2 + 1$



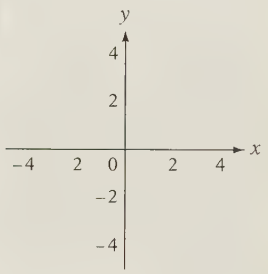
4. $y = x^2 - 1$



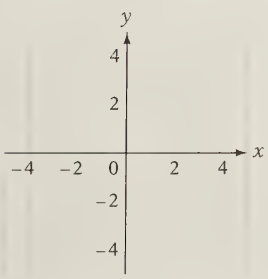
5. $y = 2x^2$



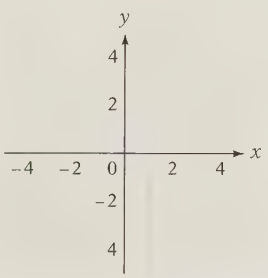
6. $y = \frac{1}{2}x^2$



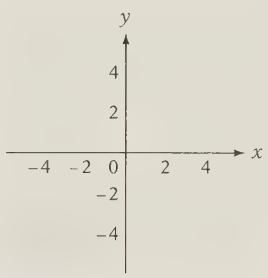
7. $y = -\frac{1}{2}x^2 + 1$



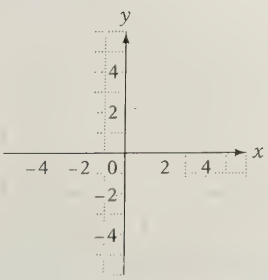
8. $y = 2x^2 - 1$



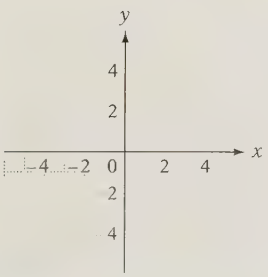
9. $y = x^2 - 4x$



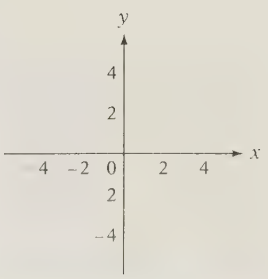
10. $y = x^2 + 4x$



11. $y = x^2 - 2x + 3$

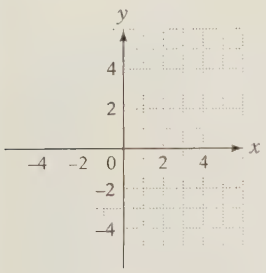


12. $y = x^2 - 4x + 2$

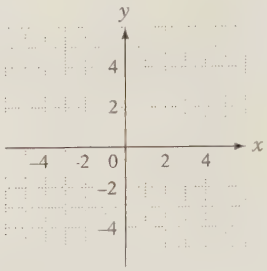


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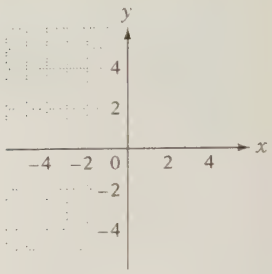
13. $y = -x^2 + 2x + 3$



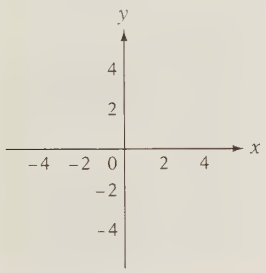
14. $y = -x^2 - 2x + 3$



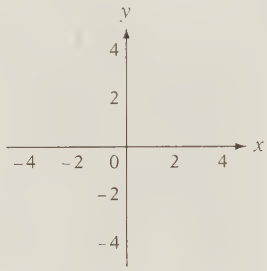
15. $y = -x^2 + 4x - 4$



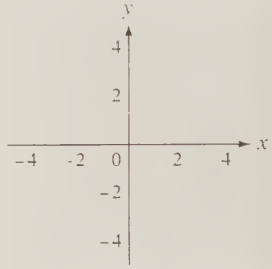
16. $y = -x^2 + 6x - 9$



17. $y = (x - 2)^2$



18. $y = -(x + 1)^2$



APPLYING THE CONCEPTS

Determine whether the graph of the equation opens up or down.

19. $y = -\frac{1}{3}x^2 + 5$

20. $y = x^2 - 2x + 3$

21. $y = -x^2 + 4x - 1$

Show that the equation is a quadratic equation in two variables by writing it in the form $y = ax^2 + bx + c$.

22. $y + 1 = (x - 4)^2$

23. $y - 2 = 3(x + 1)^2$

24. $y - 4 = 2(x - 3)^2$

The x-intercepts of the graph of $y = ax^2 + bx + c$ occur when $y = 0$. Therefore, the x-coordinate of an x-intercept is a solution of $ax^2 + bx + c = 0$. Determine the x-intercepts of the graphs of the following equations.

25. $y = x^2 - 9$

26. $y = x^2 - 4x$

27. $y = 2x^2 - x - 1$

Evaluate the function.

28. Find $f(3)$ when $f(x) = x^2 + 3$.

29. Find $g(2)$ when $g(x) = x^2 - 2x - 3$.

30. Find $S(-2)$ when $S(t) = 2t^2 - 3t - 1$.

31. Find $P(-3)$ when $P(x) = 3x^2 - 6x - 7$.

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11.5 Application Problems

Objective A To solve application problems

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The application problems in this section are varieties of those problems solved earlier in the text. Each of the strategies for the problems in this section will result in a quadratic equation.

➡ In 5 h, two campers rowed 12 mi down a stream and then rowed back to their campsite. The rate of the stream's current was 1 mph. Find the rate at which the campers rowed.

Strategy for Solving an Application Problem

1. Determine the type of problem. For example, is it a distance-rate problem, a geometry problem, or a work problem?

The problem is a distance-rate problem.

2. Choose a variable to represent the unknown quantity. Write numerical or variable expressions for all the remaining quantities. These results can be recorded in a table.

The unknown rate of the campers: r

	Distance	÷	*Rate*	=	*Time*
Downstream	12	÷	$r + 1$	=	$\frac{12}{r+1}$
Upstream	12	÷	$r - 1$	=	$\frac{12}{r-1}$

3. Determine how the quantities are related.

The total time of the trip was 5 h. $\frac{12}{r+1} + \frac{12}{r-1} = 5$

$$(r+1)(r-1)\left(\frac{12}{r+1} + \frac{12}{r-1}\right) = (r+1)(r-1)5$$
$$(r-1)12 + (r+1)12 = (r^2-1)5$$
$$12r - 12 + 12r + 12 = 5r^2 - 5$$
$$24r = 5r^2 - 5$$
$$0 = 5r^2 - 24r - 5$$
$$0 = (5r+1)(r-5)$$
$$5r + 1 = 0 \qquad r - 5 = 0$$
$$5r = -1 \qquad r = 5$$
$$r = -\frac{1}{5}$$

The rowing rate was 5 mph.

TAKE NOTE

The time going downstream plus the time going upstream is equal to the time of the entire trip.

TAKE NOTE

The solution $r = -\frac{1}{5}$ is not possible, because the rate cannot be a negative number.

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Example 1

A painter and the painter's apprentice working together can paint a room in 2 h. The apprentice working alone requires 3 more hours to paint the room than the painter requires working alone. How long does it take the painter working alone to paint the room?

Strategy

This is a work problem.

Time for the painter to paint the room: t

Time for the apprentice to paint the room: $t + 3$

	Rate	*Time*	*Part*
Painter	$\frac{1}{t}$	2	$\frac{2}{t}$
Apprentice	$\frac{1}{t+3}$	2	$\frac{2}{t+3}$

The sum of the parts of the task completed must equal 1.

Solution

$$\frac{2}{t} + \frac{2}{t+3} = 1$$

$$t(t+3)\left(\frac{2}{t} + \frac{2}{t+3}\right) = t(t+3) \cdot 1$$

$$(t+3)2 + t(2) = t(t+3)$$

$$2t + 6 + 2t = t^2 + 3t$$

$$0 = t^2 - t - 6$$

$$0 = (t-3)(t+2)$$

$$t - 3 = 0 \qquad t + 2 = 0$$

$$t = 3 \qquad t = -2$$

The solution $t = -2$ is not possible.

The time is 3 h.

You Try It 1

The length of a rectangle is 2 m more than the width. The area is 15 m^2. Find the width.

Your strategy

Your solution

Solution on p. S26

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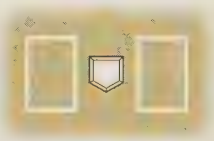
11.5 Exercises

Objective A Application Problems

1. The height of a triangle is 2 m more than twice the length of the base. The area of the triangle is 20 m^2. Find the height of the triangle and the length of the base.

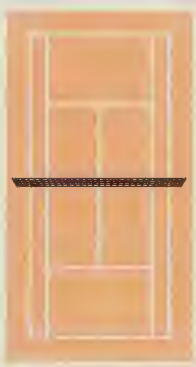
2. The length of a rectangle is 4 ft more than twice the width. The area of the rectangle is 160 ft^2. Find the length and width of the rectangle.

3. The area of the batter's box on a major league baseball field is 24 ft^2. The length of the batter's box is 2 ft more than the width. Find the length and width of the batter's box. (Area = LW)



4. The length of the batter's box on a softball field is 1 ft less than twice the width. The area of the batter's box is 15 ft^2. Find the length and width of the batter's box.

5. The length of a swimming pool is twice the width. The area of the pool is 5000 ft^2. Find the length and width of the pool. (Area = LW)



6. The length of the singles tennis court is 24 ft more than twice the width. The area of the tennis court is 2106 ft^2. Find the length and width of the court. (Area = LW)

7. One computer takes 21 min longer to calculate the value of a complex equation than a second computer. Working together, these computers complete the calculation in 10 min. How long would it take each computer, working separately, to calculate the value?

8. A tank has two drains. One drain takes 16 min longer to empty the tank than does a second drain. With both drains open, the tank is emptied in 6 min. How long would it take each drain, working alone, to empty the tank?

9. Using one engine of a ferryboat, it takes 6 h longer to cross a channel than it does using a second engine alone. Using both engines, the ferryboat can make the crossing in 4 h. How long would it take each engine, working alone, to power the ferryboat across the channel?

10. An apprentice mason takes 8 h longer to build a small fireplace than an experienced mason. Working together, they can build the fireplace in 3 h. How long would it take each mason, working alone, to complete the fireplace?

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11. It took a small plane 2 h more to fly 375 mi against the wind than to fly the same distance with the wind. The rate of the wind was 25 mph. Find the rate of the plane in calm air.

12. It took a motorboat 1 h more to travel 36 mi against the current than to go 36 mi with the current. The rate of the current was 3 mph. Find the rate of the boat in calm water.

APPLYING THE CONCEPTS

13. The sum of the squares of four consecutive integers is 86. Find the four integers.

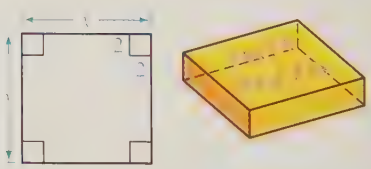
14. The hypotenuse of a right triangle is $\sqrt{13}$ cm. One leg is 1 cm shorter than twice the length of the other leg. Find the lengths of the legs of the right triangle.

15. The radius of a large pizza is 1 in. less than twice the radius of a small pizza. The difference between the areas of the two pizzas is 33π in^2. Find the radius of the large pizza.

16. The perimeter of a rectangular garden is 54 ft. The area of the garden is 180 ft^2. Find the length and width of the garden.

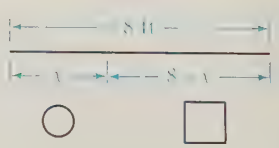
17. The distance, s, a car needs to come to a stop on a certain surface depends on the velocity, in feet per second, v, of the car when the brakes are applied. The equation is given by $s = 0.0344v^2 - 0.758v$. What is the maximum velocity a car can have when the brakes are applied and stop within 150 ft?

18. A square piece of cardboard is to be formed into a box to transport pizzas. The box is formed by cutting 2-inch square corners from the cardboard and folding them up as shown in the figure at the right. If the volume of the box is 512 in^3, what are the dimensions of the cardboard?



19. The hang time of a football that is kicked on the opening kickoff is given by $s = -16t^2 + 88t + 1$, where s is the height of the football t seconds after leaving the kicker's foot. What is the hang time of a kickoff that hits the ground without being caught?

20. A wire 8 ft long is cut into two pieces. A circle is formed from one piece and a square is formed from the other. The total area of both figures is given by $A = \frac{1}{16}(8 - x)^2 + \frac{x^2}{4\pi}$. What is the length of each piece of wire if the total area is 4.5 ft^2?



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Focus on Problem Solving

Algebraic Manipulation and Graphing Techniques

Problem solving is often easier when we have both algebraic manipulation and graphing techniques at our disposal. Solving quadratic equations and graphing quadratic equations in two variables are used here to solve problems involving profit.

A company's revenue, R, is the total amount of money the company earned by selling its products. The cost, C, is the total amount of money the company spent to manufacture and sell its products. A company's profit, P, is the difference between the revenue and cost: $P = R - C$. A company's revenue and cost may be represented by equations.

A company manufactures and sells woodstoves. The total monthly cost, in dollars, to produce n woodstoves is $C = 30n + 2000$. Write a variable expression for the company's monthly profit if the revenue, in dollars, obtained from selling all n woodstoves is $R = 150n - 0.4n^2$.

$$P = R - C$$
$$P = 150n - 0.4n^2 - (30n + 2000)$$
$$P = -0.4n^2 + 120n - 2000$$

• Replace R by $150n - 0.4n^2$ and C by $30n + 2000$. Then simplify.

How many woodstoves must the company manufacture and sell in order to make a profit of \$6000 a month?

$$P = -0.4n^2 + 120n - 2000$$
$$6000 = -0.4n^2 + 120n - 2000$$

• Substitute 6000 for P.

$$0 = -0.4n^2 + 120n - 8000$$

• Write the equation in standard form.

$$0 = n^2 - 300n + 20{,}000$$

• Divide each side of the equation by -0.4.

$$0 = (n - 100)(n - 200)$$

• Factor.

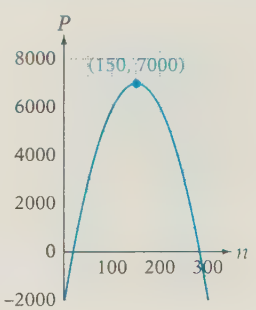
$$n - 100 = 0 \qquad n - 200 = 0$$
$$n = 100 \qquad n = 200$$

• Solve for n.

The company will make a monthly profit of \$6000 if either 100 or 200 woodstoves are manufactured and sold.

The graph of $P = -0.4n^2 + 120n - 2000$ is shown at the right. Note that when $P = 6000$, the values of n are 100 and 200.

Also note that the coordinates of the highest point on the graph are (150, 7000). This means that the company makes a *maximum* profit of \$7000 per month when 150 woodstoves are manufactured and sold.



1. The total cost, in dollars, for a company to produce and sell n guitars per month is $C = 240n + 1200$. The company's revenue, in dollars, from selling all n guitars is $R = 400n - 2n^2$.
   - **a.** How many guitars must the company produce and sell each month in order to make a monthly profit of \$1200?
   - **b.** Graph the profit equation. What is the maximum monthly profit the company can make?

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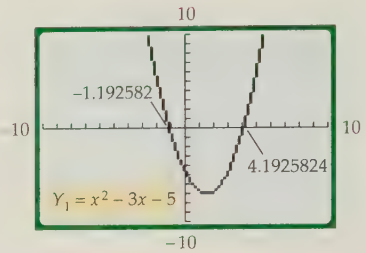
Projects and Group Activities

Graphical Solutions of Quadratic Equations

TAKE NOTE

Suggestions for graphing equations and finding the x-intercepts can be found in the appendix.

The *real* number solutions of quadratic equations can be approximated by graphing. Recall that to find an x-intercept for a graph, replace y by 0 and solve for x. For instance, to find the x-intercepts of $y = x^2 - 3x - 5$, first replace y by zero. That produces the quadratic equation $0 = x^2 - 3x - 5$. The solutions of this equation can be found by using the quadratic formula or by graphing $y = x^2 - 3x - 5$ and approximating the x-intercepts.



The approximate solutions are -1.192582 and 4.1925824.

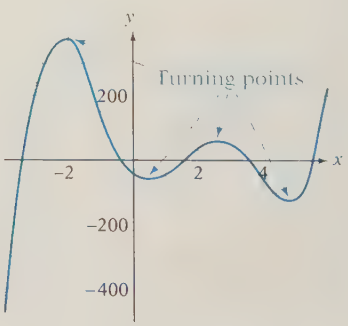
1. Graphically approximate the solutions of $2x^2 - 3x - 7 = 0$.
2. Graphically approximate the solutions of $x^2 + 5x = 1$. *Suggestion:* First write the equation in standard form.
3. Graphically approximate the solution of $\pi x^2 - \sqrt{17}x - \frac{1}{\sqrt{3}} = 0$.
4. The solution of $0.5x^2 - 2x + 2 = 0$ is a double root. Graphically approximate the solution and explain how the graph of $y = 0.5x^2 - 2x + 2$ confirms that statement.
5. There are no real number solutions of $x^2 + 4x + 5 = 0$. Explain how the graph of $y = x^2 + 4x + 5$ confirms that statement.

Properties of Polynomials

TAKE NOTE

Suggestions for graphing equations can be found in the appendix. For this particular activity, it may be necessary to adjust the viewing window so that you can get a reasonably accurate graph of the equation. Try Ymax = 1000, Ymin = −1000, and Yscl = 100.

The graph of a fifth-degree polynomial with 4 *turning points* is shown at the right. In this project you will graph various polynomials and try to make a conjecture as to the relationship between the degree of a polynomial and the number of turning points in its graph. For each of the following, graph the equation. Record the degree of the equation and the number of turning points.



1. Graph: $y = x^3 + 1$
2. Graph: $y = x^3 + 2x^2 - 5x - 6$
3. Graph: $y = x^4 - x^3 - 11x^2 - x - 12$
4. Graph: $y = x^4 - 2x^3 - 13x^2 + 14x + 24$
5. Graph: $y = x^5 - 2$
6. Graph: $y = x^5 - 3x^4 - 11x^3 + 27x^2 + 10x - 24$
7. Graph: $y = x^5 - 2x^4 - 10x^3 + 10x^2 - 11x + 12$

Make a conjecture as to a relationship between the degree of a polynomial and the number of turning points in its graph. Graph a few more polynomials of your choosing and see whether your conjecture is valid for those graphs. If not, refine your conjecture and test it again.

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Iterative Calculations

Consider the three expressions

$$p + 3p(1 - p) \qquad p(4 - 3p) \qquad 4p - 3p^2$$

When these expressions are written in simplest form, they each become $4p - 3p^2$. It would seem that if you used a graphing calculator to evaluate these expressions, each one should give the same result. However, that is not the case, as we show in this project.

TAKE NOTE

To perform these calculations with a TI calculator, first store 0.05 in P and then store the expression in P. The screen should appear as below.

```
0.05->P
                .05
P+3P(1-P)->P
                .1925
```

Each time you press the ENTER key, the expression is evaluated with the previous result.

```
0.05->P
                .05
P+3P(1-P)->P
                .1925
            .65883125
           1.333149152
7.366232839E-4
```

We begin with $p = 0.5$. Evaluating each of the expressions gives the same result, 0.1925. Now replace p in each expression with the current value in the display, 0.1925 in this case. This is called *feedback* because we are feeding our outputs back into each expression as inputs. Each new evaluation is called an *iteration*. The table below shows the results of performing the calculations on a *TI*-83 calculator.

Iteration	$p + 3p(1 - p)$	$p(4 - 3p)$	$4p - 3p^2$
1	0.1925	0.1925	0.1925
2	0.65883125	0.65883125	0.65883125
3	1.333149152	1.333149152	1.333149152
4	7.366232839E-4	7.366232839E-4	7.366232838E-4
5	0.0029448653	0.0029448653	0.0029448653
6	0.0117534445	0.0117534445	0.0117534445
7	0.0465993476	0.0465993476	0.0465993475
20	1.121356187	1.121356207	1.121356084
30	0.9471633048	0.9471899467	0.9470331284
40	0.5648166575	0.5352103994	0.7113151932

Note that for the 40th iteration the values are approximately 0.03 different between columns 2 and 3, and 0.15 different between columns 2 and 4.

1. Use a calculator to find the first 40 iterations for $p = 0.025$ for $4p - 3p^2$ and $p + 3p(1 - p)$.
2. Use a calculator to find the first 40 iterations for $p = 0.5$ for $4p - 3p^2$ and $p + 3p(1 - p)$.
3. Expressions of the form $p + rp(1 - p)$ [which is the form of $p + 3p(1 - p)$ with $r = 3$] are called Verhulst population models. Make up your own expression of this form, choosing some other value of r and a value of p.
4. Write a short paragraph on Verhulst population models.

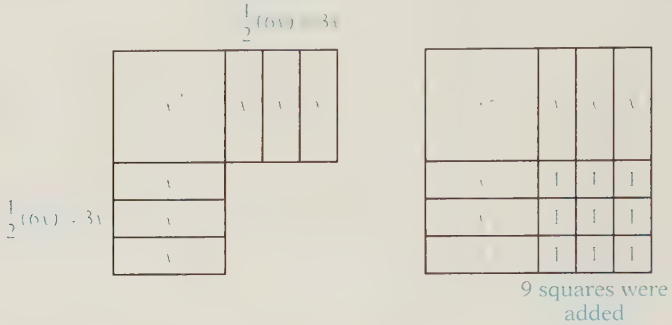
Geometric Construction of Completing the Square

Completing the square as a method of solving a quadratic equation has been known for centuries. The Persian mathematician Al-Khwarismi used this method in a textbook written around 825 A.D. The method was very geometric. That is, Al-Khwarismi literally completed a square. To understand how this method works, consider the following geometric shapes: a square whose area is x^2, a rectangle whose area is x, and another square whose area is 1.



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Now consider the expression $x^2 + 6x$. From our discussion in this chapter, to complete the square, we added $\left(\frac{1}{2} \cdot 6\right)^2 = 3^2 = 9$ to the expression. Here is the geometric construction that Al-Khwarismi used.



Note that it is necessary to add 9 squares to the figure to "complete the square." One of the difficulties of using a geometric method such as this is that it cannot easily be extended to $x^2 - 6x$. There is no way to draw an area of $-6x$! That really did not bother Al-Khwarismi much. Negative numbers were not a significant part of mathematics until well into the 13th century.

1. Show how Al-Khwarismi would have completed the square for $x^2 + 4x$.
2. Show how Al-Khwarismi would have completed the square for $x^2 + 10x$.
3. Do the geometric constructions for Exercises 1 and 2 correspond to the algebraic method shown in this chapter?

Chapter Summary

Key Words

A *quadratic equation* is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$. A quadratic equation is also called a *second-degree equation*.

A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero.

A *quadratic equation in two variables* is given by $y = ax^2 + bx + c$, where $a \neq 0$.

A *quadratic function* is given by $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is a *parabola*.

Essential Rules

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Square Root Property

If $x^2 = a$, then $x = \pm\sqrt{a}$.

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Chapter Review

1. Solve by factoring: $6x^2 + 13x - 28 = 0$

2. Solve by taking square roots: $49x^2 = 25$

3. Solve by completing the square: $x^2 + 2x - 24 = 0$

4. Solve by using the quadratic formula: $x^2 + 5x - 6 = 0$

5. Solve by completing the square: $2x^2 + 5x = 12$

6. Solve by factoring: $12x^2 + 10 = 29x$

7. Solve by taking square roots: $(x + 2)^2 - 24 = 0$

8. Solve by using the quadratic formula: $2x^2 + 3 = 5x$

9. Solve by factoring: $6x(x + 1) = x - 1$

10. Solve by taking square roots: $4y^2 + 9 = 0$

11. Solve by completing the square: $x^2 - 4x + 1 = 0$

12. Solve by using the quadratic formula: $x^2 - 3x - 5 = 0$

13. Solve by completing the square: $x^2 + 6x + 12 = 0$

14. Solve by factoring: $(x + 9)^2 = x + 11$

15. Solve by taking square roots: $\left(x - \frac{1}{2}\right)^2 = \frac{9}{4}$

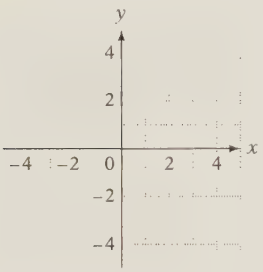
16. Solve by completing the square: $4x^2 + 16x = 7$

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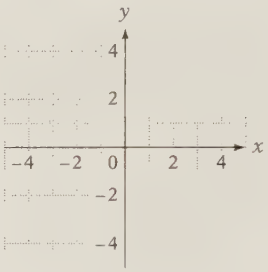
17. Solve by using the quadratic formula: $x^2 - 4x + 8 = 0$

18. Solve by using the quadratic formula: $2x^2 + 5x + 2 = 0$

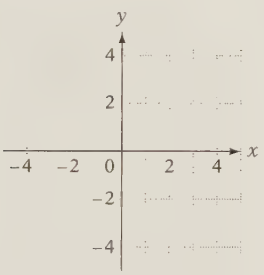
19. Graph $y = -3x^2$.



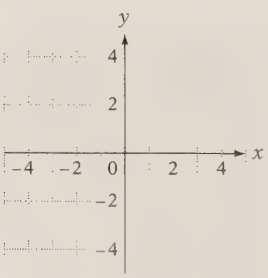
20. Graph $y = -\frac{1}{4}x^2$.



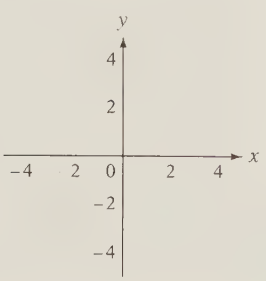
21. Graph $y = 2x^2 + 1$.



22. Graph $y = x^2 - 4x + 3$.



23. Graph $y = -x^2 + 4x - 5$.



24. It took a hawk half an hour more to fly 70 mi against the wind than to go 40 mi with the wind. The rate of the wind was 5 mph. Find the rate of the hawk in calm air.

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Chapter Test

1. Solve by factoring: $x^2 - 5x - 6 = 0$

2. Solve by factoring: $3x^2 + 7x = 20$

3. Solve by taking square roots:
$2(x - 5)^2 - 50 = 0$

4. Solve by taking square roots:
$3(x + 4)^2 - 60 = 0$

5. Solve by completing the square:
$x^2 + 4x - 16 = 0$

6. Solve by completing the square:
$x^2 + 3x = 8$

7. Solve by completing the square:
$2x^2 - 6x + 1 = 0$

8. Solve by completing the square:
$2x^2 + 8x = 3$

9. Solve by using the quadratic formula:
$x^2 + 4x + 2 = 0$

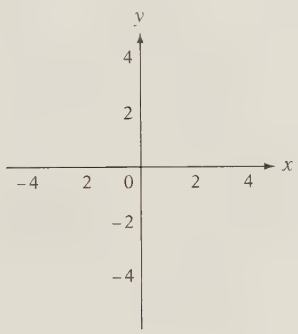
10. Solve by using the quadratic formula:
$x^2 - 3x = 6$

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11. Solve by using the quadratic formula:
$2x^2 - 5x - 3 = 0$

12. Solve by using the quadratic formula:
$3x^2 - x = 1$

13. Graph $y = x^2 + 2x - 4$.



14. The length of a rectangle is 2 ft less than twice the width. The area of the rectangle is 40 ft^2. Find the length and width of the rectangle.

15. It took a motorboat 1 h more to travel 60 mi against a current than it took the boat to travel 60 mi with the current. The rate of the current was 1 mph. Find the rate of the boat in calm water.

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Cumulative Review

1. Simplify: $2x - 3[2x - 4(3 - 2x) + 2] - 3$

2. Solve: $-\frac{3}{5}x = -\frac{9}{10}$

3. Solve: $2x - 3(4x - 5) = -3x - 6$

4. Simplify: $(2a^2b)^2(-3a^4b^2)$

5. Simplify: $(x^2 - 8) \div (x - 2)$

6. Factor: $3x^3 + 2x^2 - 8x$

7. Simplify: $\frac{3x^2 - 6x}{4x - 6} \div \frac{2x^2 + x - 6}{6x^3 - 24x}$

8. Simplify: $\frac{x}{2(x - 1)} - \frac{1}{(x - 1)(x + 1)}$

9. Simplify: $\dfrac{1 - \frac{7}{x} + \frac{12}{x^2}}{2 - \frac{1}{x} - \frac{15}{x^2}}$

10. Find the x- and y-intercepts for the graph of the line $4x - 3y = 12$.

11. Find the equation of the line that contains the point $(-3, 2)$ and has slope $-\frac{4}{3}$.

12. Solve by substitution:
$3x - y = 5$
$y = 2x - 3$

13. Solve by the addition method:
$3x + 2y = 2$
$5x - 2y = 14$

14. Solve: $2x - 3(2 - 3x) > 2x - 5$

15. Simplify: $(\sqrt{a} - \sqrt{2})(\sqrt{a} + \sqrt{2})$

16. Simplify: $\frac{\sqrt{108a^7b^3}}{\sqrt{3a^4b}}$

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17. Simplify: $\dfrac{\sqrt{3}}{5 + 2\sqrt{3}}$

18. Solve: $3 = 8 - \sqrt{5x}$

19. Solve by factoring: $6x^2 - 17x = -5$

20. Solve by taking square roots: $2(x - 5)^2 = 36$

21. Solve by completing the square: $3x^2 + 7x = -3$

22. Solve by using the quadratic formula: $2x^2 - 3x - 2 = 0$

23. Find the cost per pound of a mixture made from 20 lb of cashews that cost \$3.50 per pound and 50 lb of peanuts that cost \$1.75 per pound.

24. A stock investment of 100 shares paid a dividend of \$215. At this rate, how many additional shares are required for the investor to earn a dividend of \$752.50?

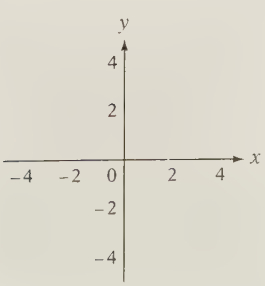
25. A 720-mile trip from one city to another takes 3 h when a plane is flying with the wind. The return trip, against the wind, takes 4.5 h. Find the rate of the plane in still air and the rate of the wind.

26. A student received a 70, a 91, an 85, and a 77 on four tests in a mathematics class. What scores on the last test will enable the student to receive a minimum of 400 points?

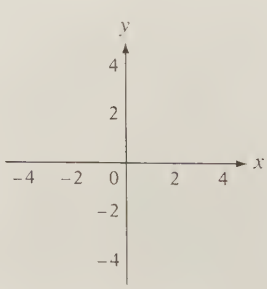
27. The sum of the squares of three consecutive odd integers is 83. Find the middle odd integer.

28. A jogger ran 7 mi at a constant rate, and then reduced the rate by 3 mph and ran an additional 8 mi at the reduced rate. The total time spent jogging the 15 mi was 3 h. Find the jogger's rate for the last 8 mi.

29. Graph $2x - 3y > 6$.



30. Graph $y = x^2 - 2x - 3$.



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Final Exam

1. Evaluate $-|-3|$.

2. Subtract: $-15 - (-12) - 3$

3. Simplify: $-2^4 \cdot (-2)^4$

4. Simplify: $-7 - \frac{12 - 15}{2 - (-1)} \cdot (-4)$

5. Evaluate $\frac{a^2 - 3b}{2a - 2b^2}$ when $a = 3$ and $b = -2$.

6. Simplify: $6x - (-4y) - (-3x) + 2y$

7. Simplify: $(-15z)\left(-\frac{2}{5}\right)$

8. Simplify: $-2[5 - 3(2x - 7) - 2x]$

9. Solve: $20 = -\frac{2}{5}x$

10. Solve: $4 - 2(3x + 1) = 3(2 - x) + 5$

11. Write $\frac{1}{8}$ as a percent.

12. Find 19% of 80.

13. Simplify: $(2x^2 - 5x + 1) - (5x^2 - 2x - 7)$

14. Simplify: $(-3xy^3)^4$

15. Simplify: $(3x^2 - x - 2)(2x + 3)$

16. Simplify: $\frac{(-2x^2y^3)^3}{(-4xy^4)^2}$

17. Simplify: $\frac{12x^2y - 16x^3y^2 - 20y^2}{4xy^2}$

18. Simplify: $(5x^2 - 2x - 1) \div (x + 2)$

19. Simplify: $(4x^{-2}y)^2(2xy^{-2})^{-2}$

20. Given $f(t) = \frac{t}{t + 1}$, find $f(3)$.

21. Factor: $x^2 - 5x - 6$

22. Factor: $6x^2 - 5x - 6$

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23. Factor: $8x^3 - 28x^2 + 12x$

24. Factor: $25x^2 - 16$

25. Factor: $2a(4 - x) - 6(x - 4)$

26. Factor: $75y - 12x^2y$

27. Solve: $2x^2 = 7x - 3$

28. Simplify: $\frac{2x^2 - 3x + 1}{4x^2 - 2x} \cdot \frac{4x^2 + 4x}{x^2 - 2x + 1}$

29. Simplify: $\frac{5}{x + 3} - \frac{3x}{2x - 5}$

30. Simplify: $x - \frac{1}{1 - \frac{1}{x}}$

31. Solve: $\frac{5x}{3x - 5} - 3 = \frac{7}{3x - 5}$

32. Solve $a = 3a - 2b$ for a.

33. Find the slope of the line that contains the points $(-1, -3)$ and $(2, -1)$.

34. Find the equation of the line that contains the point $(3, -4)$ and has slope $-\frac{2}{3}$.

35. Solve by substitution:
$y = 4x - 7$
$y = 2x + 5$

36. Solve by the addition method:
$4x - 3y = 11$
$2x + 5y = -1$

37. Solve: $4 - x \geq 7$

38. Solve: $2 - 2(y - 1) \leq 2y - 6$

39. Simplify: $\sqrt{49x^6}$

40. Simplify: $2\sqrt{27a} + 8\sqrt{48a}$

41. Simplify: $\frac{\sqrt{3}}{\sqrt{5} - 2}$

42. Solve: $\sqrt{2x - 3} + 4 = 5$

43. Solve by factoring:
$3x^2 - x = 4$

44. Solve by using the quadratic formula:
$4x^2 - 2x - 1 = 0$

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45. Translate and simplify "the sum of twice a number and three times the difference between the number and two."

46. Because of depreciation, the value of an office machine is now \$2400. This is 80% of its original value. Find the original value.

47. The manufacturer's cost for a laser printer is \$900. The manufacturer then sells the printer for \$1485. What is the markup rate?

48. An investment of \$3000 is made at an annual simple interest rate of 8%. How much additional money must be invested at 11% so that the total interest earned is 10% of the total investment?

49. A grocer mixes 4 lb of peanuts that cost \$2 per pound with 2 lb of walnuts that cost \$5 per pound. What is the cost per pound of the resulting mixture?

50. A pharmacist mixes together 20 L of a solution that is 60% acid and 30 L of a solution that is 20% acid. What is the percent concentration of the acid in the mixture?

51. At 2 P.M. a small plane had been flying 1 h when a change of wind direction doubled its average ground speed. The pilot completed the 860-kilometer trip in 2.5 h. How far did the plane travel in the first hour?

52. The angles of a triangle are such that the second angle is 10° more than the first angle and the third angle is 10° more than the second angle. Find the measure of each of the three angles.

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53. The sum of the squares of three consecutive integers is 50. Find the middle integer.

54. The length of a rectangle is 5 m more than the width. The area of the rectangle is 50 m^2. Find the dimensions of the rectangle.

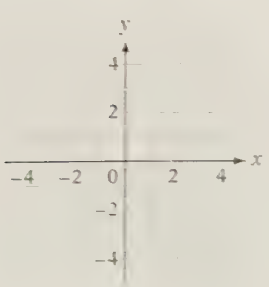
55. A paint formula requires 2 oz of dye for every 15 oz of base paint. How many ounces of dye are required for 120 oz of base paint?

56. It takes a chef 1 h to prepare a dinner. The chef's apprentice can prepare the dinner in 1.5 h. How long would it take the chef and the apprentice, working together, to prepare the dinner?

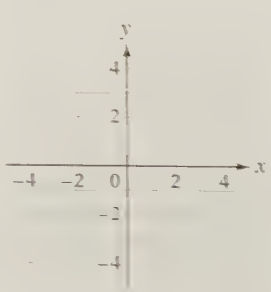
57. With the current, a motorboat travels 50 mi in 2.5 h. Against the current, it takes twice as long to travel 50 mi. Find the rate of the boat in calm water and the rate of the current.

58. Flying against the wind, it took a pilot $\frac{1}{2}$ h longer to travel 500 mi than it took flying with the wind. The rate of the plane in calm air is 225 mph. Find the rate of the wind.

59. Graph the line that has slope $-\frac{1}{2}$ and y-intercept $(0, -3)$.



60. Graph $y = x^2 - 4x + 3$.



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Appendix: Guidelines for Using Graphing Calculators

TEXAS INSTRUMENTS TI-83

To evaluate an expression

a. Press the [Y=] key. A menu showing \Y1 = through \Y7 = will be displayed vertically with a blinking cursor to the right of \Y1 =. Press [CLEAR], if necessary, to delete an unwanted expression.

b. Input the expression to be evaluated. For example, to input the expression $-3a^2b - 4c$, use the following keystrokes:

[(−)] 3 [ALPHA] A [^] 2 [ALPHA] B [−] 4 [ALPHA] C [2nd] QUIT

Note the difference between the keys for a negative sign [(−)] and a *minus* sign [−].

c. Store the value of each variable that will be used in the expression. For example, to evaluate the expression above when $a = 3$, $b = -2$, and $c = -4$, use the following keystrokes:

3 [STO▷] [ALPHA] A [ENTER] [(−)] 2 [STO▷] [ALPHA] B [ENTER] [(−)] 4 [STO▷] [ALPHA] C [ENTER]

These steps store the value of each variable.

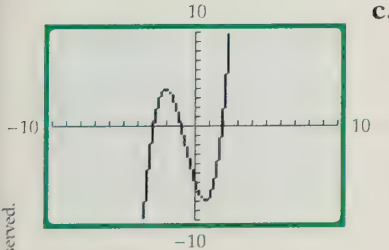
d. Press [VARS] [▷] [1] [1] [ENTER]. The value for the expression, Y1, for the given values is displayed; in this case, Y1 = 70.

To graph a function

a. Press the [Y=] key. A menu showing \Y1 = through \Y7 = will be displayed vertically with a blinking cursor to the right of \Y1 =. Press [CLEAR], if necessary, to delete an unwanted expression.

b. Input the expression for each function that is to be graphed. Press [X,T,θ,n] to input x. For example, to input $y = x^3 + 2x^2 - 5x - 6$, use the following keystrokes:

[X,T,θ,n] [^] 3 [+] 2 [X,T,θ,n] [^] 2 [−] 5 [X,T,θ,n] [−] 6



c. Set the domain and range by pressing [WINDOW]. Enter the values for the minimum x-value (Xmin), the maximum x-value (Xmax), the distance between tick marks on the x-axis (Xscl), the minimum y-value (Ymin), the maximum y-value (Ymax), and the distance between tick marks on the y-axis (Yscl). Now press [GRAPH]. For the graph shown at the left, Xmin = −10, Xmax = 10, Xscl = 1, Ymin = −10, Ymax = 10, and Yscl = 1. This is called the standard viewing rectangle. Pressing [ZOOM] [6] is a quick way to set the calculator to the standard viewing rectangle. *Note:* This will also immediately graph the function in that window.

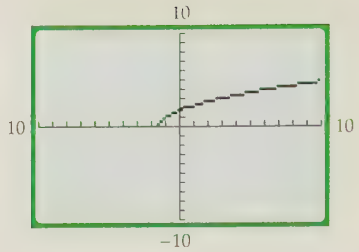
d. Press the [Y=] key. The equal sign has a black rectangle around it. This indicates that the function is active and will be graphed when the [GRAPH] key is pressed. A function is deactivated by using the arrow keys. Move the cursor over the equal sign and press [ENTER]. When the cursor is moved to the right, the black rectangle will not be present and that equation will not be active.

e. Graphing some radical equations requires special care. To graph the function $y = \sqrt{2x + 3}$, enter the following keystrokes:

[Y=] [2nd] [√] 2 [X,T,θ,n] [+] 3 [)]

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The graph is shown below.



To display the x-coordinates of rectangular coordinates as integers

a. Set the viewing window as follows: Xmin = −47, Xmax = 47, Xscl = 10, Ymin = −31, Ymax = 31, Yscl = 10.

b. Graph the function and use the TRACE feature. Press [TRACE] and then move the cursor with the [◁] and [▷] keys. The values of x and $y = f(x)$ displayed on the bottom of the screen are the coordinates of a point on the graph.

To display the x-coordinates of rectangular coordinates in tenths

a. Set the viewing window as follows: [ZOOM] [4]

b. Graph the function and use the [TRACE] feature. Press [TRACE] and then move the cursor with the [◁] and [▷] keys. The values of x and $y = f(x)$ displayed on the bottom of the screen are the coordinates of a point on the graph.

To evaluate a function for a given value of x, or to produce ordered pairs of a function

a. Input the equation; for example, input $y_1 = 2x^3 - 3x + 2$.

b. Press [2nd] QUIT.

c. Input a value for x; for example, to input 3 press 3 [STO▷] [X,T,θ,n] [ENTER].

d. Press [VARS] [▷] [1] [1] [ENTER]. The value for the expression, Y_1, for the given x-value is displayed, in this case, $Y_1 = 47$. An ordered pair of the function is (3, 47).

e. Repeat steps **c.** and **d.** to produce as many pairs as desired. The TABLE feature of the *TI-83* can also be used to determine pairs.

ZOOM FEATURES

To zoom in or out on a graph

a. Here are two methods of using ZOOM. The first method uses the built-in features of the calculator. Move the cursor to a point on the graph that is of interest. Press [ZOOM]. The ZOOM menu will appear. Press [2] [ENTER] to zoom in on the graph by the amount shown under the SET FACTORS menu. The center of the new graph is the location at which you placed the cursor. Press [ZOOM] [3] [ENTER] to zoom out on the graph by the amount under the SET FACTORS menu. (The SET FACTORS menu is accessed by pressing [ZOOM] [▷] [4].)

b. The second method uses the ZBOX option under the ZOOM menu. To use this method, press [ZOOM] [1]. A cursor will appear on the graph. Use the arrow keys to move the cursor to a portion of the graph that is of interest. Press [ENTER]. Now use the arrow keys to draw a box around the portion of the graph you wish to see. Press [ENTER]. The portion of the graph defined by the box will be drawn.

c. Pressing [ZOOM] [6] resets the window to the standard 10 × 10 viewing window.

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SOLVING EQUATIONS

This discussion is based on the fact that the solution of an equation can be related to the x-intercepts of a graph. For instance, the real solutions of the equation $x^2 = x + 1$ are the x-intercepts of the graph of $f(x) = x^2 - x - 1$, which are the zeros of f.

To solve $x^2 = x + 1$, rewrite the equation with all terms on one side. The equation is now $x^2 - x - 1 = 0$. Think of this equation as $Y_1 = x^2 - x - 1$. The x-intercepts of the graph of Y_1 are the solutions of the equation $x^2 = x + 1$.

a. Enter $x^2 - x - 1$ into Y_1.

b. Graph the equation. You may need to adjust the viewing window so that the x-intercepts are visible.

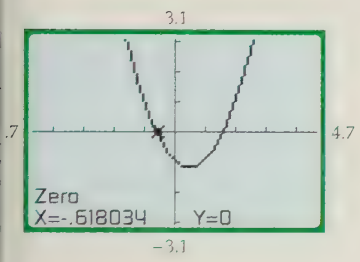
c. Press 2nd CALC 2.

d. Move the cursor to a point on the curve that is to the left of an x-intercept. Press ENTER.

e. Move the cursor to a point on the curve that is to the right of an x-intercept. Press ENTER.

f. Press ENTER.

g. The root is shown as the x-coordinate on the bottom of the screen; in this case, the root is approximately -0.618034. To find the next intercept, repeat steps **c.** through **f.** The SOLVER feature under the MATH menu can also be used to find solutions of equations.



SOLVING SYSTEMS OF EQUATIONS IN TWO VARIABLES

To solve a system of equations

To solve $\begin{cases} y = x^2 - 1 \\ \frac{1}{2}x + y = 1 \end{cases}$,

a. Solve each equation for y.

b. Enter the first equation as Y_1. For instance, $Y_1 = x^2 - 1$.

c. Enter the second equation as Y_2. For instance, $Y_2 = 1 - \frac{1}{2}x$.

d. Graph both equations. (*Note:* The point of intersection must appear on the screen. It may be necessary to adjust the viewing window so that the point(s) of intersection are displayed.)

e. Press 2nd CALC 5.

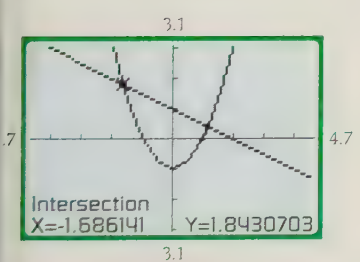
f. Move the cursor to the left of the first point of intersection. Press ENTER.

g. Move the cursor to the right of the first point of intersection. Press ENTER.

h. Press ENTER.

i. The first point of intersection is $(-1.686141, 1.8430703)$.

j. Repeat steps **e.** through **h.** for each point of intersection.



FINDING MINIMUM OR MAXIMUM VALUES OF A FUNCTION

a. Enter the function into Y_1. The equation $y = x^2 - x - 1$ is used here.

b. Graph the equation. You may need to adjust the viewing window so that the maximum or minimum points are visible.

c. Press 2nd CALC 3 to determine a minimum value or press 2nd CALC 4 to determine a maximum value.

d. Move the cursor to a point on the curve that is to the left of the minimum (maximum). Press ENTER.

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e. Move the cursor to a point on the curve that is to the right of the minimum (maximum). Press [ENTER].

f. Press [ENTER].

g. The minimum (maximum) is shown as the y-coordinate on the bottom of the screen; in this case the minimum value is -1.25.

SHARP EL-9600

To evaluate an expression

a. The SOLVER mode of the calculator is used to evaluate expressions. To enter SOLVER mode, press [2ndF] SOLVER [CL]. The expression $-3a^2b - 4c$ must be entered as the equation $-3a^2b - 4c = t$. The letter t can be any letter other than one used in the expression. Use the following keystrokes to input $-3a^2b - 4c = t$:

[(−)] 3 [ALPHA] A [a^b] 2 [▷] [ALPHA] B [−] 4 [ALPHA] C [ALPHA] [=] [ALPHA] T [ENTER]

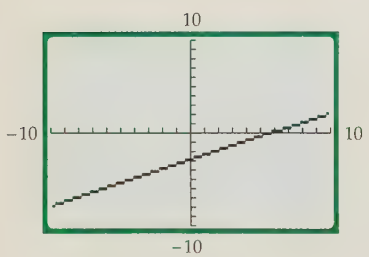
Note the difference between the keys for a *negative* sign (−) and a *minus* sign.

b. After you press [ENTER], variables used in the equation will be displayed on the screen. To evaluate the expression for $a = 3$, $b = -2$, and $c = -4$, input each value, pressing [ENTER] after each number. When the cursor moves to T, press [2ndF] EXE. T = 70 will appear on the screen. This is the value of the expression. To evaluate the expression again for different values of a, b, and c, press [2ndF] QUIT and then [2ndF] SOLVER.

c. Press [+ −; × ÷] to return to normal operation.

To graph a function

a. Press the [Y=] key. The screen will show Y_1 through Y_8.

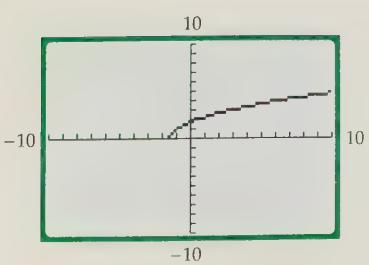


b. Input the expression for a function that is to be graphed. Press [X/T/θ/n] to enter an x. For example, to input $y = \frac{1}{2}x - 3$, use the following keystrokes:

[Y=] [CL] [(] 1 [÷] 2 [)] [X/T/θ/n] [−] 3 [ENTER]

c. Set the viewing window by pressing [WINDOW]. Enter the values for the minimum x-value (Xmin), the maximum x-value (Xmax), the distance between tick marks on the x-axis (Xscl), the minimum y-value (Ymin), the maximum y-value (Ymax), and the distance between tick marks on the y-axis (Yscl). Press [ENTER] after each entry. Press [GRAPH]. For the graph shown at the left, enter Xmin = −10, Xmax = 10, Ymin = −10, Ymax = 10. Press [GRAPH].

d. Press [Y=] to return to the equation. The equal sign has a black rectangle around it. This indicates that the function is active and will be graphed when the [GRAPH] key is pressed. A function is deactivated by using the arrow keys. Move the cursor over the equal sign and press [ENTER]. When the cursor is moved to the right, the black rectangle will not be present and that equation will not be active.



e. Graphing some radical equations requires special care. To graph the function $y = \sqrt{2x + 3}$, enter the following keystrokes:

[Y=] [CL] [2ndF] [$\sqrt{\ }$] 2 [X/T/θ/n] [+] 3 [▷] [GRAPH]

The graph is shown at the left.

To display the xy-coordinates as integers

a. Press [ZOOM] [▷] 8.

b. Graph the function. Press [TRACE]. Use the left and right arrow keys to trace

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along the graph of the function. The x- and y-coordinates of the function are shown on the bottom of the screen.

To display the xy-coordinates in tenths

a. Press [ZOOM] [▷] 7.

b. Graph the function. Press [TRACE].Use the left and right arrow keys to trace along the graph of the function. The x- and y-coordinates of the function are shown on the bottom of the screen.

To evaluate a function for a given value of x, or to produce ordered pairs of the function

a. Press [Y=]. Input the expression. For instance, input [Y-] [CL] 2 [X/T/θ/n] [^] 3 [−] 3 [X/T/θ/n] [+] 2. Press [ENTER].

b. Press [+ −; × ÷]. Store the x-coordinate of the ordered pair you want in [X/T/θ/n]. For instance, enter 3 [STO] [X/T/θ/n] [ENTER].

c. Press [VARS] [ENTER] 1 [ENTER]. The value of y, 47, will be displayed on the screen. The ordered pair is (3, 47). The TABLE feature of the calculator can also be used to find many ordered pairs for a function.

ZOOM FEATURES OF THE SHARP EL-9600

To zoom in or out on a graph

a. Here are two methods of using ZOOM. The first method uses the built-in features of the calculator. Move the cursor to a point on the graph that is of interest. Press [ZOOM]. The ZOOM menu will appear. Press 3 to zoom in on the graph by the amount shown by FACTOR. The center of the new graph is the location at which you placed the cursor. Press [ZOOM] 4 to zoom out on the graph by the amount shown in FACTOR.

b. The second method uses the BOX option under the ZOOM menu. To use this method, press [ZOOM] 2. A cursor will appear on the screen. Use the arrow keys to move the cursor to a portion of the graph that is of interest. Press [ENTER]. Use the arrow keys to draw a box around the portion of the graph you wish to see. Press [ENTER].

SOLVING EQUATIONS OR SYSTEMS OF EQUATIONS IN TWO VARIABLES

This discussion is based on the fact that the real solutions of an equation can be related to the x-intercepts of a graph. For instance, the real solutions of $x^2 = x + 1$ are the x-intercepts of the graph of $f(x) = x^2 - x - 1$, which are the zeros of f.

To solve $x^2 = x + 1$, rewrite the equation with all terms on one side of the equation. The equation is now $x^2 - x - 1 = 0$. Think of this equation as $Y_1 = x^2 - x - 1$. The x-intercepts of the graph of Y_1 are the solutions of the equation $x^2 = x + 1$.

a. Enter $x^2 - x - 1$ into Y_1.

b. Graph the equation. You may need to adjust the viewing window so that the x-intercepts are visible.

c. Press [2ndF] CALC 5

d. A solution is shown as the x-coordinate at the bottom of the screen. To find the next intercept, move the cursor to the right of the first x-intercept. Then press [2ndF] CALC 5.

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SOLVING SYSTEMS OF EQUATIONS

To solve a system of equations

a. Solve each equation for y.

b. Press [Y=] and then enter both equations.

c. Graph the equations. You may need to adjust the viewing window so that the point of intersection is visible.

d. Press [2ndF] CALC 2 to find the point of intersection. Pressing [2ndF] CALC 2 again will find the next point of intersection.

e. The x- and y-coordinates at the bottom of the screen are the coordinates for the point of intersection.

FINDING MAXIMUM AND MINIMUM VALUES OF A FUNCTION

a. Press [Y=] and then enter the function.

b. Graph the equation. You may need to adjust the viewing window so that the maximum (minimum) are visible.

c. Press [2ndF] CALC 3 for the minimum value of the function or [2ndF] CALC 4 for the maximum value of the fuction.

d. The y-coordinate at the bottom of the screen is the maximum (minimum).

CASIO *CFX-9850G*

To evaluate an expression

a. Press [MENU] [5]. Use the arrow keys to highlight Y1.

b. Input the expression to be evaluated. For example, to input the expression $-3A^2B - 4C$, use the following keystrokes:

[(−)] 3 [ALPHA] A [x^2] [ALPHA] B [−] 4 [ALPHA] C [EXE]

Note the difference between the keys for a *negative* sign [(−)] and a *minus* sign [−].

c. Press [MENU] 1. Store the value of each variable that will be used in the expression. For example, to evaluate the expression above when $A = 3$, $B = -2$, and $C = -4$, use the following keystrokes:

3 → [ALPHA] A [EXE] [(−)] 2 → [ALPHA] B [EXE] [(−)] 4 → [ALPHA] C [EXE]

These steps store the value of each variable.

d. Press [VARS] [F4] [F1] 1 [EXE].

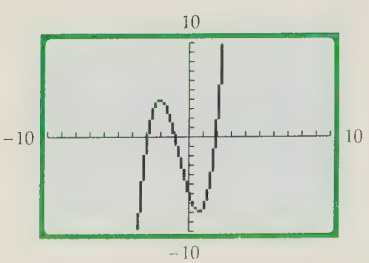
The value of the expression, Y1, for the given values is displayed; in this case, Y1 = 70.

To graph a function

a. Press Menu [5] to obtain the GRAPH FUNCTION Menu.

b. Input the function that you desire to graph. Press [X,θ,T] to input the variable x. For example, to input $y = x^3 + 2x^2 - 5x - 6$, use the following keystrokes:

[X,θ,T] [^] 3 [+] 2 [X,θ,T] [x^2] [−] 5 [X,θ,T] [−] 6 [EXE]



c. Set the viewing window by pressing [SHIFT] [F3] and the Range Parameter Menu will appear. Enter the values for the minimum x-value (Xmin), maximum x-value (Xmax), units between tick marks on the x-axis (Xscl), minimum y-value (Ymin), maximum y-value (Ymax), and the units between tick marks on the y-axis (Yscl). Press [EXE] after each of the 6 entries above. Press [EXIT], or [SHIFT] [QUIT], to leave the Range Parameter Menu.

d. Press [F6] to draw the graph. For the graph shown at the left, Xmin = −10, Xmax = 10, Xscl = 1, Ymin = −10, Ymax = 10, Yscl = 1.

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e. In the equation for Y_1, there is a rectangle around the equal sign. This indicates that this function is *active* and will be graphed when the [F6] key is pressed. A function is deactivated by using the [F1] key. After using this key once, the rectangle around the equal sign will not be present and that function will not be graphed.

To display the x-coordinates of rectangular coordinates as integers

a. Set the Range as follows: For example, set Xmin = −63, Xmax = 63, Xscl = 10, Ymin = −32, Ymax = 32, Yscl = 10.

b. Graph a function and use the Trace feature. Press [F1] and then move the cursor with the [◁] and the [▷] keys. The values of x and $y = f(x)$ displayed on the bottom of the screen are the coordinates of a point on the graph. Observe that the x-value is given as an integer.

To display the x-coordinates of rectangular coordinates in tenths

a. Set the Range as follows: For example, set Xmin = −6.3, Xmax = 6.3. A quick way to choose these range parameter settings is to press [F1] from the V-Window Menu.

b. Graph a function and use the Trace feature. Press [F1] and then move the cursor with the [◁] and the [▷] keys. The values of x and $y = f(x)$ displayed on the bottom of the screen are the coordinates of a point on the graph. Observe that the x-value is given as a decimal that terminates in the first decimal place (tenths).

To evaluate a function for a given value of x, or to produce ordered pairs of the function

a. Press [MENU] [5].

b. Input the function to be evaluated. For example, input $2x^3 - 3x + 2$ into Y_1.

c. Press [MENU] 1.

d. Input a value for x; for example, to input 3 press
3 → [X,θ,T] [EXE]

e. Press [VARS] [F4] [F1] 1 [EXE].
The value of Y_1 for the given value $x = 3$ is displayed. In this case, $Y_1 = 47$.

ZOOM FEATURES

To zoom in or out on a graph

a. After drawing a graph, press [SHIFT] Zoom to display the Zoom/Auto Range menu. To zoom in on a graph by a factor of 2 on the x-axis and a factor of 1.5 on the y-axis:
Press [F2] to display the Factor Input Screen. Input the zoom factors for each axis: 2 [EXE] 1 [·] 5 [EXE] [EXIT]. Press [F3] to redraw the graph according to the factors specified above. To specify the center point of the enlarged (reduced) display after pressing [SHIFT] Zoom use the arrow keys to move the pointer to the position you wish to become the center of the next display. You can repeat the zoom procedures as needed. If you wish to see the original graph, press [F6] [F1]. This procedure resets the range parameters to their original values and redraws the graph.

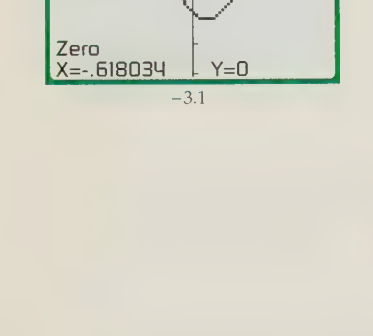
b. A second method of zooming makes use of the Box Zoom Function. To use this method, first draw a graph. Then press [SHIFT] Zoom [F1]. Now use the arrow (cursor) keys to move the pointer. Once the pointer is located at a portion of the graph that is of interest, press [EXE]. Now use the arrow keys to draw a box around the portion of the graph you wish to see. Press [EXE]. The portion of the graph defined by the box will be drawn.

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SOLVING EQUATIONS

This discussion is based on the fact that the real solutions of an equation can be related to the x-intercepts of a graph. For instance, the real solutions of $x^2 = x + 1$ are the x-intercepts of the graph of $f(x) = x^2 - x - 1$, which are the zeros of f.

To solve $x^2 = x + 1$, rewrite the equation with all terms on one side. The equation is now $x^2 - x - 1 = 0$. Think of this equation as $Y_1 = x^2 - x - 1$. The x-intercepts of the graph of Y_1 are the solutions of the equation $x^2 = x + 1$.



a. Enter $x^2 - x - 1$ into Y_1.

b. Graph the equation. You may need to adjust the viewing window so that the x-intercept is visible.

c. Press SHIFT G-SOLV F1.

d. The root is shown as the x-coordinate on the bottom of the screen; in this case, the root is approximately -0.618034. To find the next x-intercept, press the right arrow key.

The EQUA Mode (Press MENU ALPHA A) can also be used to find solutions of linear, quadratic, and cubic equations.

SOLVING SYSTEMS OF TWO EQUATIONS IN TWO VARIABLES

The following discussion is based on the concept that the solutions of a system of two equations are represented by the point(s) of intersections of the graphs.

The system of equations $\begin{matrix} y = x^2 - 1 \\ \frac{1}{2}x + y = 1 \end{matrix}$ will be solved.

a. Solve each equation for y.

b. Enter the first equation in the Graph Menu as Y_1. For instance, let $Y_1 = x^2 - 1$.

c. Enter the second equation as Y_2. For instance, let $Y_2 = 1 - \frac{1}{2}x$.

d. Graph both equations. (*Note:* The point of intersection must appear on the screen. It may be necessary to adjust the viewing window so that the point of intersection that is of interest is the only intersection point that is displayed.)

e. Press SHIFT G-SOL F5 EXE.

f. The display will show that the graphs intersect at $(-1.686141, 1.8430703)$. To find the next intersect, repeat step **e.**

FINDING MINIMUM OR MAXIMUM VALUES OF A FUNCTION

a. Enter the function into the graphing menu. For this example we have used $y = x^2 - x - 1$.

b. Graph the function. Adjust the viewing window so that the maximum or minimum is visible.

c. Press SHIFT G-SOL F2 EXE for a maximum and F3 EXE for a minimum.

d. The local maximum (minimum) is shown as the y-coordinate on the bottom of the screen; in this case, the minimum value is -1.25.

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Table of Symbols

$+$	add	$<$	is less than
$-$	subtract	$\leq$	is less than or equal to
$\cdot, \times, (a)(b)$	multiply	$>$	is greater than
$\frac{a}{b}, \div, a\overline{)b}$	divide	$\geq$	is greater than or equal to
$( )$	parentheses, a grouping symbol	(a, b)	an ordered pair whose first component is a and whose second component is b
$[ ]$	brackets, a grouping symbol	$^\circ$	degree (for angles)
π	pi, a number approximately equal to $\frac{22}{7}$ or 3.14	$\sqrt{a}$	the principal square root of a
$-a$	the opposite, or additive inverse, of a	$\varnothing, \{ \}$	the empty set
$\frac{1}{a}$	the reciprocal, or multiplicative inverse, of a	$\lvert a \rvert$	the absolute value of a
$=$	is equal to	$\cup$	union of two sets
$\approx$	is approximately equal to	$\cap$	intersection of two sets
$\neq$	is not equal to	$\in$	is an element of (for sets)

Table of Measurement Abbreviations

U.S. Customary System

Length		Capacity		Weight		Area	
in.	inches	oz	fluid ounces	oz	ounces	in^2	square inches
ft	feet	c	cups	lb	pounds	ft^2	square feet
yd	yards	qt	quarts			yd^2	square yards
mi	miles	gal	gallons			mi^2	square miles

Metric System

Length		Capacity		Weight/Mass		Area	
mm	millimeter (0.001 m)	ml	milliliter (0.001 L)	mg	milligram (0.001 g)	cm^2	square centimeters
cm	centimeter (0.01 m)	cl	centiliter (0.01 L)	cg	centigram (0.01 g)	m^2	square meters
dm	decimeter (0.1 m)	dl	deciliter (0.1 L)	dg	decigram (0.1 g)		
m	meter	L	liter	g	gram		
dam	decameter (10 m)	dal	decaliter (10 L)	dag	decagram (10 g)		
hm	hectometer (100 m)	hl	hectoliter (100 L)	hg	hectogram (100 g)		
km	kilometer (1000 m)	kl	kiloliter (1000 L)	kg	kilogram (1000 g)		

Time

h	hours	min	minutes	s	seconds

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Solutions to Chapter 1 "You Try It"

SECTION 1.1

You Try It 1 $A = \{1, 2, 3, 4, 5, 6\}$

You Try It 2
$-5 < -1$
$-1 = -1$
$5 > -1$

The element 5 is greater than -1.

You Try It 3
$|-5| = 5$
$-|-23| = -23$

You Try It 4
$-(-11) = 11$
$-0 = 0$
$-(8) = -8$

You Try It 5
$|-37| = 37$
$|0| = 0$
$|29| = 29$

SECTION 1.2

You Try It 1 $100 + (-43) = 57$

You Try It 2
$$\begin{aligned}(-51) + 42 + 17 + (-102) &= -9 + 17 + (-102)\\ &= 8 + (-102)\\ &= -94\end{aligned}$$

You Try It 3
$$\begin{aligned}19 - (-32) &= 19 + 32\\ &= 51\end{aligned}$$

You Try It 4
$$\begin{aligned}-9 - (-12) - 17 - 4 &= -9 + 12 + (-17) + (-4)\\ &= 3 + (-17) + (-4)\\ &= -14 + (-4)\\ &= -18\end{aligned}$$

You Try It 5

Strategy To find the difference between the two average temperatures, subtract the smaller number (-130) from the larger number (-17).

Solution $-17 - (-130) = -17 + 130 = 113$

The difference is 113°F.

SECTION 1.3

You Try It 1
$$\begin{aligned}8(-9)10 &= -72(10)\\ &= -720\end{aligned}$$

You Try It 2
$$\begin{aligned}(-2)3(-8)7 &= -6(-8)7\\ &= 48(7)\\ &= 336\end{aligned}$$

You Try It 3 $(-135) \div (-9) = 15$

You Try It 4 $\dfrac{-72}{4} = -18$

You Try It 5
$$\begin{aligned}-\frac{36}{-12} &= -(-3)\\ &= 3\end{aligned}$$

You Try It 6

Strategy

To find the average low temperature:
- Add the seven temperature readings.
- Divide the sum by 7.

Solution

$-6 + (-7) + 0 + (-5) + (-8) + (-1) + (-1) = -28$
$-28 \div 7 = -4$

The average daily low temperature was −4°C.

SECTION 1.4

You Try It 1
$$\begin{array}{r} 0.444\ldots \\ 9\overline{)4.000} \\ -3\,6 \\ \hline 40 \\ -36 \\ \hline 40 \\ -36 \\ \hline 4 \end{array} \qquad \frac{4}{9} = 0.\overline{4}$$

You Try It 2
$125\% = 125\left(\frac{1}{100}\right) = \frac{125}{100} = 1\frac{1}{4}$
$125\% = 125(0.01) = 1.25$

You Try It 3 $16\frac{2}{3}\% = 16\frac{2}{3}\left(\frac{1}{100}\right) = \frac{50}{3}\left(\frac{1}{100}\right) = \frac{1}{6}$

You Try It 4 $\dfrac{9}{16} = \dfrac{9}{16}(100\%) = \dfrac{900}{16}\% = 56.25\%$ or $56\frac{1}{4}\%$

You Try It 5 $0.043 = 0.043(100\%) = 4.3\%$

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You Try It 6 The LCM of 9 and 12 is 36.

$$\frac{5}{9} - \frac{11}{12} = \frac{20}{36} - \frac{33}{36} = \frac{20}{36} + \frac{-33}{36}$$
$$= \frac{20 + (-33)}{36} = \frac{-13}{36} = -\frac{13}{36}$$

You Try It 7 The LCM of 8, 6, and 4 is 24.

$$-\frac{7}{8} - \frac{5}{6} + \frac{3}{4} = -\frac{21}{24} - \frac{20}{24} + \frac{18}{24}$$
$$= \frac{-21}{24} + \frac{-20}{24} + \frac{18}{24}$$
$$= \frac{-21 + (-20) + 18}{24}$$
$$= \frac{-23}{24} = -\frac{23}{24}$$

You Try It 8

$$16.127 - 67.91 = 16.127 + (-67.91)$$
$$= -51.783$$

You Try It 9 The product is negative.

$$-\frac{7}{12} \times \frac{9}{14} = -\frac{7 \cdot 9}{12 \cdot 14}$$
$$= -\frac{\overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{3}} \cdot 3}{2 \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 2 \cdot \underset{1}{\cancel{7}}}$$
$$= -\frac{3}{8}$$

You Try It 10 The quotient is positive.

$$-\frac{3}{8} \div \left(-\frac{5}{12}\right) = \frac{3}{8} \times \frac{12}{5}$$
$$= \frac{3 \cdot 12}{8 \cdot 5} = \frac{3 \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot 5} = \frac{9}{10}$$

You Try It 11 The product is negative.
Multiply the absolute values.

$$-5.44 \times 3.8 = -20.672$$

You Try It 12 The quotient is negative.
Divide the absolute values.

$$\begin{array}{r} 0.231 \\ 17.\overline{)03.940} \\ -34 \\ \hline 54 \\ -51 \\ \hline 30 \\ -17 \\ \hline 13 \end{array}$$

$$-0.394 \div 1.7 \approx -0.23$$

You Try It 13

Strategy To find the percent of the population that is in the baby-boomer generation:
- Find the total population by adding the numbers in all four generations.
- Divide the number of people in the baby-boomer generation (77.6 million) by the total population.

Solution

$$72.4 + 44.6 + 77.6 + 68.3 = 262.9$$

The population of the United States is 262.9 million.

$$\frac{77.6}{262.9} \approx 0.295 \approx 30\%$$

The baby-boomers make up about 30% of the population of the United States.

SECTION 1.5

You Try It 1

$$-6^3 = -(6 \cdot 6 \cdot 6) = -216$$

You Try It 2

$$(-3)^4 = (-3)(-3)(-3)(-3) = 81$$

You Try It 3

$$(3^3)(-2)^3 = (3)(3)(3) \cdot (-2)(-2)(-2)$$
$$= 27(-8) = -216$$

You Try It 4

$$\left(-\frac{2}{5}\right)^2 = \left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right) = \frac{4}{25}$$

You Try It 5

$$-3(0.3)^3 = -3(0.3)(0.3)(0.3)$$
$$= -0.9(0.3)(0.3)$$
$$= -0.27(0.3) = -0.081$$

You Try It 6

$$(6.97 - 4.72)^2 \cdot 4.5 \div 0.05$$
$$= (2.25)^2 \cdot 4.5 \div 0.05$$
$$= 5.0625 \cdot 4.5 \div 0.05$$
$$= 22.78125 \div 0.05$$
$$= 455.625$$

You Try It 7

$$18 - 5[8 - 2(2 - 5)] \div 10$$
$$= 18 - 5[8 - 2(-3)] \div 10$$
$$= 18 - 5[8 + 6] \div 10$$
$$= 18 - 5[14] \div 10$$
$$= 18 - 70 \div 10$$
$$= 18 - 7$$
$$= 11$$

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You Try It 8

$$\begin{aligned}36 \div (8-5)^2 - (-3)^2 \cdot 2 &= 36 \div (3)^2 - (-3)^2 \cdot 2 \\ &= 36 \div 9 - 9 \cdot 2 \\ &= 4 - 9 \cdot 2 \\ &= 4 - 18 \\ &= -14\end{aligned}$$

You Try It 9

$$\begin{aligned}\frac{5}{8} \div \left(\frac{1}{3} - \frac{3}{4}\right) + \frac{7}{12} &= \frac{5}{8} \div \left(-\frac{5}{12}\right) + \frac{7}{12} \\ &= \frac{5}{8} \cdot \left(-\frac{12}{5}\right) + \frac{7}{12} \\ &= -\frac{3}{2} + \frac{7}{12} \\ &= -\frac{18}{12} + \frac{7}{12} \\ &= -\frac{11}{12}\end{aligned}$$

Solutions to Chapter 2 "You Try It"

SECTION 2.1

You Try It 1 -4 is the constant term.

You Try It 2

$$\begin{aligned}&2xy + y^2 \\ &2(-4)(2) + (2)^2 \\ &\quad = 2(-4)(2) + 4 \\ &\quad = (-8)(2) + 4 \\ &\quad = (-16) + 4 \\ &\quad = -12\end{aligned}$$

You Try It 3

$$\frac{a^2 + b^2}{a + b}$$

$$\begin{aligned}\frac{5^2 + (-3)^2}{5 + (-3)} &= \frac{25 + 9}{5 + (-3)} \\ &= \frac{34}{2} \\ &= 17\end{aligned}$$

You Try It 4

$$\begin{aligned}&x^3 - 2(x + y) + z^2 \\ &(2)^3 - 2[2 + (-4)] + (-3)^2 \\ &\quad = 8 - 2(-2) + 9 \\ &\quad = 8 + 4 + 9 \\ &\quad = 12 + 9 \\ &\quad = 21\end{aligned}$$

SECTION 2.2

You Try It 1 $3a - 2b - 5a + 6b = -2a + 4b$

You Try It 2 $-3y^2 + 7 + 8y^2 - 14 = 5y^2 - 7$

You Try It 3 $-5(4y^2) = -20y^2$

You Try It 4 $-7(-2a) = 14a$

You Try It 5 $(-5x)(-2) = 10x$

You Try It 6 $5(3 + 7b) = 15 + 35b$

You Try It 7 $(3a - 1)5 = 15a - 5$

You Try It 8 $-8(-2a + 7b) = 16a - 56b$

You Try It 9 $3(12x^2 - x + 8) = 36x^2 - 3x + 24$

You Try It 10 $3(-a^2 - 6a + 7) = -3a^2 - 18a + 21$

You Try It 11

$$\begin{aligned}3y - 2(y - 7x) &= 3y - 2y + 14x \\ &= y + 14x\end{aligned}$$

You Try It 12

$$\begin{aligned}-2(x - 2y) - (-x + 3y) &= -2x + 4y + x - 3y \\ &= -x + y\end{aligned}$$

You Try It 13

$$\begin{aligned}3y - 2[x - 4(2 - 3y)] &= 3y - 2[x - 8 + 12y] \\ &= 3y - 2x + 16 - 24y \\ &= -2x - 21y + 16\end{aligned}$$

SECTION 2.3

You Try It 1 the difference between twice n and one-third of n

$$2n - \frac{1}{3}n$$

You Try It 2 the quotient of 7 less than b and 15

$$\frac{b - 7}{15}$$

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You Try It 3 an unknown number: n
the cube of the number: n^3
the total of ten and the cube of the number: $10 + n^3$

$-4(10 + n^3)$

You Try It 4 the unknown number: x
the difference between the number and sixty: $x - 60$

$5(x - 60) = 5x - 300$

You Try It 5 the speed of the older model: s
the new jet operates at twice the speed of the older model: $2s$

You Try It 6 the length of the longer piece: y
the length of the shorter piece: $6 - y$

Solutions to Chapter 3 "You Try It"

SECTION 3.1

You Try It 1

$$\begin{array}{c|c} 5 - 4x = 8x + 2 & \\ \hline 5 - 4\left(\frac{1}{4}\right) & 8\left(\frac{1}{4}\right) + 2 \\ 5 - 1 & 2 + 2 \\ 4 = 4 & \end{array}$$

Yes, $\frac{1}{4}$ is a solution.

You Try It 2

$$\begin{array}{c|c} 10x - x^2 = 3x - 10 & \\ \hline 10(5) - (5)^2 & 3(5) - 10 \\ 50 - 25 & 15 - 10 \\ 25 \neq 5 & \end{array}$$

No, 5 is not a solution.

You Try It 3

$$\frac{5}{6} = y - \frac{3}{8}$$
$$\frac{5}{6} + \frac{3}{8} = y - \frac{3}{8} + \frac{3}{8}$$
$$\frac{29}{24} = y$$

The solution is $\frac{29}{24}$.

You Try It 4

$$-\frac{2}{5}x = 6$$
$$\left(-\frac{5}{2}\right)\left(-\frac{2}{5}x\right) = \left(-\frac{5}{2}\right)(6)$$
$$x = -15$$

The solution is -15.

You Try It 5

$$4x - 8x = 16$$
$$-4x = 16$$
$$\frac{-4x}{-4} = \frac{16}{-4}$$
$$x = -4$$

The solution is -4.

You Try It 6

$$P \cdot B = A$$
$$\frac{1}{6}B = 18 \qquad \bullet\ 16\frac{2}{3}\% = \frac{1}{6}$$
$$6 \cdot \frac{1}{6}B = 6 \cdot 18$$
$$B = 108$$

18 is $16\frac{2}{3}\%$ of 108.

You Try It 7

Strategy To find the percent, solve the basic percent equation using $B = 47.1$ and $A = 23.1$. The percent is unknown.

Solution

$$P \cdot B = A$$
$$P(47.1) = 23.1$$
$$\frac{P(47.1)}{47.1} = \frac{23.1}{47.1}$$
$$P \approx 49.0\%$$

The March 1997 deficit was 49.0% of the March 1996 deficit.

SECTION 3.2

You Try It 1

$$5x + 7 = 10$$
$$5x + 7 - 7 = 10 - 7$$
$$5x = 3$$
$$\frac{5x}{5} = \frac{3}{5}$$
$$x = \frac{3}{5}$$

The solution is $\frac{3}{5}$.

You Try It 2

$$2 = 11 + 3x$$
$$2 - 11 = 11 - 11 + 3x$$
$$-9 = 3x$$

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$$\frac{-9}{3} = \frac{3x}{3}$$
$$-3 = x$$

The solution is -3.

You Try It 3

$$\begin{aligned} x - 5 + 4x &= 25 \\ 5x - 5 &= 25 \\ 5x - 5 + 5 &= 25 + 5 \\ 5x &= 30 \\ \frac{5x}{5} &= \frac{30}{5} \\ x &= 6 \end{aligned}$$

The solution is 6.

You Try It 4

Strategy

Given: $S = 986$
$r = 45\% = 0.45$
Unknown: C

Solution

$$\begin{aligned} S &= C + rC \\ 986 &= C + 0.45C \\ 986 &= 1.45C \\ 680 &= C \end{aligned}$$

The cost of the outboard motor is \$680.

You Try It 5

Strategy

Given: $S = 159$
$r = 25\% = 0.25$
Unknown: R

Solution

$$\begin{aligned} S &= R - rR \\ 159 &= R - 0.25R \\ 159 &= 0.75R \\ 212 &= R \end{aligned}$$

The regular price of the garage door opener is \$212.

You Try It 6

Strategy

To find the depth, replace P with the given value and solve for D.

Solution

$$\begin{aligned} P &= 15 + \frac{1}{2}D \\ 45 &= 15 + \frac{1}{2}D \\ 45 - 15 &= 15 - 15 + \frac{1}{2}D \\ 30 &= \frac{1}{2}D \\ 2(30) &= 2 \cdot \frac{1}{2}D \\ 60 &= D \end{aligned}$$

The depth is 60 ft.

SECTION 3.3

You Try It 1

$$\begin{aligned} 5x + 4 &= 6 + 10x \\ 5x - 10x + 4 &= 6 + 10x - 10x \\ -5x + 4 &= 6 \\ -5x + 4 - 4 &= 6 - 4 \\ -5x &= 2 \\ \frac{-5x}{-5} &= \frac{2}{-5} \\ x &= -\frac{2}{5} \end{aligned}$$

The solution is $-\frac{2}{5}$.

You Try It 2

$$\begin{aligned} 5x - 10 - 3x &= 6 - 4x \\ 2x - 10 &= 6 - 4x \\ 2x + 4x - 10 &= 6 - 4x + 4x \\ 6x - 10 &= 6 \\ 6x - 10 + 10 &= 6 + 10 \\ 6x &= 16 \\ \frac{6x}{6} &= \frac{16}{6} \\ x &= \frac{8}{3} \end{aligned}$$

The solution is $\frac{8}{3}$.

You Try It 3

$$\begin{aligned} 5x - 4(3 - 2x) &= 2(3x - 2) + 6 \\ 5x - 12 + 8x &= 6x - 4 + 6 \\ 13x - 12 &= 6x + 2 \\ 13x - 6x - 12 &= 6x - 6x + 2 \\ 7x - 12 &= 2 \\ 7x - 12 + 12 &= 2 + 12 \\ 7x &= 14 \\ \frac{7x}{7} &= \frac{14}{7} \\ x &= 2 \end{aligned}$$

The solution is 2.

You Try It 4

$$\begin{aligned} -2[3x - 5(2x - 3)] &= 3x - 8 \\ -2[3x - 10x + 15] &= 3x - 8 \\ -2[-7x + 15] &= 3x - 8 \\ 14x - 30 &= 3x - 8 \\ 14x - 3x - 30 &= 3x - 3x - 8 \\ 11x - 30 &= -8 \\ 11x - 30 + 30 &= -8 + 30 \\ 11x &= 22 \\ \frac{11x}{11} &= \frac{22}{11} \\ x &= 2 \end{aligned}$$

The solution is 2.

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You Try It 5 Solve $2x = 5x + 6$ for x.

$$\begin{aligned} 2x &= 5x + 6 \\ -3x &= 6 \\ x &= -2 \end{aligned}$$

Evaluate $-2x + 7$ for $x = -2$.

$$\begin{aligned} &-2x + 7 \\ &-2(-2) + 7 = 4 + 7 \\ &\qquad = 11 \end{aligned}$$

You Try It 6

Strategy To find the location of the fulcrum when the system balances, replace the variables F_1, F_2, and d in the lever system equation by the given values and solve for x.

Solution

$$\begin{aligned} F_1 \cdot x &= F_2 \cdot (d - x) \\ 45x &= 80(25 - x) \\ 45x &= 2000 - 80x \\ 45x + 80x &= 2000 - 80x + 80x \\ 125x &= 2000 \\ \frac{125x}{125} &= \frac{2000}{125} \\ x &= 16 \end{aligned}$$

The fulcrum is 16 ft from the 45-pound force.

SECTION 3.4

You Try It 1

The smaller number: n
The larger number: $12 - n$

The total of three times the smaller number and six	amounts to	seven less than the product of four and the larger number

$$\begin{aligned} 3n + 6 &= 4(12 - n) - 7 \\ 3n + 6 &= 48 - 4n - 7 \\ 3n + 6 &= 41 - 4n \\ 3n + 4n + 6 &= 41 - 4n + 4n \\ 7n + 6 &= 41 \\ 7n + 6 - 6 &= 41 - 6 \\ 7n &= 35 \\ \frac{7n}{7} &= \frac{35}{7} \\ n &= 5 \end{aligned}$$

$12 - n = 12 - 5 = 7$

The smaller number is 5.
The larger number is 7.

You Try It 2

Strategy
- First consecutive integer: n
  Second consecutive integer: $n + 1$
  Third consecutive integer: $n + 2$
- The sum of the three integers is -6.

Solution

$$\begin{aligned} n + (n + 1) + (n + 2) &= -6 \\ 3n + 3 &= -6 \\ 3n &= -9 \\ n &= -3 \end{aligned}$$

$n + 1 = -3 + 1 = -2$
$n + 2 = -3 + 2 = -1$

The three consecutive integers are -3, -2, and -1.

You Try It 3

Strategy
To find the number of tickets that you are purchasing, write and solve an equation using x to represent the number of tickets purchased.

Solution

$3.50 plus $17.50 for each ticket	equals	$161

$$\begin{aligned} 3.50 + 17.50x &= 161 \\ 3.50 - 3.50 + 17.50x &= 161 - 3.50 \\ 17.50x &= 157.50 \\ \frac{17.50x}{17.50} &= \frac{157.50}{17.50} \\ x &= 9 \end{aligned}$$

You purchased 9 tickets.

You Try It 4

Strategy
To find the length, write and solve an equation using x to represent the length of the shorter piece and $22 - x$ to represent the length of the longer piece.

Solution

The longer piece	is	4 in. more than twice the shorter piece

$$\begin{aligned} 22 - x &= 2x + 4 \\ 22 - x - 2x &= 2x - 2x + 4 \\ 22 - 3x &= 4 \\ 22 - 22 - 3x &= 4 - 22 \\ -3x &= -18 \\ \frac{-3x}{-3} &= \frac{-18}{-3} \\ x &= 6 \end{aligned}$$

$22 - x = 22 - 6 = 16$

The shorter piece is 6 in.
The longer piece is 16 in.

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SECTION 3.5

You Try It 1

Strategy To find the length of each side, use the formula for the perimeter of a square. Substitute 52 for P and solve for s.

Solution
$$P = 4s$$
$$52 = 4s$$
$$13 = s$$

The length of each side of the patio is 13 ft.

You Try It 2

Strategy To find the supplement, let x represent the supplement of a 107° angle. Use the fact that supplementary angles are two angles whose sum is 180° to write an equation. Solve for x.

Solution
$$x + 107° = 180°$$
$$x = 73°$$

The supplement of a 107° angle is a 73° angle.

You Try It 3

Strategy The angles labeled are adjacent angles of intersecting lines and are, therefore, supplementary angles. To find x, write an equation and solve for x.

Solution
$$x + (3x + 20°) = 180°$$
$$4x + 20° = 180°$$
$$4x = 160°$$
$$x = 40°$$

You Try It 4

Strategy $2x = y$ because alternate exterior angles have the same measure. $(x + 15°) + y = 180°$ because adjacent angles of intersecting lines are supplementary angles. Substitute $2x$ for y and solve for x.

Solution
$$(x + 15°) + 2x = 180°$$
$$3x + 15° = 180°$$
$$3x = 165°$$
$$x = 55°$$

You Try It 5

Strategy

- To find the measure of angle a, use the fact that $\angle a$ and $\angle y$ are vertical angles.
- To find the measure of angle b, use the fact that the sum of the measures of the interior angles of a triangle is 180°.
- To find the measure of angle d, use the fact that the sum of an interior and an exterior angle is 180°.

Solution
$$\angle a = \angle y = 55°$$
$$\angle a + \angle b + 90° = 180°$$
$$55° + \angle b + 90° = 180°$$
$$\angle b + 145° = 180°$$
$$\angle b = 35°$$
$$\angle d + \angle b = 180°$$
$$\angle d + 35° = 180°$$
$$\angle d = 145°$$

You Try It 6

Strategy To find the measure of the third angle, use the fact that the sum of the measures of the interior angles of a triangle is 180°. Write an equation using x to represent the measure of the third angle. Solve the equation for x.

Solution
$$x + 90° + 27° = 180°$$
$$x + 117° = 180°$$
$$x = 63°$$

The measure of the third angle is 63°.

SECTION 3.6

You Try It 1

Strategy

- Pounds of \$.55 fertilizer: x

	Amount	*Cost*	*Value*
\$.80 fertilizer	20	\$.80	0.80(20)
\$.55 fertilizer	x	\$.55	$0.55x$
\$.75 fertilizer	$20 + x$	\$.75	$0.75(20 + x)$

- The sum of the values before mixing equals the value after mixing.

Solution
$$0.80(20) + 0.55x = 0.75(20 + x)$$
$$16 + 0.55x = 15 + 0.75x$$
$$16 - 0.20x = 15$$
$$-0.20x = -1$$
$$x = 5$$

5 lb of the \$.55 fertilizer must be added.

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You Try It 2

Strategy • Liters of 6% solution: x

	Amount	*Percent*	*Quantity*
6% solution	x	0.06	$0.06x$
12% solution	5	0.12	$5(0.12)$
8% solution	$x + 5$	0.08	$0.08(x + 5)$

• The sum of the quantities before mixing equals the quantity after mixing.

Solution
$$0.06x + 5(0.12) = 0.08(x + 5)$$
$$0.06x + 0.60 = 0.08x + 0.40$$
$$0.06x + 0.20 = 0.08x$$
$$0.20 = 0.02x$$
$$10 = x$$

The pharmacist adds 10 L of the 6% solution to the 12% solution to get an 8% solution.

You Try It 3

Strategy • Rate of the first train: r
Rate of the second train: $2r$

	Rate	*Time*	*Distance*
1st train	r	3	$3r$
2nd train	$2r$	3	$3(2r)$

• The sum of the distances traveled by each train equals 288 mi.

Solution
$$3r + 3(2r) = 288$$
$$3r + 6r = 288$$
$$9r = 288$$
$$r = 32$$
$$2r = 2(32) = 64$$

The first train is traveling at 32 mph. The second train is traveling at 64 mph.

You Try It 4

Strategy • Time spent flying out: t
Time spent flying back: $5 - t$

	Rate	*Time*	*Distance*
Out	150	t	$150t$
Back	100	$5 - t$	$100(5 - t)$

• The distance out equals the distance back.

Solution
$$150t = 100(5 - t)$$
$$150t = 500 - 100t$$
$$250t = 500$$
$$t = 2 \quad \text{(The time out was 2 h.)}$$

$$\text{The distance} = 150t = 150(2)$$
$$= 300 \text{ mi}$$

The parcel of land was 300 mi away.

Solutions to Chapter 4 "You Try It"

SECTION 4.1

You Try It 1
$$(-4x^3 + 2x^2 - 8) + (4x^3 + 6x^2 - 7x + 5)$$
$$= (-4x^3 + 4x^3) + (2x^2 + 6x^2) + (-7x) + (-8 + 5)$$
$$= 8x^2 - 7x - 3$$

You Try It 2
$$\begin{array}{r} 6x^3 \quad + 2x + 8 \\ -9x^3 + 2x^2 - 12x - 8 \\ \hline -3x^3 + 2x^2 - 10x \end{array}$$

You Try It 3
$$(-4w^3 + 8w - 8) - (3w^3 - 4w^2 - 2w - 1)$$
$$= (-4w^3 + 8w - 8)$$
$$+ (-3w^3 + 4w^2 + 2w + 1)$$
$$= -7w^3 + 4w^2 + 10w - 7$$

You Try It 4
$$\begin{array}{r} 13y^3 \quad - 6y - 7 \\ - 4y^2 + 6y + 9 \\ \hline 13y^3 - 4y^2 \quad + 2 \end{array}$$

SECTION 4.2

You Try It 1
$$(8m^3n)(-3n^5) = [8(-3)](m^3)(n \cdot n^5)$$
$$= -24m^3n^6$$

You Try It 2
$$(12p^4q^3)(-3p^5q^2) = [12(-3)](p^4 \cdot p^5)(q^3 \cdot q^2)$$
$$= -36p^9q^5$$

You Try It 3
$$(-3a^4bc^2)^3 = (-3)^{1\cdot3}a^{4\cdot3}b^{1\cdot3}c^{2\cdot3}$$
$$= (-3)^3a^{12}b^3c^6$$
$$= -27a^{12}b^3c^6$$

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You Try It 4

$$\begin{aligned}(-xy^4)(-2x^3y^2)^2 &= (-xy^4)[(-2)^{1\cdot 2}x^{3\cdot 2}y^{2\cdot 2}]\\ &= (-xy^4)[(-2)^2x^6y^4]\\ &= (-xy^4)(4x^6y^4)\\ &= -4x^7y^8\end{aligned}$$

SECTION 4.3

You Try It 1 $(-2y + 3)(-4y) = 8y^2 - 12y$

You Try It 2

$$-a^2(3a^2 + 2a - 7) = -3a^4 - 2a^3 + 7a^2$$

You Try It 3

$$\begin{array}{r} 2y^3 + 2y^2 \quad - 3 \\ \hline 3y - 1 \\ \hline -2y^3 - 2y^2 \quad + 3 \\ 6y^4 + 6y^3 \quad - 9y \\ \hline 6y^4 + 4y^3 - 2y^2 - 9y + 3 \end{array}$$

You Try It 4

$$\begin{aligned}(4y - 5)(2y - 3) &= 8y^2 - 12y - 10y + 15\\ &= 8y^2 - 22y + 15\end{aligned}$$

You Try It 5

$$\begin{aligned}(3b + 2)(3b - 5) &= 9b^2 - 15b + 6b - 10\\ &= 9b^2 - 9b - 10\end{aligned}$$

You Try It 6 $(2a + 5c)(2a - 5c) = 4a^2 - 25c^2$

You Try It 7 $(3x + 2y)^2 = 9x^2 + 12xy + 4y^2$

You Try It 8

Strategy To find the area, replace the variable r in the equation $A = \pi r^2$ by $(x - 4)$ and solve for A.

Solution

$$\begin{aligned}A &= \pi r^2\\ A &= \pi(x - 4)^2\\ A &= \pi(x^2 - 8x + 16)\\ A &= \pi x^2 - 8\pi x + 16\pi\end{aligned}$$

The area of the circle is $(\pi x^2 - 8\pi x + 16\pi)$ ft^2.

SECTION 4.4

You Try It 1

$$\begin{aligned}(-2x^2)(x^{-3}y^{-4})^{-2} &= (-2x^2)(x^6y^8)\\ &= -2x^8y^8\end{aligned}$$

You Try It 2

$$\begin{aligned}\frac{(6a^{\ 2}b^3)^{-1}}{(4a^3b^{-2})^{-2}} &= \frac{6^{-1}a^2b^{-3}}{4^{-2}a^{-6}b^4}\\ &= 4^2(6^{-1}a^8b^{-7})\\ &= \frac{16a^8}{6b^7} = \frac{8a^8}{3b^7}\end{aligned}$$

You Try It 3

$$\begin{aligned}\left[\frac{6r^3s^{-3}}{9r^3s^{-1}}\right]^{-2} &= \left[\frac{2r^0s^{-2}}{3}\right]^{-2}\\ &= \frac{2^{-2}s^4}{3^{-2}} = \frac{9s^4}{4}\end{aligned}$$

You Try It 4 $0.000000961 = 9.61 \times 10^{-7}$

You Try It 5 $7.329 \times 10^6 = 7{,}329{,}000$

SECTION 4.5

You Try It 1

$$\begin{aligned}\frac{24x^2y^2 - 18xy + 6y}{6xy} &= \frac{24x^2y^2}{6xy} - \frac{18xy}{6xy} + \frac{6y}{6xy}\\ &= 4xy - 3 + \frac{1}{x}\end{aligned}$$

You Try It 2

$$\begin{array}{r} x^2 + 2x - 1 \\ 2x - 3\overline{)2x^3 + x^2 - 8x - 3} \\ \underline{2x^3 - 3x^2} \\ 4x^2 - 8x \\ \underline{4x^2 - 6x} \\ -2x - 3 \\ \underline{-2x + 3} \\ -6 \end{array}$$

$$\begin{aligned}&(2x^3 + x^2 - 8x - 3) \div (2x - 3)\\ &= x^2 + 2x - 1 - \frac{6}{2x - 3}\end{aligned}$$

You Try It 3

$$\begin{array}{r} x^2 + \ x - 1 \\ x - 1\overline{)x^3 + 0x^2 - 2x + 1} \\ \underline{x^3 - \ x^2} \\ x^2 - 2x \\ \underline{x^2 - \ x} \\ -x + 1 \\ \underline{-x + 1} \\ 0 \end{array}$$

$$(x^3 - 2x + 1) \div (x - 1) = x^2 + x - 1$$

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Solutions to Chapter 5 "You Try It"

SECTION 5.1

You Try It 1 The GCF is $7a^2$.

$$14a^2 - 21a^4b = 7a^2(2) + 7a^2(-3a^2b)$$
$$= 7a^2(2 - 3a^2b)$$

You Try It 2 The GCF is 9.

$$27b^2 + 18b + 9$$
$$= 9(3b^2) + 9(2b) + 9(1)$$
$$= 9(3b^2 + 2b + 1)$$

You Try It 3
The GCF is $3x^2y^2$.

$$6x^4y^2 - 9x^3y^2 + 12x^2y^4$$
$$= 3x^2y^2(2x^2) + 3x^2y^2(-3x) + 3x^2y^2(4y^2)$$
$$= 3x^2y^2(2x^2 - 3x + 4y^2)$$

You Try It 4
$$2y(5x - 2) - 3(2 - 5x)$$
$$= 2y(5x - 2) + 3(5x - 2)$$
$$= (5x - 2)(2y + 3)$$

You Try It 5
$$a^2 - 3a + 2ab - 6b$$
$$= (a^2 - 3a) + (2ab - 6b)$$
$$= a(a - 3) + 2b(a - 3)$$
$$= (a - 3)(a + 2b)$$

You Try It 6
$$2mn^2 - n + 8mn - 4$$
$$= (2mn^2 - n) + (8mn - 4)$$
$$= n(2mn - 1) + 4(2mn - 1)$$
$$= (2mn - 1)(n + 4)$$

You Try It 7
$$2xy - 6y - 12 + 4x$$
$$= (2xy - 6y) - (12 - 4x)$$
$$= 2y(x - 3) - 4(3 - x)$$
$$= 2y(x - 3) + 4(x - 3)$$
$$= (x - 3)(2y + 4)$$

SECTION 5.2

You Try It 1
Find the positive factors of 20 whose sum is 9.

Factors	*Sum*
1, 20	21
2, 10	12
4, 5	9

$$x^2 + 9x + 20 = (x + 4)(x + 5)$$

You Try It 2
Find the factors of −18 whose sum is 7.

Factors	*Sum*
+1, −18	−17
−1, +18	17
+2, −9	−7
−2, +9	7
+3, −6	−3
−3, +6	+3

$$x^2 + 7x - 18 = (x + 9)(x - 2)$$

You Try It 3
The GCF is $-2x$.

$$-2x^3 + 14x^2 - 12x = -2x(x^2 - 7x + 6)$$

Factor the trinomial $x^2 - 7x + 6$. Find two negative factors of 6 whose sum is −7.

Factors	*Sum*
−3, −2	−5
−6, −1	−7

$$-2x^3 + 14x^2 - 12x = -2x(x - 6)(x - 1)$$

You Try It 4
The GCF is 3.

$$3x^2 - 9xy - 12y^2 = 3(x^2 - 3xy - 4y^2)$$

Factor the trinomial.

Find the factors of −4 whose sum is −3.

Factors	*Sum*
+1, −4	−3
−1, +4	3
+2, −2	0

$$3x^2 - 9xy - 12y^2 = 3(x + y)(x - 4y)$$

SECTION 5.3

You Try It 1
Factor the trinomial $2x^2 - x - 3$.

Positive factors of 2: 1, 2

Factors of −3: +1, −3
−1, +3

Trial Factors	*Middle Term*
$(x + 1)(2x - 3)$	$-3x + 2x = -x$
$(x - 3)(2x + 1)$	$x - 6x = -5x$
$(x - 1)(2x + 3)$	$3x - 2x = x$
$(x + 3)(2x - 1)$	$-x + 6x = 5x$

$$2x^2 - x - 3 = (x + 1)(2x - 3)$$

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You Try It 2
The GCF is $-3y$.

$$-45y^3 + 12y^2 + 12y = -3y(15y^2 - 4y - 4)$$

Factor the trinomial $15y^2 - 4y - 4$.

Positive factors of 15:	Factors of -4:
1, 15	$-1, \quad 4$
3, 5	$1, \quad -4$
	$-2, \quad 2$

Trial Factors	*Middle Term*
$(y - 1)(15y + 4)$	$4y - 15y = -11y$
$(y + 4)(15y - 1)$	$-y + 60y = 59y$
$(y + 1)(15y - 4)$	$-4y + 15y = 11y$
$(y - 4)(15y + 1)$	$y - 60y = -59y$
$(y - 2)(15y + 2)$	$2y - 30y = -28y$
$(y + 2)(15y - 2)$	$-2y + 30y = 28y$
$(3y - 1)(5y + 4)$	$12y - 5y = 7y$
$(3y + 4)(5y - 1)$	$-3y + 20y = 17y$
$(3y + 1)(5y - 4)$	$-12y + 5y = -7y$
$(3y - 4)(5y + 1)$	$3y - 20y = -17y$
$(3y - 2)(5y + 2)$	$6y - 10y = -4y$
$(3y + 2)(5y - 2)$	$-6y + 10y = 4y$

$$-45y^3 + 12y^2 + 12y = -3y(3y - 2)(5y + 2)$$

You Try It 3

Factors of -14 [$2(-7)$]	*Sum*
$-1, +14$	13
$1, -14$	-13
$2, \quad -7$	-5
$-2, \quad 7$	5

$$\begin{aligned} 2a^2 + 13a - 7 &= 2a^2 - a + 14a - 7 \\ &= (2a^2 - a) + (14a - 7) \\ &= a(2a - 1) + 7(2a - 1) \\ &= (2a - 1)(a + 7) \end{aligned}$$

$$2a^2 + 13a - 7 = (2a - 1)(a + 7)$$

You Try It 4
The GCF is $5x$.

$$15x^3 + 40x^2 - 80x = 5x(3x^2 + 8x - 16)$$

Factors of -48 [$3(-16)$]	*Sum*
$-1, +48$	47
$+1, -48$	-47
$-2, +24$	22
$+2, -24$	-22
$-3, +16$	13
$+3, -16$	-13
$-4, +12$	8

$$\begin{aligned} 3x^2 + 8x - 16 &= 3x^2 - 4x + 12x - 16 \\ &= (3x^2 - 4x) + (12x - 16) \\ &= x(3x - 4) + 4(3x - 4) \\ &= (3x - 4)(x + 4) \end{aligned}$$

$$\begin{aligned} 15x^3 + 40x^2 - 80x &= 5x(3x^2 + 8x - 16) \\ &= 5x(3x - 4)(x + 4) \end{aligned}$$

SECTION 5.4

You Try It 1
$$25a^2 - b^2 = (5a)^2 - b^2 = (5a + b)(5a - b)$$

You Try It 2
$$\begin{aligned} n^4 - 81 &= (n^2)^2 - 9^2 = (n^2 + 9)(n^2 - 9) \\ &= (n^2 + 9)(n + 3)(n - 3) \end{aligned}$$

You Try It 3 Because $16y^2 = (4y)^2$, $1 = 1^2$, and $8y = 2(4y)(1)$, the trinomial is a perfect-square trinomial.

$$16y^2 + 8y + 1 = (4y + 1)^2$$

You Try It 4 Because $x^2 = (x)^2$, $36 = 6^2$, and $15x \neq 2(x)(6)$, the trinomial is not a perfect-square trinomial. Try to factor the trinomial by another method.

$$x^2 + 15x + 36 = (x + 3)(x + 12)$$

You Try It 5
$$\begin{aligned} &(x^2 - 6x + 9) - y^2 \\ &= (x - 3)^2 - y^2 \\ &= (x - 3 - y)(x - 3 + y) \end{aligned}$$

You Try It 6 The GCF is $3x$.

$$\begin{aligned} 12x^3 - 75x &= 3x(4x^2 - 25) \\ &= 3x(2x + 5)(2x - 5) \end{aligned}$$

You Try It 7
Factor by grouping.

$$\begin{aligned} a^2b - 7a^2 - b + 7 &= (a^2b - 7a^2) - (b - 7) \\ &= a^2(b - 7) - (b - 7) \\ &= (b - 7)(a^2 - 1) \\ &= (b - 7)(a + 1)(a - 1) \end{aligned}$$

You Try It 8
The GCF is $4x$.

$$\begin{aligned} 4x^3 + 28x^2 - 120x &= 4x(x^2 + 7x - 30) \\ &= 4x(x + 10)(x - 3) \end{aligned}$$

SECTION 5.5

You Try It 1
$$2x(x + 7) = 0$$

$$\begin{aligned} 2x &= 0 & x + 7 &= 0 \\ x &= 0 & x &= -7 \end{aligned}$$

The solutions are 0 and -7.

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You Try It 2

$$4x^2 - 9 = 0$$
$$(2x - 3)(2x + 3) = 0$$

$2x - 3 = 0$ $\qquad$ $2x + 3 = 0$
$2x = 3$ $\qquad$ $2x = -3$
$x = \frac{3}{2}$ $\qquad$ $x = -\frac{3}{2}$

The solutions are $\frac{3}{2}$ and $-\frac{3}{2}$.

You Try It 3

$$(x + 2)(x - 7) = 52$$
$$x^2 - 5x - 14 = 52$$
$$x^2 - 5x - 66 = 0$$
$$(x + 6)(x - 11) = 0$$

$x + 6 = 0$ $\qquad$ $x - 11 = 0$
$x = -6$ $\qquad$ $x = 11$

The solutions are -6 and 11.

You Try It 4

Strategy

First positive consecutive integer: n
Second positive consecutive integer: $n + 1$

The sum of the squares of two positive consecutive integers is 61.

Solution

$$n^2 + (n + 1)^2 = 61$$
$$n^2 + n^2 + 2n + 1 = 61$$
$$2n^2 + 2n + 1 = 61$$
$$2n^2 + 2n - 60 = 0$$
$$2(n^2 + n - 30) = 0$$
$$2(n - 5)(n + 6) = 0$$

$n - 5 = 0$ $\qquad$ $n + 6 = 0$
$n = 5$ $\qquad$ $n = -6$

Since -6 is not a positive integer, it is not a solution.

$n = 5$
$n + 1 = 5 + 1 = 6$

The two integers are 5 and 6.

You Try It 5

Strategy

Width $= x$
Length $= 2x + 4$

The area of a rectangle is 96 in^2. Use the equation $A = L \cdot W$.

Solution

$$A = L \cdot W$$
$$96 = (2x + 4)x$$
$$96 = 2x^2 + 4x$$
$$0 = 2x^2 + 4x - 96$$
$$0 = 2(x^2 + 2x - 48)$$
$$0 = 2(x + 8)(x - 6)$$

$x + 8 = 0$ $\qquad$ $x - 6 = 0$
$x = -8$ $\qquad$ $x = 6$

Since the width cannot be a negative number, -8 is not a solution.

$x = 6$
$2x + 4 = 2(6) + 4 = 12 + 4 = 16$

The length is 16 in. The width is 6 in.

Solutions to Chapter 6 "You Try It"

SECTION 6.1

You Try It 1

$$\frac{6x^5y}{12x^2y^3} = \frac{\overset{1}{2} \cdot \overset{1}{3} \cdot x^5y}{\underset{1}{2} \cdot 2 \cdot \underset{1}{3} \cdot x^2y^3} = \frac{x^3}{2y^2}$$

You Try It 2

$$\frac{x^2 + 2x - 24}{16 - x^2} = \frac{\overset{-1}{\cancel{(x - 4)}}(x + 6)}{\underset{1}{\cancel{(4 - x)}}(4 + x)} = -\frac{x + 6}{x + 4}$$

You Try It 3

$$\frac{x^2 + 4x - 12}{x^2 - 3x + 2} = \frac{\overset{1}{\cancel{(x - 2)}}(x + 6)}{(x - 1)\underset{1}{\cancel{(x - 2)}}} = \frac{x + 6}{x - 1}$$

You Try It 4

$$\frac{12x^2 + 3x}{10x - 15} \cdot \frac{8x - 12}{9x + 18} = \frac{3x(4x + 1)}{5(2x - 3)} \cdot \frac{4(2x - 3)}{9(x + 2)}$$
$$= \frac{\overset{1}{3}x(4x + 1) \cdot 2 \cdot 2\overset{1}{\cancel{(2x - 3)}}}{5\underset{1}{\cancel{(2x - 3)}} \cdot \underset{1}{3} \cdot 3(x + 2)}$$
$$= \frac{4x(4x + 1)}{15(x + 2)}$$

You Try It 5

$$\frac{x^2 + 2x - 15}{9 - x^2} \cdot \frac{x^2 - 3x - 18}{x^2 - 7x + 6}$$
$$= \frac{(x - 3)(x + 5)}{(3 - x)(3 + x)} \cdot \frac{(x + 3)(x - 6)}{(x - 1)(x - 6)}$$
$$= \frac{\overset{-1}{\cancel{(x - 3)}}(x + 5) \cdot \overset{1}{\cancel{(x + 3)}}\overset{1}{\cancel{(x - 6)}}}{\underset{1}{\cancel{(3 - x)}}\underset{1}{\cancel{(3 + x)}} \cdot (x - 1)\underset{1}{\cancel{(x - 6)}}} = -\frac{x + 5}{x - 1}$$

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You Try It 6

$$\frac{a^2}{4bc^2 - 2b^2c} \div \frac{a}{6bc - 3b^2} = \frac{a^2}{4bc^2 - 2b^2c} \cdot \frac{6bc - 3b^2}{a}$$

$$= \frac{a^2 \cdot 3\overset{1}{b}(2c - b)}{2\underset{1}{bc}(2c - b) \cdot a} = \frac{3a}{2c}$$

You Try It 7

$$\frac{3x^2 + 26x + 16}{3x^2 - 7x - 6} \div \frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$$

$$= \frac{3x^2 + 26x + 16}{3x^2 - 7x - 6} \cdot \frac{x^2 + 2x - 15}{2x^2 + 9x - 5}$$

$$= \frac{\overset{1}{(3x + 2)}(x + 8) \cdot \overset{1}{(x + 5)}\overset{1}{(x - 3)}}{\underset{1}{(3x + 2)}\underset{1}{(x - 3)} \cdot (2x - 1)\underset{1}{(x + 5)}} = \frac{x + 8}{2x - 1}$$

SECTION 6.2

You Try It 1

$8uv^2 = 2 \cdot 2 \cdot 2 \cdot u \cdot v \cdot v$

$12uw = 2 \cdot 2 \cdot 3 \cdot u \cdot w$

$\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot u \cdot v \cdot v \cdot w = 24uv^2w$

You Try It 2

$m^2 - 6m + 9 = (m - 3)(m - 3)$

$m^2 - 2m - 3 = (m + 1)(m - 3)$

$\text{LCM} = (m - 3)(m - 3)(m + 1)$

You Try It 3

The LCM is $36xy^2z$.

$$\frac{x - 3}{4xy^2} = \frac{x - 3}{4xy^2} \cdot \frac{9z}{9z} = \frac{9xz - 27z}{36xy^2z}$$

$$\frac{2x + 1}{9y^2z} = \frac{2x + 1}{9y^2z} \cdot \frac{4x}{4x} = \frac{8x^2 + 4x}{36xy^2z}$$

You Try It 4

The LCM is $(x + 2)(x - 5)(x + 5)$.

$$\frac{x + 4}{x^2 - 3x - 10} = \frac{x + 4}{(x + 2)(x - 5)} \cdot \frac{x + 5}{x + 5}$$

$$= \frac{x^2 + 9x + 20}{(x + 2)(x - 5)(x + 5)}$$

$$\frac{2x}{25 - x^2} = \frac{2x}{-(x^2 - 25)} = -\frac{2x}{(x - 5)(x + 5)} \cdot \frac{x + 2}{x + 2}$$

$$= -\frac{2x^2 + 4x}{(x + 2)(x - 5)(x + 5)}$$

SECTION 6.3

You Try It 1

$$\frac{2x^2}{x^2 - x - 12} - \frac{7x + 4}{x^2 - x - 12}$$

$$= \frac{2x^2 - (7x + 4)}{x^2 - x - 12} = \frac{2x^2 - 7x - 4}{x^2 - x - 12}$$

$$= \frac{(2x + 1)\overset{1}{(x - 4)}}{(x + 3)\underset{1}{(x - 4)}} = \frac{2x + 1}{x + 3}$$

You Try It 2

$$\frac{x^2 - 1}{x^2 - 8x + 12} - \frac{2x + 1}{x^2 - 8x + 12} + \frac{x}{x^2 - 8x + 12}$$

$$= \frac{(x^2 - 1) - (2x + 1) + x}{x^2 - 8x + 12} = \frac{x^2 - 1 - 2x - 1 + x}{x^2 - 8x + 12}$$

$$= \frac{x^2 - x - 2}{x^2 - 8x + 12} = \frac{(x + 1)\overset{1}{(x - 2)}}{\underset{1}{(x - 2)}(x - 6)} = \frac{x + 1}{x - 6}$$

You Try It 3

The LCM of the denominators is $24y$.

$$\frac{z}{8y} - \frac{4z}{3y} + \frac{5z}{4y} = \frac{z}{8y} \cdot \frac{3}{3} - \frac{4z}{3y} \cdot \frac{8}{8} + \frac{5z}{4y} \cdot \frac{6}{6}$$

$$= \frac{3z}{24y} - \frac{32z}{24y} + \frac{30z}{24y}$$

$$= \frac{3z - 32z + 30z}{24y} = \frac{z}{24y}$$

You Try It 4

$2 - x = -(x - 2)$

Therefore, $\frac{3}{2 - x} = \frac{-3}{x - 2}$.

The LCM is $x - 2$.

$$\frac{5x}{x - 2} - \frac{3}{2 - x} = \frac{5x}{x - 2} - \frac{-3}{x - 2}$$

$$= \frac{5x - (-3)}{x - 2} = \frac{5x + 3}{x - 2}$$

You Try It 5

The LCM is $(3x - 1)(x + 4)$.

$$\frac{4x}{3x - 1} - \frac{9}{x + 4} = \frac{4x}{3x - 1} \cdot \frac{x + 4}{x + 4} - \frac{9}{x + 4} \cdot \frac{3x - 1}{3x - 1}$$

$$= \frac{4x^2 + 16x}{(3x - 1)(x + 4)} - \frac{27x - 9}{(3x - 1)(x + 4)}$$

$$= \frac{(4x^2 + 16x) - (27x - 9)}{(3x - 1)(x + 4)}$$

$$= \frac{4x^2 + 16x - 27x + 9}{(3x - 1)(x + 4)}$$

$$= \frac{4x^2 - 11x + 9}{(3x - 1)(x + 4)}$$

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You Try It 6 The LCM is $x - 3$.

$$2 - \frac{1}{x-3} = 2 \cdot \frac{x-3}{x-3} - \frac{1}{x-3}$$
$$= \frac{2x-6}{x-3} - \frac{1}{x-3}$$
$$= \frac{2x-6-1}{x-3}$$
$$= \frac{2x-7}{x-3}$$

You Try It 7

$$\frac{2}{5-x} = \frac{-2}{x-5}$$

The LCM is $(x + 5)(x - 5)$.

$$\frac{2x-1}{x^2-25} + \frac{2}{5-x} = \frac{2x-1}{(x+5)(x-5)} + \frac{-2}{x-5}$$
$$= \frac{2x-1}{(x+5)(x-5)} + \frac{-2}{x-5} \cdot \frac{x+5}{x+5}$$
$$= \frac{2x-1}{(x+5)(x-5)} + \frac{-2(x+5)}{(x+5)(x-5)}$$
$$= \frac{2x-1+(-2)(x+5)}{(x+5)(x-5)}$$
$$= \frac{2x-1-2x-10}{(x+5)(x-5)}$$
$$= \frac{-11}{(x+5)(x-5)}$$
$$= -\frac{11}{(x+5)(x-5)}$$

You Try It 8
The LCM is $(3x + 2)(x - 1)$.

$$\frac{2x-3}{3x^2-x-2} + \frac{5}{3x+2} - \frac{1}{x-1}$$
$$= \frac{2x-3}{(3x+2)(x-1)} + \frac{5}{3x+2} \cdot \frac{x-1}{x-1} - \frac{1}{x-1} \cdot \frac{3x+2}{3x+2}$$
$$= \frac{2x-3}{(3x+2)(x-1)} + \frac{5x-5}{(3x+2)(x-1)} - \frac{3x+2}{(3x+2)(x-1)}$$
$$= \frac{(2x-3)+(5x-5)-(3x+2)}{(3x+2)(x-1)}$$
$$= \frac{2x-3+5x-5-3x-2}{(3x+2)(x-1)}$$
$$= \frac{4x-10}{(3x+2)(x-1)} = \frac{2(2x-5)}{(3x+2)(x-1)}$$

SECTION 6.4

You Try It 1
The LCM of 3, x, 9, and x^2 is $9x^2$.

$$\frac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}} = \frac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}} \cdot \frac{9x^2}{9x^2} = \frac{\frac{1}{3} \cdot 9x^2 - \frac{1}{x} \cdot 9x^2}{\frac{1}{9} \cdot 9x^2 - \frac{1}{x^2} \cdot 9x^2}$$
$$= \frac{3x^2-9x}{x^2-9} = \frac{3x\cancel{(x-3)}^{1}}{\cancel{(x-3)}_{1}(x+3)} = \frac{3x}{x+3}$$

You Try It 2
The LCM of x and x^2 is x^2.

$$\frac{1+\frac{4}{x}+\frac{3}{x^2}}{1+\frac{10}{x}+\frac{21}{x^2}} = \frac{1+\frac{4}{x}+\frac{3}{x^2}}{1+\frac{10}{x}+\frac{21}{x^2}} \cdot \frac{x^2}{x^2}$$
$$= \frac{1 \cdot x^2 + \frac{4}{x} \cdot x^2 + \frac{3}{x^2} \cdot x^2}{1 \cdot x^2 + \frac{10}{x} \cdot x^2 + \frac{21}{x^2} \cdot x^2}$$
$$= \frac{x^2+4x+3}{x^2+10x+21} = \frac{(x+1)\cancel{(x+3)}^{1}}{\cancel{(x+3)}_{1}(x+7)}$$
$$= \frac{x+1}{x+7}$$

You Try It 3
The LCM is $x - 5$.

$$\frac{x+3-\frac{20}{x-5}}{x+8+\frac{30}{x-5}} = \frac{x+3-\frac{20}{x-5}}{x+8+\frac{30}{x-5}} \cdot \frac{x-5}{x-5}$$
$$= \frac{(x+3)(x-5) - \frac{20}{x-5} \cdot (x-5)}{(x+8)(x-5) + \frac{30}{x-5} \cdot (x-5)}$$
$$= \frac{x^2-2x-15-20}{x^2+3x-40+30} = \frac{x^2-2x-35}{x^2+3x-10}$$
$$= \frac{\cancel{(x+5)}^{1}(x-7)}{(x-2)\cancel{(x+5)}_{1}} = \frac{x-7}{x-2}$$

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SECTION 6.5

You Try It 1

$$\frac{x}{x+6} = \frac{3}{x} \quad \text{The LCM is } x(x+6).$$

$$\frac{x(x+6)}{1} \cdot \frac{x}{x+6} = \frac{x(x+6)}{1} \cdot \frac{3}{x}$$

$$x^2 = (x+6)3$$
$$x^2 = 3x + 18$$
$$x^2 - 3x - 18 = 0$$
$$(x+3)(x-6) = 0$$

$$x + 3 = 0 \qquad x - 6 = 0$$
$$x = -3 \qquad x = 6$$

Both -3 and 6 check as solutions.
The solutions are -3 and 6.

You Try It 2

$$\frac{5x}{x+2} = 3 - \frac{10}{x+2} \quad \text{The LCM is } x+2.$$

$$\frac{(x+2)}{1} \cdot \frac{5x}{x+2} = \frac{(x+2)}{1}\left(3 - \frac{10}{x+2}\right)$$

$$\frac{x+2}{1} \cdot \frac{5x}{x+2} = \frac{x+2}{1} \cdot 3 - \frac{x+2}{1} \cdot \frac{10}{x+2}$$

$$5x = (x+2)3 - 10$$
$$5x = 3x + 6 - 10$$
$$5x = 3x - 4$$
$$2x = -4$$
$$x = -2$$

-2 does not check as a solution.
The equation has no solution.

SECTION 6.6

You Try It 1

$$\frac{2}{x+3} = \frac{6}{5x+5}$$

$$\frac{(x+3)(5x+5)}{1} \cdot \frac{2}{x+3} = \frac{(x+3)(5x+5)}{1} \cdot \frac{6}{5x+5}$$

$$\frac{(x+3)(5x+5)}{1} \cdot \frac{2}{x+3} = \frac{(x+3)(5x+5)}{1} \cdot \frac{6}{5x+5}$$

$$(5x+5)2 = (x+3)6$$
$$10x + 10 = 6x + 18$$
$$4x + 10 = 18$$
$$4x = 8$$
$$x = 2$$

The solution is 2.

You Try It 2

Strategy To find the total area that 256 ceramic tiles will cover, write and solve a proportion using x to represent the number of square feet that 256 tiles will cover.

Solution

$$\frac{9}{16} = \frac{x}{256}$$
$$256\left(\frac{9}{16}\right) = 256\left(\frac{x}{256}\right)$$
$$144 = x$$

A 144-square-foot area can be tiled using 256 ceramic tiles.

You Try It 3

Strategy To find the area of triangle AOB:
- Solve a proportion to find the length of AO (the height of triangle AOB).
- Use the formula for the area of a triangle. AB is the base and AO is the height.

Solution

$$\frac{CD}{AB} = \frac{DO}{AO}$$
$$\frac{4}{10} = \frac{3}{AO}$$
$$10 \cdot AO \cdot \frac{4}{10} = 10 \cdot AO \cdot \frac{3}{AO}$$
$$4(AO) = 30$$
$$AO = 7.5$$

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(10)(7.5)$$
$$= 37.5$$

The area of triangle AOB is 37.5 cm^2.

SECTION 6.7

You Try It 1

$$5x - 2y = 10$$
$$5x - 5x - 2y = -5x + 10$$
$$-2y = -5x + 10$$
$$\frac{-2y}{-2} = \frac{-5x + 10}{-2}$$
$$y = \frac{5}{2}x - 5$$

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You Try It 2

$$s = \frac{A + L}{2}$$
$$2 \cdot s = 2\left(\frac{A + L}{2}\right)$$
$$2s = A + L$$
$$2s - A = A - A + L$$
$$2s - A = L$$

You Try It 3

$$S = a + (n - 1)d$$
$$S = a + nd - d$$
$$S - a = a - a + nd - d$$
$$S - a = nd - d$$
$$S - a + d = nd - d + d$$
$$S - a + d = nd$$
$$\frac{S - a + d}{d} = \frac{nd}{d}$$
$$\frac{S - a + d}{d} = n$$

You Try It 4

$$S = C + rC$$
$$S = (1 + r)C$$
$$\frac{S}{1 + r} = \frac{(1 + r)C}{1 + r}$$
$$\frac{S}{1 + r} = C$$

SECTION 6.8

You Try It 1

Strategy

- Time for one printer to complete the job: t

	Rate	*Time*	*Part*
1st printer	$\frac{1}{t}$	2	$\frac{2}{t}$
2nd printer	$\frac{1}{t}$	5	$\frac{5}{t}$

- The sum of the parts of the task completed must equal 1.

Solution

$$\frac{2}{t} + \frac{5}{t} = 1$$
$$t\left(\frac{2}{t} + \frac{5}{t}\right) = t \cdot 1$$
$$2 + 5 = t$$
$$7 = t$$

Working alone, one printer takes 7 h to print the payroll.

You Try It 2

Strategy

- Rate sailing across the lake: r
  Rate sailing back: $3r$

	Distance	*Rate*	*Time*
Across	6	r	$\frac{6}{r}$
Back	6	$3r$	$\frac{6}{3r}$

- The total time for the trip was 2 h.

Solution

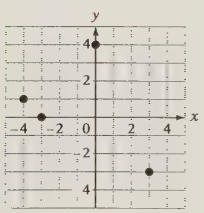
$$\frac{6}{r} + \frac{6}{3r} = 2$$
$$3r\left(\frac{6}{r} + \frac{6}{3r}\right) = 3r(2)$$
$$3r \cdot \frac{6}{r} + 3r \cdot \frac{6}{3r} = 6r$$
$$18 + 6 = 6r$$
$$24 = 6r$$
$$4 = r$$

The rate across the lake was 4 km/h.

Solutions to Chapter 7 "You Try It"

SECTION 7.1

You Try It 1



You Try It 2

$A(4, -2)$, $B(-2, 4)$.
The abscissa of D is 0.
The ordinate of C is 0.

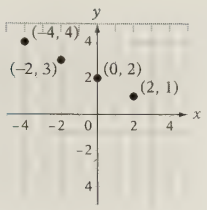
You Try It 3

$x - 3y = -14$	
$-2 - 3(4)$	-14
$-2 - 12$	-14
$-14 = -14$	

Yes, $(-2, 4)$ is a solution of $x - 3y = -14$.

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You Try It 4



You Try It 5

{(145, 140), (140, 125), (150, 130), (165, 150), (140, 130), (165, 160)}

No, the relation is not a function. The two ordered pairs (140, 125) and (140, 130) have the same first coordinate but different second coordinates.

You Try It 6 Determine the ordered pairs defined by the equation. Replace x in $y = \frac{1}{2}x + 1$ by the given values and solve for y. $\{(-4, -1), (0, 1), (2, 2)\}$ Yes, y is a function of x.

You Try It 7

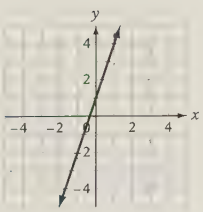
$$H(x) = \frac{x}{x - 4}$$

$$H(8) = \frac{8}{8 - 4}$$

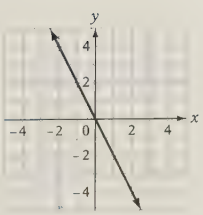
$$H(8) = \frac{8}{4} = 2$$

SECTION 7.2

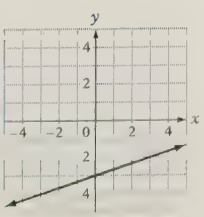
You Try It 1



You Try It 2

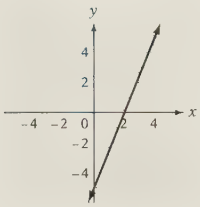


You Try It 3



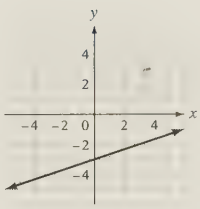
You Try It 4

$$5x - 2y = 10$$
$$-2y = -5x + 10$$
$$y = \frac{5}{2}x - 5$$

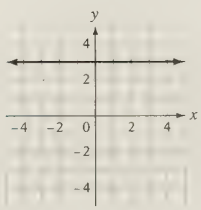


You Try It 5

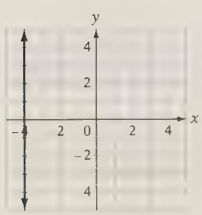
$$x - 3y = 9$$
$$-3y = -x + 9$$
$$y = \frac{1}{3}x - 3$$



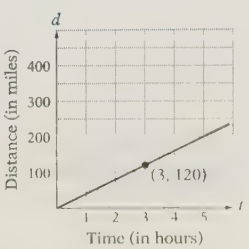
You Try It 6



You Try It 7



You Try It 8



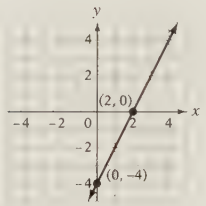
The ordered pair (3, 120) means that in 3 h the car will travel 120 mi.

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SECTION 7.3

You Try It 1

x-intercept:	y-intercept:
$y = 2x - 4$	$(0, b)$
$0 = 2x - 4$	$b = -4$
$-2x = -4$	$(0, -4)$
$x = 2$	
$(2, 0)$	



You Try It 2 Let $P_1 = (1, 4)$ and $P_2 = (-3, 8)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{-3 - 1} = \frac{4}{-4} = -1$$

The slope is -1.

You Try It 3 Let $P_1 = (-1, 2)$ and $P_2 = (4, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{4 - (-1)} = \frac{0}{5} = 0$$

The slope is 0.

You Try It 4

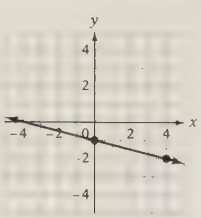
$$m = \frac{8650 - 6100}{1 - 4} = \frac{2550}{-3}$$

$$m = -850$$

A slope of -850 means that the value of the car is decreasing at a rate of \$850 per year.

You Try It 5 y-intercept $= (0, b) = (0, -1)$

$$m = -\frac{1}{4}$$

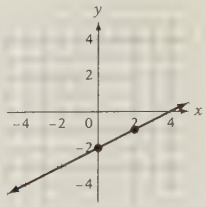


You Try It 6 Solve the equation for y.

$$x - 2y = 4$$
$$-2y = -x + 4$$
$$y = \frac{1}{2}x - 2$$

y-intercept $= (0, b) = (0, -2)$

$$m = \frac{1}{2}$$



SECTION 7.4

You Try It 1 Because the slope and y-intercept are known, use the slope-intercept formula, $y = mx + b$.

$$y = mx + b$$
$$y = \frac{5}{3}x + 2$$

You Try It 2

$$m = \frac{3}{4} \qquad (x_1, y_1) = (4, -2)$$
$$y - y_1 = m(x - x_1)$$
$$y - (-2) = \frac{3}{4}(x - 4)$$
$$y + 2 = \frac{3}{4}x - 3$$
$$y = \frac{3}{4}x - 5$$

The equation of the line is $y = \frac{3}{4}x - 5$.

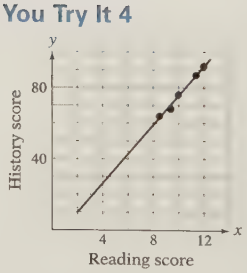
You Try It 3 Find the slope of the line between the two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{3 - (-6)} = \frac{2}{9}$$

Use the point-slope formula.

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = \frac{2}{9}[x - (-6)]$$
$$y + 1 = \frac{2}{9}x + \frac{4}{3}$$
$$y = \frac{2}{9}x + \frac{1}{3}$$

You Try It 4



The slope of the line means that the grade on the history test increases 8.3 points for each 1-point increase in the grade on the reading test.

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Solutions to Chapter 8 "You Try It"

SECTION 8.1

You Try It 1

$2x - 5y = 8$

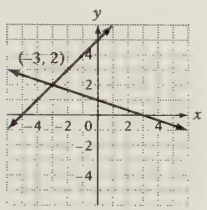
$2(-1) - 5(-2)$	8
$-2 + 10$	8
$8 = 8$	

$-x + 3y = -5$

$-(-1) + 3(-2)$	-5
$1 + (-6)$	-5
$-5 = -5$	

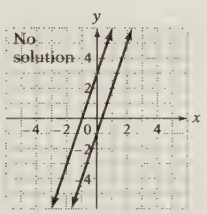
Yes. $(-1, -2)$ is a solution of the system of equations.

You Try It 2



The solution is $(-3, 2)$.

You Try It 3



The lines are parallel. The system of equations is inconsistent and therefore does not have a solution.

SECTION 8.2

You Try It 1

(1) $7x - y = 4$
(2) $3x + 2y = 9$

Solve Equation (1) for y.

$$\begin{aligned} 7x - y &= 4 \\ -y &= -7x + 4 \\ y &= 7x - 4 \end{aligned}$$

Substitute in Equation (2).

$$\begin{aligned} 3x + 2y &= 9 \\ 3x + 2(7x - 4) &= 9 \\ 3x + 14x - 8 &= 9 \\ 17x - 8 &= 9 \\ 17x &= 17 \\ x &= 1 \end{aligned}$$

Substitute in Equation (1).

$$\begin{aligned} 7x - y &= 4 \\ 7(1) - y &= 4 \\ 7 - y &= 4 \\ -y &= -3 \\ y &= 3 \end{aligned}$$

The solution is $(1, 3)$.

You Try It 2

(1) $3x - y = 4$
(2) $y = 3x + 2$

$$\begin{aligned} 3x - y &= 4 \\ 3x - (3x + 2) &= 4 \\ 3x - 3x - 2 &= 4 \\ -2 &= 4 \end{aligned}$$

This is not a true equation. The system of equations is inconsistent and therefore does not have a solution.

You Try It 3

(1) $y = -2x + 1$
(2) $6x + 3y = 3$

$$\begin{aligned} 6x + 3y &= 3 \\ 6x + 3(-2x + 1) &= 3 \\ 6x - 6x + 3 &= 3 \\ 3 &= 3 \end{aligned}$$

The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $y = -2x + 1$.

You Try It 4

Strategy

- Amount invested at 6.5%: x
  Amount invested at 4.5%: y

	Principal	*Rate*	*Interest*
Amount at 6.5%	x	0.065	$0.065x$
Amount at 4.5%	y	0.045	$0.045y$

- The sum of the two investments is \$330,000: $x + y = 330{,}000$.
  The interest earned at 6.5% equals the interest earned at 4.5%: $0.065x = 0.045y$

Solution

(1) $x + y = 330{,}000$
(2) $0.065x = 0.045y$

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Solve Equation (2) for y.

$$(3) \qquad y = \frac{13}{9}x$$

Replace y by $\frac{13}{9}x$ in Equation (1) and solve for x.

$$\begin{aligned} x + y &= 330{,}000 \\ x + \frac{13}{9}x &= 330{,}000 \\ \frac{22}{9}x &= 330{,}000 \\ x &= 135{,}000 \end{aligned}$$

Replace x by 135,000 in Equation (3) and solve for y.

$$\begin{aligned} y &= \frac{13}{9}x \\ &= \frac{13}{9}(135{,}000) = 195{,}000 \end{aligned}$$

$135,000 should be invested at 6.5% and $195,000 should be invested at 4.5%.

SECTION 8.3

You Try It 1

(1) $x - 2y = 1$
(2) $2x + 4y = 0$

Eliminate y.

$$\begin{aligned} 2(x - 2y) &= 2 \cdot 1 \\ 2x + 4y &= 0 \end{aligned}$$

$$\begin{aligned} 2x - 4y &= 2 \\ 2x + 4y &= 0 \end{aligned}$$

Add the equations.

$$\begin{aligned} 4x &= 2 \\ x &= \frac{2}{4} = \frac{1}{2} \end{aligned}$$

Replace x in Equation (2).

$$\begin{aligned} 2\left(\frac{1}{2}\right) + 4y &= 0 \\ 1 + 4y &= 0 \\ 4y &= -1 \\ y &= -\frac{1}{4} \end{aligned}$$

The solution is $\left(\frac{1}{2}, -\frac{1}{4}\right)$.

You Try It 2

(1) $2x - 3y = 4$
(2) $-4x + 6y = -8$

Eliminate y.

$$\begin{aligned} 2(2x - 3y) &= 2 \cdot 4 \\ -4x + 6y &= -8 \end{aligned}$$

$$\begin{aligned} 4x - 6y &= 8 \\ -4x + 6y &= -8 \end{aligned}$$

Add the equations.

$$\begin{aligned} 0x + 0y &= 0 \\ 0 &= 0 \end{aligned}$$

The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $2x - 3y = 4$.

You Try It 3

(1) $4x + 5y = 11$
(2) $3y = x + 10$

Write equation (2) in the form $Ax + By = C$.

$$\begin{aligned} 3y &= x + 10 \\ -x + 3y &= 10 \end{aligned}$$

Eliminate x.

$$\begin{aligned} 4x + 5y &= 11 \\ 4(-x + 3y) &= 4 \cdot 10 \end{aligned}$$

$$\begin{aligned} 4x + 5y &= 11 \\ -4x + 12y &= 40 \end{aligned}$$

Add the equations.

$$\begin{aligned} 17y &= 51 \\ y &= 3 \end{aligned}$$

Replace y in Equation (1).

$$\begin{aligned} 4x + 5y &= 11 \\ 4x + 5 \cdot 3 &= 11 \\ 4x + 15 &= 11 \\ 4x &= -4 \\ x &= -1 \end{aligned}$$

The solution is $(-1, 3)$.

SECTION 8.4

You Try It 1

Strategy

- Rate of the current: c
  Rate of the canoeist in calm water: r

	Rate	*Time*	*Distance*
With current	$r + c$	3	$3(r + c)$
Against current	$r - c$	5	$5(r - c)$

- The distance traveled with the current is 15 mi.
  The distance traveled against the current is 15 mi.

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Solution

$3(r + c) = 15$ $\quad \frac{1}{3} \cdot 3(r + c) = \frac{1}{3} \cdot 15$

$5(r - c) = 15$ $\quad \frac{1}{5} \cdot 5(r - c) = \frac{1}{5} \cdot 15$

$$r + c = 5$$
$$r - c = 3$$
$$2r = 8$$
$$r = 4$$

$$r + c = 5$$
$$4 + c = 5$$
$$c = 1$$

The rate of the current is 1 mph.
The rate of the canoeist in calm water is 4 mph.

You Try It 2

Strategy

- Cost of an orange tree: x
  Cost of a grapefruit tree: y
  First purchase:

	Amount	*Unit Cost*	*Value*
Orange trees	25	x	$25x$
Grapefruit trees	20	y	$20y$

Second purchase:

	Amount	*Unit Cost*	*Value*
Orange trees	20	x	$20x$
Grapefruit trees	30	y	$30y$

- The total of the first purchase was \$290.
  The total of the second purchase was \$330.

Solution

$25x + 20y = 290$ $\quad 4(25x + 20y) = 4 \cdot 290$

$20x + 30y = 330$ $\quad -5(20x + 30y) = -5 \cdot 330$

$$100x + 80y = 1160$$
$$-100x - 150y = -1650$$
$$-70y = -490$$
$$y = 7$$

$$25x + 20y = 290$$
$$25x + 20(7) = 290$$
$$25x + 140 = 290$$
$$25x = 150$$
$$x = 6$$

The cost of an orange tree is \$6.
The cost of a grapefruit tree is \$7.

Solutions to Chapter 9 "You Try It"

SECTION 9.1

You Try It 1 $A = \{-9, -7, -5, -3, -1\}$

You Try It 2 $A = \{1, 3, 5, \ldots\}$

You Try It 3 $A \cup B = \{-2, -1, 0, 1, 2, 3, 4\}$

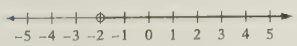
You Try It 4 $C \cap D = \{10, 16\}$

You Try It 5 $A \cap B = \varnothing$

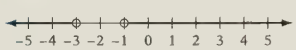
You Try It 6 $\{x \mid x < 59, x \in \text{positive even integers}\}$

You Try It 7 $\{x \mid x > -3, x \in \text{real numbers}\}$

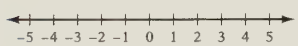
You Try It 8 The solution set is the numbers greater than -2.



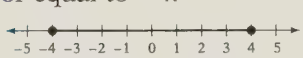
You Try It 9 The solution set is the numbers greater than -1 and the numbers less than -3.



You Try It 10 The solution set is the real numbers.



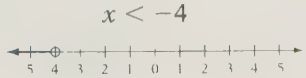
You Try It 11 The solution set is the numbers less than or equal to 4 and greater than or equal to -4.



SECTION 9.2

You Try It 1

$$x + 2 < -2$$
$$x + 2 - 2 < -2 - 2$$
$$x < -4$$



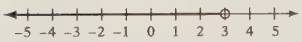
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You Try It 2

$$\begin{aligned} 5x + 3 &> 4x + 5 \\ 5x - 4x + 3 &> 4x - 4x + 5 \\ x + 3 &> 5 \\ x + 3 - 3 &> 5 - 3 \\ x &> 2 \end{aligned}$$

You Try It 3

$$\begin{aligned} -3x &> -9 \\ \frac{-3x}{-3} &< \frac{-9}{-3} \\ x &< 3 \end{aligned}$$



You Try It 4

$$\begin{aligned} -\frac{3}{4}x &\geq 18 \\ -\frac{4}{3}\left(-\frac{3}{4}x\right) &\leq -\frac{4}{3}(18) \\ x &\leq -24 \end{aligned}$$

You Try It 5

Strategy To find the selling prices, write and solve an inequality using p to represent the possible selling prices.

Solution

$$\begin{aligned} 0.70p &> 314 \\ p &> 448.571 \end{aligned}$$

The dealer will make a profit if the selling price is greater than or equal to \$448.58.

SECTION 9.3

You Try It 1

$$\begin{aligned} 5 - 4x &> 9 - 8x \\ 5 - 4x + 8x &> 9 - 8x + 8x \\ 5 + 4x &> 9 \\ 5 - 5 + 4x &> 9 - 5 \\ 4x &> 4 \\ \frac{4x}{4} &> \frac{4}{4} \\ x &> 1 \end{aligned}$$

You Try It 2

$$\begin{aligned} 8 - 4(3x + 5) &\leq 6(x - 8) \\ 8 - 12x - 20 &\leq 6x - 48 \\ -12 - 12x &\leq 6x - 48 \\ -12 - 12x - 6x &\leq 6x - 6x - 48 \\ -12 - 18x &\leq -48 \\ -12 + 12 - 18x &\leq -48 + 12 \\ -18x &\leq -36 \\ \frac{-18x}{-18} &\geq \frac{-36}{-18} \\ x &\geq 2 \end{aligned}$$

You Try It 3

Strategy To find the maximum number of miles:

- Write an expression for the cost of each car, using x to represent the number of miles driven during the week.
- Write and solve an inequality.

Solution

Cost of a Company A car	is less than	cost of a Company B car

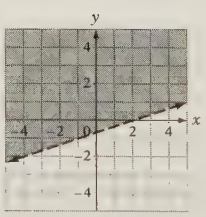
$$\begin{aligned} 8(7) + 0.10x &< 10(7) + 0.08x \\ 56 + 0.10x &< 70 + 0.08x \\ 56 + 0.10x - 0.08x &< 70 + 0.08x - 0.08x \\ 56 + 0.02x &< 70 \\ 56 - 56 + 0.02x &< 70 - 56 \\ 0.02x &< 14 \\ \frac{0.02x}{0.02} &< \frac{14}{0.02} \\ x &< 700 \end{aligned}$$

The maximum number of miles is 699.

SECTION 9.4

You Try It 1

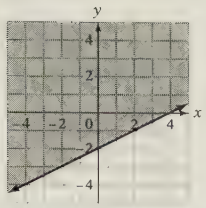
$$\begin{aligned} x - 3y &< 2 \\ x - x - 3y &< -x + 2 \\ -3y &< -x + 2 \\ \frac{-3y}{-3} &> \frac{-x + 2}{-3} \\ y &> \frac{1}{3}x - \frac{2}{3} \end{aligned}$$



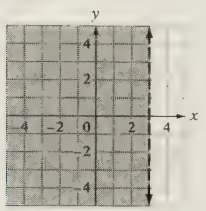
You Try It 2

$$\begin{aligned} 2x - 4y &\leq 8 \\ 2x - 2x - 4y &\leq -2x + 8 \\ -4y &\leq -2x + 8 \\ \frac{-4y}{-4} &\geq \frac{-2x + 8}{-4} \\ y &\geq \frac{1}{2}x - 2 \end{aligned}$$

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You Try It 3 $x < 3$



Solutions to Chapter 10 "You Try It"

SECTION 10.1

You Try It 1

$$-5\sqrt{32} = -5\sqrt{2^5} = -5\sqrt{2^4 \cdot 2} = -5\sqrt{2^4}\sqrt{2}$$
$$= -5 \cdot 2^2\sqrt{2} = -20\sqrt{2}$$

You Try It 2

$$\sqrt{216} = \sqrt{2^3 \cdot 3^3} = \sqrt{2^2 \cdot 3^2(2 \cdot 3)}$$
$$= \sqrt{2^2 \cdot 3^2}\sqrt{2 \cdot 3} = 2 \cdot 3\sqrt{2 \cdot 3} = 6\sqrt{6}$$

You Try It 3 $\sqrt{y^{19}} = \sqrt{y^{18} \cdot y} = \sqrt{y^{18}}\sqrt{y} = y^9\sqrt{y}$

You Try It 4

$$\sqrt{45b^7} = \sqrt{3^2 \cdot 5 \cdot b^7} = \sqrt{3^2b^6(5 \cdot b)}$$
$$= \sqrt{3^2b^6}\sqrt{5b} = 3b^3\sqrt{5b}$$

You Try It 5

$$3a\sqrt{28a^9b^{18}} = 3a\sqrt{2^2 \cdot 7 \cdot a^9b^{18}}$$
$$= 3a\sqrt{2^2a^8b^{18}(7a)}$$
$$= 3a\sqrt{2^2a^8b^{18}}\sqrt{7a}$$
$$= 3a \cdot 2 \cdot a^4b^9\sqrt{7a} = 6a^5b^9\sqrt{7a}$$

You Try It 6

$$\sqrt{25(a+3)^2} = \sqrt{5^2(a+3)^2} = 5(a+3)$$
$$= 5a + 15$$

You Try It 7 $\sqrt{x^2 + 14x + 49} = \sqrt{(x+7)^2} = x + 7$

SECTION 10.2

You Try It 1 $9\sqrt{3} + 3\sqrt{3} - 18\sqrt{3} = -6\sqrt{3}$

You Try It 2

$$2\sqrt{50} - 5\sqrt{32} = 2\sqrt{2 \cdot 5^2} - 5\sqrt{2^5}$$
$$= 2\sqrt{5^2}\sqrt{2} - 5\sqrt{2^4}\sqrt{2}$$
$$= 2 \cdot 5\sqrt{2} - 5 \cdot 2^2\sqrt{2}$$
$$= 10\sqrt{2} - 20\sqrt{2}$$
$$= -10\sqrt{2}$$

You Try It 3

$$y\sqrt{28y} + 7\sqrt{63y^3}$$
$$= y\sqrt{2^2 \cdot 7y} + 7\sqrt{3^2 \cdot 7 \cdot y^3}$$
$$= y\sqrt{2^2}\sqrt{7y} + 7\sqrt{3^2 \cdot y^2}\sqrt{7y}$$
$$= y \cdot 2\sqrt{7y} + 7 \cdot 3 \cdot y\sqrt{7y}$$
$$= 2y\sqrt{7y} + 21y\sqrt{7y}$$
$$= 23y\sqrt{7y}$$

You Try It 4

$$2\sqrt{27a^5} - 4a\sqrt{12a^3} + a^2\sqrt{75a}$$
$$= 2\sqrt{3^3 \cdot a^5} - 4a\sqrt{2^2 \cdot 3 \cdot a^3} + a^2\sqrt{3 \cdot 5^2 \cdot a}$$
$$= 2\sqrt{3^2 \cdot a^4}\sqrt{3a} - 4a\sqrt{2^2 \cdot a^2}\sqrt{3a}$$
$$+ a^2\sqrt{5^2}\sqrt{3a}$$
$$= 2 \cdot 3 \cdot a^2\sqrt{3a} - 4a \cdot 2 \cdot a\sqrt{3a} + a^2 \cdot 5\sqrt{3a}$$
$$= 6a^2\sqrt{3a} - 8a^2\sqrt{3a} + 5a^2\sqrt{3a} = 3a^2\sqrt{3a}$$

SECTION 10.3

You Try It 1

$$\sqrt{5a}\sqrt{15a^3b^4}\sqrt{3b^5}$$
$$= \sqrt{225a^4b^9} = \sqrt{3^2 5^2 a^4 b^9}$$
$$= \sqrt{3^2 5^2 a^4 b^8}\sqrt{b} = 3 \cdot 5a^2b^4\sqrt{b}$$
$$= 15a^2b^4\sqrt{b}$$

You Try It 2

$$\sqrt{5x}(\sqrt{5x} - \sqrt{25y})$$
$$= \sqrt{5^2x^2} - \sqrt{5^3xy}$$
$$= \sqrt{5^2x^2} - \sqrt{5^2}\sqrt{5xy} = 5x - 5\sqrt{5xy}$$

You Try It 3

$$(2\sqrt{x} + 7)(2\sqrt{x} - 7) = 4(\sqrt{x})^2 - 7^2$$
$$= 4x - 49$$

You Try It 4

$$(3\sqrt{x} - \sqrt{y})(5\sqrt{x} - 2\sqrt{y})$$
$$= 15(\sqrt{x})^2 - 6\sqrt{xy} - 5\sqrt{xy} + 2(\sqrt{y})^2$$
$$= 15(\sqrt{x})^2 - 11\sqrt{xy} + 2(\sqrt{y})^2$$
$$= 15x - 11\sqrt{xy} + 2y$$

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You Try It 5

$$\frac{\sqrt{15x^6y^7}}{\sqrt{3x^7y^9}} = \sqrt{\frac{15x^6y^7}{3x^7y^9}} = \sqrt{\frac{5}{xy^2}} = \frac{\sqrt{5}}{\sqrt{xy^2}}$$
$$= \frac{\sqrt{5}}{y\sqrt{x}} = \frac{\sqrt{5}}{y\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{5x}}{xy}$$

You Try It 6

$$\frac{\sqrt{3}}{\sqrt{3}-\sqrt{6}} = \frac{\sqrt{3}}{\sqrt{3}-\sqrt{6}} \cdot \frac{\sqrt{3}+\sqrt{6}}{\sqrt{3}+\sqrt{6}}$$
$$= \frac{3+\sqrt{18}}{3-6} = \frac{3+3\sqrt{2}}{-3}$$
$$= \frac{3(1+\sqrt{2})}{-3} = -1(1+\sqrt{2})$$
$$= -1-\sqrt{2}$$

You Try It 7

$$\frac{5+\sqrt{y}}{1-2\sqrt{y}} = \frac{5+\sqrt{y}}{1-2\sqrt{y}} \cdot \frac{1+2\sqrt{y}}{1+2\sqrt{y}}$$
$$= \frac{5+10\sqrt{y}+\sqrt{y}+2(\sqrt{y})^2}{1-4y}$$
$$= \frac{5+11\sqrt{y}+2y}{1-4y}$$

SECTION 10.4

You Try It 1

$$\sqrt{4x}+3 = 7$$
$$\sqrt{4x} = 4$$
$$(\sqrt{4x})^2 = 4^2$$
$$4x = 16$$
$$x = 4$$

Check:

$\sqrt{4x}+3=7$	
$\sqrt{4\cdot 4}+3$	7
$\sqrt{4^2}+3$	7
$4+3$	7
$7=7$	

The solution is 4.

You Try It 2

$$\sqrt{x}+\sqrt{x+9} = 9$$
$$\sqrt{x} = 9-\sqrt{x+9}$$
$$(\sqrt{x})^2 = (9-\sqrt{x+9})^2$$
$$x = 81-18\sqrt{x+9}+(x+9)$$
$$-90 = -18\sqrt{x+9}$$
$$5 = \sqrt{x+9}$$
$$5^2 = (\sqrt{x+9})^2$$
$$25 = x+9$$
$$16 = x$$

Check:

$\sqrt{x}+\sqrt{x+9}=9$	
$\sqrt{16}+\sqrt{16+9}$	9
$4+5$	9
$9=9$	

The solution is 16.

You Try It 3

Strategy To find the distance, use the Pythagorean Theorem. The hypotenuse is the length of the ladder. One leg is the distance from the bottom of the ladder to the base of the building. The distance along the building from the ground to the top of the ladder is the unknown leg.

Solution

$$a^2 = \sqrt{c^2-b^2}$$
$$= \sqrt{(8)^2-(3)^2}$$
$$= \sqrt{64-9}$$
$$= \sqrt{55}$$
$$\approx 7.42$$

The distance is approximately 7.42 ft.

You Try It 4

Strategy To find the length of the pendulum, replace T in the equation with the given value and solve for L.

Solution

$$T = 2\pi\sqrt{\frac{L}{32}}$$
$$2.5 = 2(3.14)\sqrt{\frac{L}{32}}$$
$$2.5 = 6.28\sqrt{\frac{L}{32}}$$
$$\frac{2.5}{6.28} = \sqrt{\frac{L}{32}}$$
$$\left(\frac{2.5}{6.28}\right)^2 = \left(\sqrt{\frac{L}{32}}\right)^2$$
$$\frac{6.25}{39.4384} = \frac{L}{32}$$
$$(32)\left(\frac{6.25}{39.4384}\right) = (32)\left(\frac{L}{32}\right)$$
$$\frac{200}{39.4384} = L$$
$$5.07 \approx L$$

The length of the pendulum is 5.07 ft.

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Solutions to Chapter 11 "You Try It"

SECTION 11.1

You Try It 1

$$\frac{3y^2}{2} + y - \frac{1}{2} = 0$$

$$2\left(\frac{3y^2}{2} + y - \frac{1}{2}\right) = 2(0)$$

$$3y^2 + 2y - 1 = 0$$

$$(3y - 1)(y + 1) = 0$$

$$3y - 1 = 0 \qquad y + 1 = 0$$

$$3y = 1 \qquad y = -1$$

$$y = \frac{1}{3}$$

The solutions are $\frac{1}{3}$ and -1.

You Try It 2

$$x^2 + 81 = 0$$

$$x^2 = -81$$

$$\sqrt{x^2} = \sqrt{-81}$$

$\sqrt{-81}$ is not a real number.

The equation has no real number solution.

You Try It 3

$$7(z + 2)^2 = 21$$

$$(z + 2)^2 = 3$$

$$\sqrt{(z + 2)^2} = \sqrt{3}$$

$$z + 2 = \pm\sqrt{3}$$

$$z = -2 \pm \sqrt{3}$$

The solutions are $-2 + \sqrt{3}$ and $-2 - \sqrt{3}$.

SECTION 11.2

You Try It 1

$$3x^2 - 6x - 2 = 0$$

$$3x^2 - 6x = 2$$

$$\frac{1}{3}(3x^2 - 6x) = \frac{1}{3} \cdot 2$$

$$x^2 - 2x = \frac{2}{3}$$

Complete the square.

$$x^2 - 2x + 1 = \frac{2}{3} + 1$$

$$(x - 1)^2 = \frac{5}{3}$$

$$\sqrt{(x - 1)^2} = \sqrt{\frac{5}{3}}$$

$$x - 1 = \pm\sqrt{\frac{5}{3}} = \pm\frac{\sqrt{15}}{3}$$

$$x - 1 = \frac{\sqrt{15}}{3} \qquad x - 1 = -\frac{\sqrt{15}}{3}$$

$$x = 1 + \frac{\sqrt{15}}{3} \qquad x = 1 - \frac{\sqrt{15}}{3}$$

$$= \frac{3 + \sqrt{15}}{3} \qquad = \frac{3 - \sqrt{15}}{3}$$

The solutions are $\frac{3 + \sqrt{15}}{3}$ and $\frac{3 - \sqrt{15}}{3}$.

You Try It 2

$$x^2 + 6x + 12 = 0$$

$$x^2 + 6x = -12$$

$$x^2 + 6x + 9 = -12 + 9$$

$$(x + 3)^2 = -3$$

$$\sqrt{(x + 3)^2} = \sqrt{-3}$$

$\sqrt{-3}$ is not a real number.

The quadratic equation has no real number solution.

You Try It 3

$$\frac{x^2}{8} + x + 1 = 0$$

$$8\left(\frac{x^2}{8} + x + 1\right) = 8(0)$$

$$x^2 + 8x + 8 = 0$$

$$x^2 + 8x = -8$$

$$x^2 + 8x + 16 = -8 + 16$$

$$(x + 4)^2 = 8$$

$$\sqrt{(x + 4)^2} = \sqrt{8}$$

$$x + 4 = \pm\sqrt{8} = \pm 2\sqrt{2}$$

$$x + 4 = 2\sqrt{2} \qquad x + 4 = -2\sqrt{2}$$

$$x = -4 + 2\sqrt{2} \qquad x = -4 - 2\sqrt{2}$$

$$\approx -4 + 2(1.414) \qquad \approx -4 - 2(1.414)$$

$$\approx -4 + 2.828 \qquad \approx -4 - 2.828$$

$$\approx -1.172 \qquad \approx -6.828$$

The solutions are approximately -1.172 and -6.828.

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SECTION 11.3

You Try It 1

$3x^2 + 4x - 4 = 0$

$a = 3, b = 4, c = -4$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(3)(-4)}}{2 \cdot 3}$$

$$= \frac{-4 \pm \sqrt{16 + 48}}{6}$$

$$= \frac{-4 \pm \sqrt{64}}{6} = \frac{-4 \pm 8}{6}$$

$$x = \frac{-4 + 8}{6} \qquad x = \frac{-4 - 8}{6}$$

$$= \frac{4}{6} = \frac{2}{3} \qquad = \frac{-12}{6} = -2$$

The solutions are $\frac{2}{3}$ and -2.

You Try It 2

$$\frac{x^2}{4} + \frac{x}{2} = \frac{1}{4}$$

$$4\left(\frac{x^2}{4} + \frac{x}{2}\right) = 4\left(\frac{1}{4}\right)$$

$$x^2 + 2x = 1$$

$$x^2 + 2x - 1 = 0$$

$a = 1, b = 2, c = -1$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-1)}}{2 \cdot 1}$$

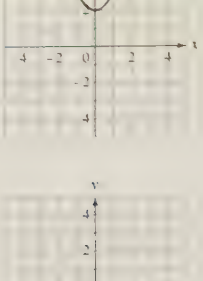
$$= \frac{-2 \pm \sqrt{4 + 4}}{2} = \frac{-2 \pm \sqrt{8}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

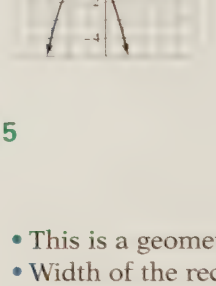
The solutions are $-1 + \sqrt{2}$ and $-1 - \sqrt{2}$.

SECTION 11.4

You Try It 1



You Try It 2



SECTION 11.5

You Try It 1

Strategy

- This is a geometry problem.
- Width of the rectangle: W
  Length of the rectangle: $W + 2$
- Use the equation $A = L \cdot W$.

Solution

$A = L \cdot W$

$15 = (W + 2)W$

$15 = W^2 + 2W$

$0 = W^2 + 2W - 15$

$0 = (W + 5)(W - 3)$

$W + 5 = 0 \qquad W - 3 = 0$

$W = -5 \qquad W = 3$

The solution -5 is not possible.
The width is 3 m.

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Answers to Chapter 1 Odd-Numbered Exercises

SECTION 1.1

1. $8 > -6$ **3.** $-12 < 1$ **5.** $42 > 19$ **7.** $0 > -31$ **9.** $53 > -46$ **11.** false **13.** true **15.** false
17. true **19.** true **21.** $\{1, 2, 3, 4, 5, 6, 7, 8\}$ **23.** $\{1, 2, 3, 4, 5, 6, 7, 8\}$ **25.** $\{-6, -5, -4, -3, -2, -1\}$ **27.** 5
29. $-23, -18$ **31.** 21, 37 **33.** $-52, -46, 0$ **35.** $-17, 0, 4, 29$ **37.** 5, 6, 7, 8, 9 **39.** $-10, -9, -8, -7, -6, -5$
41. -4 **43.** 9 **45.** 36 **47.** 40 **49.** -39 **51.** 74 **53.** -82 **55.** -81 **57.** $|22| > |-19|$
59. $|-71| < |-92|$ **61.** $|12| < |-31|$ **63.** $|-28| < |43|$ **65a.** 11, 7, 3, $-1, -5$ **b.** 11, 7, 3, 1, 5 **67.** never true

SECTION 1.2

1. -11 **3.** -5 **5.** -83 **7.** -46 **9.** 0 **11.** -5 **13.** 9 **15.** 1 **17.** -10 **19.** -18 **21.** -41
23. -12 **25.** 0 **27.** -34 **29.** 0 **31.** -61 **33.** 27 **35.** 8 **37.** -7 **39.** -9 **41.** 9 **43.** -3
45. 18 **47.** -9 **49.** 11 **51.** -18 **53.** 0 **55.** 2 **57.** -138 **59.** -8 **61.** -12 **63.** -20
65. The difference is 399°C. **67.** The difference is 7046 m. **69.** The difference between the highest and lowest elevations is greatest in Asia. **71.** The difference is 5182°C. **73.** The total net loss was -303 million dollars.
75. The difference between the net losses in 1994 and in 1992 was 544 million dollars. **77.** The difference is 16°F.
79. No. For example, $10 - (-8) = 18$.

SECTION 1.3

1. 42 **3.** -28 **5.** 60 **7.** -253 **9.** -238 **11.** -114 **13.** -105 **15.** 252 **17.** -240 **19.** 96
21. -216 **23.** -315 **25.** 420 **27.** 2880 **29.** -2772 **31.** 0 **33.** -2 **35.** 8 **37.** -7 **39.** -12
41. -6 **43.** -7 **45.** 11 **47.** -14 **49.** 15 **51.** -16 **53.** 0 **55.** -29 **57.** undefined **59.** -11
61. undefined **63.** The average daily high temperature was -26°F. **65.** The five-day moving average was 10, -10, -5, -15, 15, 15. **67.** The score for the exam was 74. **69.** -6 **71.** x is negative. Explanations will vary.

SECTION 1.4

1. $\frac{1}{3}$ **3.** $\frac{4}{11}$ **5.** $\frac{2}{3}$ **7.** $\frac{3}{2}$ **9.** 0 **11.** $\frac{3}{5}$ **13.** $\frac{11}{15}$ **15.** $\frac{1}{2}$ **17.** $0.16\overline{6}$ **19.** 0.125 **21.** $0.\overline{2}$ **23.** $0.\overline{45}$
25. $0.91\overline{6}$ **27.** 0.9375 **29.** $0.9\overline{4}$ **31.** $\frac{3}{4}$; 0.75 **33.** $\frac{16}{25}$; 0.64 **35.** $\frac{5}{4}$; 1.25 **37.** $\frac{19}{100}$; 0.19 **39.** $\frac{1}{20}$; 0.05
41. $\frac{1}{9}$ **43.** $\frac{1}{8}$ **45.** $\frac{2}{3}$ **47.** $\frac{1}{200}$ **49.** $\frac{5}{6}$ **51.** 0.073 **53.** 0.158 **55.** 0.003 **57.** 0.099 **59.** 1.212
61. 15% **63.** 5% **65.** 17.5% **67.** 115% **69.** 0.8% **71.** 54% **73.** $33\frac{1}{3}\%$ **75.** $45\frac{5}{11}\%$ **77.** 87.5%
79. $166\frac{2}{3}\%$ **81.** $\frac{13}{12}$ **83.** $-\frac{5}{24}$ **85.** $-\frac{19}{24}$ **87.** $\frac{5}{26}$ **89.** $\frac{11}{8}$ **91.** $\frac{1}{12}$ **93.** $\frac{7}{24}$ **95.** 0 **97.** $-\frac{7}{16}$
99. $\frac{11}{24}$ **101.** 1 **103.** $\frac{11}{8}$ **105.** 8.022 **107.** -38.8 **109.** -6.192 **111.** 13.355 **113.** 4.676
115. -10.03 **117.** -60.03 **119.** 11.56 **121.** -0.88 **123.** -4.73 **125.** $-\frac{3}{8}$ **127.** $\frac{1}{10}$ **129.** $-\frac{4}{9}$
131. $-\frac{7}{30}$ **133.** $\frac{15}{64}$ **135.** $-\frac{10}{9}$ **137.** $-\frac{147}{32}$ **139.** $\frac{25}{8}$ **141.** $\frac{2}{3}$ **143.** 4.164 **145.** 4.347 **147.** -4.028
149. -2.22 **151.** -1.104 **153.** 0.506 **155.** -0.2376 **157.** $-274.\overline{4}$ **159.** -2.59 **161.** -5.11
163. -2060.55 **165.** 5% of the total is contributed by corporations. **167a.** 47% of *Waterworld*'s total gross was from foreign countries.

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b. 44% of *Judge Dredd*'s total gross was its U.S. box-office gross.

c. *Judge Dredd* and *Spy Hard* grossed more than half their box-office incomes in foreign countries.

169.

$\frac{2}{3}$	$\frac{1}{6}$	0
$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$
$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{3}$

171. Answers will vary. **173.** $a = 2, b = 3, c = 6$

SECTION 1.5

1. 36 **3.** -49 **5.** 9 **7.** 81 **9.** $\frac{1}{4}$ **11.** 0.09 **13.** 12 **15.** 0.216 **17.** -12 **19.** 16 **21.** -864

23. -1008 **25.** 3 **27.** $-77{,}760$ **29.** 9 **31.** 12 **33.** 1 **35.** 8 **37.** -16 **39.** 12 **41.** 13

43. -36 **45.** 13 **47.** 4 **49.** 15 **51.** -1 **53.** 4 **55.** 0.51 **57.** 1.7 **59.** $-\frac{1}{16}$

61. No. $6 + 2(4 - 9) = 6 + 2(-5) = 6 + (-10) = -4$. Do multiplication before addition.

CHAPTER REVIEW

1. -6 [1.2A] **2.** 0.28 [1.4A] **3.** -25 [1.5A] **4.** 10 [1.5B] **5.** $-4 < 2$ [1.1A] **6.** 0.062 [1.4B]

7. -42 [1.3A] **8.** $\frac{7}{12}$ [1.4C] **9.** 60 [1.5B] **10.** -4 [1.1A] **11.** -1.068 [1.4C] **12.** 62.5% [1.4B]

13. $0.1\overline{3}$ [1.4A] **14.** -4 [1.2B] **15.** 4 [1.1B] **16.** 8 [1.3B] **17.** -20 [1.3B] **18.** $\frac{159}{200}$ [1.4B]

19. 31 [1.5B] **20.** -13 [1.2A] **21.** $54\frac{2}{7}\%$ [1.4B] **22.** $\frac{16}{81}$ [1.5A] **23.** -4.6224 [1.4D] **24.** -5 [1.1B]

25. 1 [1.2B] **26.** $-\frac{8}{15}$ [1.4D] **27.** 7, 0, 9 [1.1B] **28.** 81 [1.3A] **29.** $-|6| < |-10|$ [1.1B]

30. 1 [1.5B] **31.** The score for the exam was 98. [1.3C] **32.** 17.8% of the messages is E-mail. [1.4E]

33. The difference is 396°C. [1.2C]

CHAPTER TEST

1. 17 [1.3B] **2.** $83\frac{1}{3}\%$ [1.4B] **3.** 12 [1.5A] **4.** -5.3578 [1.4D] **5.** -14 [1.2B] **6.** $\frac{3}{8}$ [1.4B]

7. $\frac{1}{15}$ [1.4C] **8.** 8 [1.5B] **9.** 90 [1.3A] **10.** 4 [1.1B] **11.** -108 [1.5A] **12.** 4 [1.1A]

13. $-2 > -40$ [1.1A] **14.** -4 [1.1B] **15.** $\frac{9}{20}$; 0.45 [1.4B] **16.** -16 [1.2A] **17.** -48 [1.3A]

18. 17 [1.5B] **19.** 4 [1.2B] **20.** $-\frac{1}{2}$ [1.4D] **21.** 102.5% [1.4B] **22.** 9 [1.5B] **23.** $0.\overline{7}$ [1.4A]

24a. The annual losses would be $-\$12{,}560{,}000$. **b.** The average monthly loss was $-\$5{,}070{,}000$. [1.3C]

25. The price of one share was $\$62\frac{5}{8}$. [1.4E]

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Answers to Chapter 2 Odd-Numbered Exercises

SECTION 2.1

1. $2x^2$, $5x$, -8 **3.** $-a^4$, 6 **5.** $7x^2y$, $6xy^2$ **7.** $1, -9$ **9.** $1, -4, -1$ **11.** 10 **13.** 32 **15.** 21 **17.** 16 **19.** -9 **21.** 41 **23.** -7 **25.** 13 **27.** -15 **29.** 41 **31.** 1 **33.** 5 **35.** 1 **37.** 57 **39.** 5 **41.** 8 **43.** -3 **45.** -2 **47.** -4 **49.** 10 **51.** -25 **53.** 1 **55.** -23 **57a.** 2 **b.** 5 **c.** 6 **d.** 7

SECTION 2.2

1. $14x$ **3.** $5a$ **5.** $-6y$ **7.** $-3b - 7$ **9.** $5a$ **11.** $-2ab$ **13.** $5xy$ **15.** 0 **17.** $-\frac{5}{6}x$ **19.** $-\frac{1}{24}x^2$ **21.** $11x$ **23.** $7a$ **25.** $-14x^2$ **27.** $-x + 3y$ **29.** $17x - 3y$ **31.** $-2a - 6b$ **33.** $-3x - 8y$ **35.** $-4x^2 - 2x$ **37.** $12x$ **39.** $-21a$ **41.** $6y$ **43.** $8x$ **45.** $-6a$ **47.** $12b$ **49.** $-15x^2$ **51.** x^2 **53.** a **55.** x **57.** n **59.** x **61.** y **63.** $3x$ **65.** $-2x$ **67.** $-8a^2$ **69.** $8y$ **71.** $4y$ **73.** $-2x$ **75.** $6a$ **77.** $-x - 2$ **79.** $8x - 6$ **81.** $-2a - 14$ **83.** $-6y + 24$ **85.** $35 - 21b$ **87.** $2 - 5y$ **89.** $15x^2 + 6x$ **91.** $2y - 18$ **93.** $-15x - 30$ **95.** $-6x^2 - 28$ **97.** $-6y^2 + 21$ **99.** $3x^2 - 3y^2$ **101.** $-4x + 12y$ **103.** $-6a^2 + 7b^2$ **105.** $4x^2 - 12x + 20$ **107.** $x - 3y + 4$ **109.** $-12a^2 - 20a + 28$ **111.** $12x^2 - 9x + 12$ **113.** $10x^2 - 20xy - 5y^2$ **115.** $-8b^2 + 6b - 9$ **117.** $a - 7$ **119.** $-11x + 13$ **121.** $-4y - 4$ **123.** $-2x - 16$ **125.** $14y - 45$ **127.** $a + 7b$ **129.** $6x + 28$ **131.** $5x - 75$ **133.** $4x - 4$ **135.** $2x - 9$ **137a.** False. $8 \div 4 \neq 4 \div 8$ **b.** False. $(8 \div 4) \div 2 \neq 8 \div (4 \div 2)$ **c.** False. $(7 - 5) - 1 \neq 7 - (5 - 1)$ **d.** False. $6 - 3 \neq 3 - 6$ **139.** No. 0 does not have a multiplicative inverse. **141.** Answers will vary.

SECTION 2.3

1. $8 + y$ **3.** $t + 10$ **5.** $z + 14$ **7.** $x^2 - 20$ **9.** $\frac{3}{4}n + 12$ **11.** $8 + \frac{n}{4}$ **13.** $3(y + 7)$ **15.** $t(t + 16)$ **17.** $\frac{1}{2}x^2 + 15$ **19.** $5n^3 + n^2$ **21.** $r - \frac{r}{3}$ **23.** $x^2 - (x + 17)$ **25.** $9(z + 4)$ **27.** $12 - x$ **29.** $\frac{2}{3}x$ **31.** $\frac{2x}{9}$ **33.** $11x - 8$ **35.** $(x + 2) - 9; x - 7$ **37.** $\frac{7}{5 + x}$ **39.** $5 + \frac{1}{2}(x + 3); \frac{1}{2}x + \frac{13}{2}$ **41.** $(2x - 4) + x; 3x - 4$ **43.** $(x - 5)7; 7x - 35$ **45.** $\frac{2x + 5}{x}$ **47.** $x - (3x - 8); -2x + 8$ **49.** $3x + x; 4x$ **51.** $(x + 6) + 5; x + 11$ **53.** $x - (x + 10); -10$ **55.** $\frac{1}{6}x + \frac{4}{9}x; \frac{11}{18}x$ **57.** $\frac{x}{3} + x; \frac{4}{3}x$ **59.** average sale price of a home in Carmel: P; average sale price of a home in Vail: $P + 143{,}600$ **61.** $S, 12 - S$ **63.** distance traveled by the faster car: x; distance traveled by the slower car: $200 - x$ **65.** time to prepare Form 1040: t; time to prepare Schedule A: $\frac{1}{4}t$ **67.** world population in 1990: p; world population in 2050: $2p$ **69.** number of oxygen atoms: x; number of hydrogen atoms: $2x$ **71.** Answers will vary.

CHAPTER REVIEW

1. $3x^2 - 24x - 21$ [2.2C] **2.** $11x$ [2.2A] **3.** $8a - 4b$ [2.2A] **4.** $-5n$ [2.2B] **5.** 79 [2.1A] **6.** $10x - 35$ [2.2C] **7.** $12y^2 + 8y - 10$ [2.2C] **8.** $-6a$ [2.2B] **9.** $-42x^2$ [2.2B] **10.** $-63 - 36x$ [2.2C] **11.** $-5y$ [2.2A] **12.** -4 [2.1A] **13.** $24y + 30$ [2.2D] **14.** $9c - 5d$ [2.2A] **15.** $20x$ [2.2B] **16.** $7x + 46$ [2.2D] **17.** 29 [2.1A] **18.** $-9r + 8s$ [2.2A] **19.** $-4x^2 + 6x$ [2.2A] **20.** $-90x + 25$ [2.2D]

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21. $28a^2 - 8a + 12$ [2.2C] **22.** $-4x + 20$ [2.2D] **23.** -7 [2.1A] **24.** $36y$ [2.2B] **25.** $\frac{2}{3}(x + 10)$ [2.3A]

26. number of calories in an apple: a; number of calories in a candy bar: $2a + 8$ [2.3C]

27. $3x + 5(x - 1)$; $8x - 5$ [2.3B] **28.** $4x$ [2.3A] **29.** number of ten-dollar bills: T; number of five-dollar bills: $35 - T$ [2.3C] **30.** $2x - \frac{1}{2}x$; $\frac{3}{2}x$ [2.3B] **31.** number of American League cards: A; number of National League cards: $5A$ [2.3C] **32.** $x - 6$ [2.3A] **33.** $x + 2x$; $3x$ [2.3B]

CHAPTER TEST

1. $5x$ [2.2A] **2.** $-6x^2 + 21y^2$ [2.2C] **3.** $-x + 6$ [2.2D] **4.** $-7x + 33$ [2.2D] **5.** $-9x - 7y$ [2.2A]

6. 22 [2.1A] **7.** $2x$ [2.2B] **8.** $7x + 38$ [2.2D] **9.** $-10x^2 + 15x - 30$ [2.2C] **10.** $-2x - 5y$ [2.2A]

11. 3 [2.1A] **12.** $3x$ [2.2B] **13.** y^2 [2.2A] **14.** $-4x + 8$ [2.2C] **15.** $-10a$ [2.2B]

16. $2x + y$ [2.2D] **17.** $36y$ [2.2B] **18.** $15 - 35b$ [2.2C] **19.** $a^2 - b^2$ [2.3A]

20. $10(x - 3)$; $10x - 30$ [2.3B] **21.** $x + 2x^2$ [2.3B] **22.** speed of return: s; speed of fastball: $2s$ [2.3C]

23. $\frac{6}{x} - 3$ [2.3B] **24.** $b - 7b$ [2.3A] **25.** shorter piece: x; longer piece: $4x - 3$ [2.3C]

CUMULATIVE REVIEW

1. -7 [1.2A] **2.** 5 [1.2B] **3.** 24 [1.3A] **4.** -5 [1.3B] **5.** 1.25 [1.4A] **6.** $\frac{11}{48}$ [1.4C] **7.** $\frac{1}{6}$ [1.4D]

8. $\frac{1}{4}$ [1.4D] **9.** 75% [1.4B] **10.** -5 [1.5B] **11.** $\frac{53}{48}$ [1.5B] **12.** 16 [2.1A] **13.** $5x^2$ [2.2A]

14. $-7a - 10b$ [2.2A] **15.** $6a$ [2.2B] **16.** $30b$ [2.2B] **17.** $24 - 6x$ [2.2C] **18.** $6y - 18$ [2.2C]

19. $\frac{3}{8}$ [1.4B] **20.** 0.0105 [1.4B] **21.** $-8x^2 + 12y^2$ [2.2C] **22.** $-9y^2 + 9y + 21$ [2.2C]

23. $-7x + 14$ [2.2D] **24.** $5x - 43$ [2.2D] **25.** $17x - 24$ [2.2D] **26.** $-3x + 21y$ [2.2D]

27. $\frac{1}{2}b + b$ [2.3A] **28.** $\frac{10}{y - 2}$ [2.3A] **29.** $8 - \frac{x}{12}$ [2.3B] **30.** $x + (x + 2)$; $2x + 2$ [2.3B]

31. $(3 + x)5 + 12$; $27 + 5x$ [2.3B] **32.** speed of normal drive: s; speed of triple-speed drive: $3s$ [2.3C]

Answers to Chapter 3 Odd-Numbered Exercises

SECTION 3.1

1. yes **3.** no **5.** no **7.** yes **9.** yes **11.** yes **13.** no **15.** yes **17.** yes **19.** yes **21.** no

23. 6 **25.** 16 **27.** 7 **29.** -2 **31.** 1 **33.** 0 **35.** 3 **37.** -10 **39.** -3 **41.** -14 **43.** 2

45. 11 **47.** -9 **49.** -1 **51.** -14 **53.** -5 **55.** -1 **57.** 1 **59.** $-\frac{1}{2}$ **61.** $-\frac{3}{4}$ **63.** $\frac{1}{12}$ **65.** $-\frac{7}{12}$

67. 0.6529 **69.** -0.283 **71.** 9.257 **73.** -3 **75.** 0 **77.** -2 **79.** 9 **81.** 80 **83.** -4 **85.** 0

87. 8 **89.** -7 **91.** 12 **93.** -18 **95.** 15 **97.** -20 **99.** 0 **101.** 15 **103.** 75 **105.** $\frac{8}{3}$ **107.** $\frac{1}{3}$

109. $-\frac{1}{2}$ **111.** $-\frac{3}{2}$ **113.** $\frac{15}{7}$ **115.** 4 **117.** 3 **119.** 4.745 **121.** 2.06 **123.** -2.13 **125.** 28

127. 0.72 **129.** 64 **131.** 24% **133.** 7.2 **135.** 400 **137.** 9 **139.** 25% **141.** 5 **143.** 200%

145. 400 **147.** 7.7 **149.** 200 **151.** 400 **153.** 20 **155.** 80.34% **157.** 19% of the students are in the fine

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arts college. **159.** 65% of the amount spent by companies was spent on sports. **161.** The total electricity used for home lighting is 96.1 billion kWh. **163.** There were about 2,482 million shares. **165a.** More money was spent on new cars in 1986. **b.** $41 billion more was spent on new cars in 1986. **167a.** Answers will vary. **b.** Answers will vary. **169.** Answers will vary. **171.** It is two times its original value.

SECTION 3.2

1. 3 **3.** 6 **5.** -1 **7.** -3 **9.** 2 **11.** 2 **13.** 5 **15.** -3 **17.** 6 **19.** 3 **21.** 1 **23.** 6 **25.** -7 **27.** 0 **29.** $\frac{3}{4}$ **31.** $\frac{4}{9}$ **33.** $\frac{1}{3}$ **35.** $-\frac{1}{2}$ **37.** $-\frac{3}{4}$ **39.** $\frac{1}{3}$ **41.** $-\frac{1}{6}$ **43.** 1 **45.** 1 **47.** 0 **49.** $\frac{13}{10}$ **51.** $\frac{2}{5}$ **53.** $-\frac{4}{3}$ **55.** $-\frac{3}{2}$ **57.** 18 **59.** 8 **61.** -16 **63.** 25 **65.** 21 **67.** 15 **69.** -16 **71.** -21 **73.** $\frac{15}{2}$ **75.** $-\frac{18}{5}$ **77.** 2 **79.** 3 **81.** 1 **83.** -2 **85.** 19 **87.** -1 **89.** -11 **91.** The markup rate is 60%. **93.** The cost is $59. **95.** The markup rate is 44.4%. **97.** The cost is $8.50. **99.** The discount rate is 23.2%. **101.** The regular price is $300. **103.** The markdown rate is 38%. **105.** The regular price is $275. **107.** The initial velocity is 8 ft/s. **109.** The depreciated value will be $38,000 in 2 years. **111.** The length is approximately 31.8 in. **113.** The distance is 168 ft. **115.** The population is approximately 51,000 people. **117.** The year is 1952. **119.** The markup is $18. **121.** The regular price is $317.65. **123.** 385 **125.** Answers will vary. **127.** The distance and the speed of travel are needed.

SECTION 3.3

1. 2 **3.** 3 **5.** -1 **7.** 2 **9.** -2 **11.** -3 **13.** 0 **15.** -1 **17.** -3 **19.** -1 **21.** 4 **23.** $\frac{2}{3}$ **25.** $\frac{5}{6}$ **27.** $\frac{3}{4}$ **29.** -17 **31.** 41 **33.** 8 **35.** 1 **37.** 4 **39.** -1 **41.** -1 **43.** $-\frac{2}{3}$ **45.** $\frac{4}{3}$ **47.** $\frac{1}{2}$ **49.** $-\frac{1}{3}$ **51.** $\frac{10}{3}$ **53.** $-\frac{1}{4}$ **55.** 0 **57.** -1 **59.** The force applied must be 25 lb. **61.** The fulcrum must be 6 ft from the 180-pound person. **63.** The fulcrum must be 10 ft from the 128-pound acrobat. **65.** The minimum force is 34.6 lb. **67.** The breakeven point is 260 units. **69.** The breakeven point is 520 units. **71.** 3000 bats must be sold to break even. **73.** no solution **75.** 0 **77.** Yes. Answers will vary.

SECTION 3.4

1. $x - 15 = 7$; 22 **3.** $7x = -21$; -3 **5.** $9 - x = 7$; 2 **7.** $5 - 2x = 1$; 2 **9.** $2x + 5 = 15$; 5 **11.** $4x - 6 = 22$; 7 **13.** $3(4x - 7) = 15$; 3 **15.** $2(x - 25) = 3x$; -50 **17.** $3x - 1 = 15 - x$; 4, 11 **19.** $3x + 2(18 - x) = 44$; 8, 10 **21.** The integers are -8, -6, and -4. **23.** The integers are 11, 13, and 15. **25.** The integers are 6 and 8. **27.** The integers are 2, 4, and 6. **29.** The storage capacity is 2 gigabytes. **31.** The customer used the service for 11 min. **33.** The executive used the phone for 162 min. **35.** The family used 515 kWh of electricity. **37.** The pieces measure 6 yd and 8 yd. **39.** The value of the stock account is $6000. The value of the mutual fund account is $4000. **41.** Answers will vary. **43.** contradiction **45.** $-\frac{5}{16}$ **47.** 0

SECTION 3.5

1. The sides measure 50 ft, 50 ft, and 25 ft. **3.** The length is 13 m. The width is 8 m. **5.** The length is 40 ft. The width is 20 ft. **7.** The sides measure 40 cm, 20 cm, and 50 cm. **9.** The length is 130 ft. The width is 39 ft.

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11. The width is 12 ft. **13.** Each side measures 12 in. **15.** The complement of a 28° angle is 62°. **17.** The supplement of a 73° angle is 107°. **19.** 35° **21.** 20° **23.** 53° **25.** 121° **27.** 15° **29.** 18° **31.** 45° **33.** 49° **35.** 12° **37.** $\angle a = 122°$; $\angle b = 58°$ **39.** $\angle a = 44°$; $\angle b = 136°$ **41.** 20° **43.** 40° **45.** 128° **47.** $\angle x = 160°$, $\angle y = 145°$ **49.** $\angle a = 40°$, $\angle b = 140°$ **51.** $75° - x$ **53.** The measure of the third angle is 45°. **55.** The measure of the third angle is 73°. **57.** The measure of the third angle is 43°. **59.** The length is 9 cm. The width is 4 cm. **61.** Answers will vary.

SECTION 3.6

1. 20 oz of herbs should be used. **3.** The cost of the mixture is $1.84 per pound. **5.** 3 lb of caramel is needed. **7.** 2 c of olive oil and 8 c of vinegar are used. **9.** The cost is $3.00 per ounce. **11.** 16 oz of the $400 alloy should be used. **13.** 37 lb of almonds and 63 lb of walnuts were used. **15.** 228 adult tickets were sold. **17.** The cost is $.70 per pound. **19.** The resulting gold alloy is 24% gold. **21.** 20 gal of the 15% acid solution are used. **23.** 30 lb of the yarn that is 25% wool is used. **25.** 6.25 gal of the plant food that is 9% nitrogen is used. **27.** The resulting mixture is 19% sugar. **29.** 20 lb of the coffee that is 40% java beans are used. **31.** 100 ml of the 7% solution and 200 ml of the 4% solution are used. **33.** 150 oz of pure chocolate must be added. **35.** The resulting alloy is 50% silver. **37.** The rate of the first plane is 105 mph. The rate of the second plane is 130 mph. **39.** They will be 3000 km apart at 11 A.M. **41.** The cabin cruiser will be alongside the motorboat 2 h after the cabin cruiser leaves. **43.** The distance from the airport to the corporate offices is 120 mi. **45.** The rate of the car is 68 mph. **47.** The distance between the two airports is 300 mi. **49.** The planes will pass each other 2.5 h after the plane leaves Seattle. **51.** They will meet 1.5 h after they begin. **53.** The bus overtakes the car 180 mi from the starting point. **55.** 75 g of pure water must be added. **57.** 3.75 gal must be drained from the radiator and replaced by pure antifreeze. **59.** The cyclist's average speed for the trip was $13\frac{1}{3}$ mph.

CHAPTER REVIEW

1. 21 [3.1B] **2.** 10 [3.3B] **3.** 7 [3.2A] **4.** No [3.1A] **5.** 20 [3.1C] **6.** -2 [3.3B] **7.** 30 is 250% of 12. [3.1D] **8.** 4 [3.3A] **9.** -1 [3.3B] **10.** 4 [3.3A] **11.** The cost is $671.25. [3.2B] **12.** 68° [3.5B] **13.** A force of 24 lb must be applied to the other end. [3.3C] **14.** The average speed was 32 mph. [3.6C] **15.** The discount rate is $33\frac{1}{3}\%$. [3.2B] **16.** $\angle x = 22°$, $\angle y = 158°$ [3.5B] **17.** 7 qt of cranberry juice and 3 qt of apple juice were used. [3.6A] **18.** The integers are -1, 0, and 1. [3.4A] **19.** The angles measure 75°, 60°, and 45°. [3.5C] **20.** $5n - 4 = 16$; 4 [3.4A] **21.** The height of the Eiffel Tower is 993 ft. [3.4B] **22.** The length is 80 ft. The width is 20 ft. [3.5A] **23.** The jet overtakes the propeller-driven plane 600 mi from the starting point. [3.6C] **24.** The numbers are 8 and 13. [3.4A] **25.** The resulting mixture is 14% butterfat. [3.6B]

CHAPTER TEST

1. -5 [3.3A] **2.** -5 [3.1B] **3.** -3 [3.2A] **4.** 2 [3.3B] **5.** No [3.1A] **6.** 5 [3.2A] **7.** 0.5% of 8 is 0.04. [3.1D] **8.** $-\frac{1}{3}$ [3.3B] **9.** 2 [3.3A] **10.** -12 [3.1C] **11.** 10 lb of the $.70 rye flour and 5 lb of the $.40 wheat flour should be used. [3.6A] **12.** 19° [3.5B] **13.** The discount rate is 20%. [3.2B] **14.** 200 calculators were produced. [3.2B] **15.** The measure of one of the equal angles is 70°. [3.5C] **16.** The integers are 10, 12, and 14. [3.4A] **17.** 1.25 gal of water must be used. [3.6B] **18.** $\angle a = 138°$, $\angle b = 42°$ [3.5B] **19.** $3x - 15 = 27$; 14 [3.4A] **20.** The rate of the snowmobile was 6 mph. [3.6C] **21.** There are 110 color TVs

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made each day. [3.4B] **22.** The two numbers are 8 and 10. [3.4A] **23.** The distance between the two airports is 360 mi. [3.6C] **24.** The sides measure 5 ft, 8 ft, and 10 ft. [3.5A] **25.** The temperature of the water after mixing is 60°C. [3.3C]

CUMULATIVE REVIEW

1. 6 [1.2B] **2.** -48 [1.3A] **3.** $-\frac{19}{48}$ [1.4C] **4.** -2 [1.4D] **5.** 54 [1.5A] **6.** 24 [1.5B]

7. 6 [2.1A] **8.** $-17x$ [2.2A] **9.** $-5a - 2b$ [2.2A] **10.** $2x$ [2.2B] **11.** $36y$ [2.2B]
12. $2x^2 + 6x - 4$ [2.2C] **13.** $-4x + 14$ [2.2D] **14.** $6x - 34$ [2.2D] **15.** Yes [3.1A] **16.** No [3.1A]
17. 19.2 [3.1D] **18.** -25 [3.1C] **19.** -3 [3.2A] **20.** 3 [3.2A] **21.** 13 [3.3B] **22.** 2 [3.3B]

23. -3 [3.3A] **24.** $\frac{1}{2}$ [3.3A] **25.** The final temperature of the water is 60°C. [3.3C]

26. $12 - 5x = -18$; 6 [3.4A] **27.** The area of the garage is 600 ft^2. [3.4B] **28.** 20 lb of oat flour must be used. [3.6A] **29.** 25 g of pure gold must be added. [3.6B] **30.** The length is 12 ft. The width is 10 ft. [3.5A]
31. 131° [3.5B] **32.** The measure of one of the angles is 60°. [3.5C] **33.** The length of the track is 120 m. [3.6C]

Answers to Chapter 4 Odd-Numbered Exercises

SECTION 4.1

1. yes **3.** no **5.** yes **7.** yes **9.** binomial **11.** trinomial **13.** none **15.** binomial **17.** $-2x^2 + 3x$
19. $y^2 - 8$ **21.** $5x^2 + 7x + 20$ **23.** $x^3 + 2x^2 - 6x - 6$ **25.** $2a^3 - 3a^2 - 11a + 2$ **27.** $5x^2 + 8x$
29. $7x^2 + xy - 4y^2$ **31.** $3a^2 - 3a + 17$ **33.** $5x^3 + 10x^2 - x - 4$ **35.** $3r^3 + 2r^2 - 11r + 7$ **37.** $4x$
39. $3y^2 - 4y - 2$ **41.** $-7x - 7$ **43.** $4x^3 + 3x^2 + 3x + 1$ **45.** $y^3 + 5y^2 - 2y - 4$ **47.** $-y^2 - 13xy$
49. $2x^2 - 3x - 1$ **51.** $-2x^3 + x^2 + 2$ **53.** $3a^3 - 2$ **55.** $4y^3 + 2y^2 + 2y - 4$ **57.** $x^2 + 9x - 11$ **59.** Answers will vary. **61.** Yes. For example, $(2x^3 + 3x - 4) + (-2x^3 + 5x^2 - 6) = 5x^2 + 3x - 10$.

SECTION 4.2

1. $30x^3$ **3.** $-42c^6$ **5.** $9a^7$ **7.** x^3y^4 **9.** $-10x^9y$ **11.** $12x^7y^8$ **13.** $-6x^3y^5$ **15.** x^4y^5z **17.** $a^3b^5c^4$
19. $-30a^5b^8$ **21.** $6a^5b$ **23.** $40y^{10}z^6$ **25.** $x^3y^3z^2$ **27.** $-24a^3b^3c^3$ **29.** $8x^7yz^6$ **31.** $30x^6y^8$ **33.** $-36a^3b^2c^3$
35. x^{15} **37.** x^{14} **39.** x^8 **41.** y^{12} **43.** $-8x^6$ **45.** x^4y^6 **47.** $9x^4y^2$ **49.** $-243x^{15}y^{10}$ **51.** $-8x^7$
53. $24x^8y^7$ **55.** a^4b^6 **57.** $64x^{12}y^3$ **59.** $-18x^3y^4$ **61.** $-8a^7b^5$ **63.** $-54a^9b^3$ **65.** $12x^2$ **67.** $2x^6y^2 + 9x^4y^2$
69. 0 **71.** $17x^4y^8$ **73.** true **75.** False. $(x^2)^5 = x^{2 \cdot 5} = x^{10}$ **77.** No. $2^{(3^2)}$ is larger. **79.** Answers will vary.

SECTION 4.3

1. $x^2 - 2x$ **3.** $-x^2 - 7x$ **5.** $3a^3 - 6a^2$ **7.** $-5x^4 + 5x^3$ **9.** $-3x^5 + 7x^3$ **11.** $12x^3 - 6x^2$ **13.** $6x^2 - 12x$
15. $3x^2 + 4x$ **17.** $-x^3y + xy^3$ **19.** $2x^4 - 3x^2 + 2x$ **21.** $2a^3 + 3a^2 + 2a$ **23.** $3x^6 - 3x^4 - 2x^2$
25. $-6y^4 - 12y^3 + 14y^2$ **27.** $-2a^3 - 6a^2 + 8a$ **29.** $6y^4 - 3y^3 + 6y^2$ **31.** $x^3y - 3x^2y^2 + xy^3$ **33.** $x^3 + 4x^2 + 5x + 2$
35. $a^3 - 6a^2 + 13a - 12$ **37.** $-2b^3 + 7b^2 + 19b - 20$ **39.** $-6x^3 + 31x^2 - 41x + 10$ **41.** $x^3 - 3x^2 + 5x - 15$
43. $x^4 - 4x^3 - 3x^2 + 14x - 8$ **45.** $15y^3 - 16y^2 - 70y + 16$ **47.** $5a^4 - 20a^3 - 5a^2 + 22a - 8$
49. $y^4 + 4y^3 + y^2 - 5y + 2$ **51.** $x^2 + 4x + 3$ **53.** $a^2 + a - 12$ **55.** $y^2 - 5y - 24$ **57.** $y^2 - 10y + 21$
59. $2x^2 + 15x + 7$ **61.** $3x^2 + 11x - 4$ **63.** $4x^2 - 31x + 21$ **65.** $3y^2 - 2y - 16$ **67.** $9x^2 + 54x + 77$
69. $21a^2 - 83a + 80$ **71.** $6a^2 - 25ab + 14b^2$ **73.** $2a^2 - 11ab - 63b^2$ **75.** $100a^2 - 100ab + 21b^2$
77. $15x^2 + 56xy + 48y^2$ **79.** $14x^2 - 97xy - 60y^2$ **81.** $56x^2 - 61xy + 15y^2$ **83.** $y^2 - 25$ **85.** $4x^2 - 9$

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87. $x^2 + 2x + 1$ **89.** $9a^2 - 30a + 25$ **91.** $9x^2 - 49$ **93.** $4a^2 + 4ab + b^2$ **95.** $x^2 - 4xy + 4y^2$ **97.** $16 - 9y^2$
99. $25x^2 + 20xy + 4y^2$ **101.** The area is $(10x^2 - 35x)$ ft^2. **103.** The area is $(4x^2 + 4x + 1)$ km^2.
105. The area is $(4x^2 + 10x)$ m^2. **107.** The total area is $(60w + 3000)$ yd^2. **109.** $x^4 + 2x^3 - 5x^2 - 6x + 9$
111. $12x^2 - x - 20$ **113.** $x^3 - 7x^2 - 7$ **115.** No. Answers will vary.

SECTION 4.4

1. $\frac{1}{25}$ **3.** 64 **5.** $\frac{1}{27}$ **7.** 2 **9.** $\frac{1}{x^2}$ **11.** a^6 **13.** $\frac{4}{x^7}$ **15.** $\frac{2}{3z^2}$ **17.** $5b^8$ **19.** $\frac{x^2}{3}$ **21.** 1 **23.** -1
25. y^4 **27.** a^3 **29.** p^4 **31.** $2x^3$ **33.** $2k$ **35.** m^5n^2 **37.** $\frac{3r^2}{2}$ **39.** $-\frac{2a}{3}$ **41.** $\frac{1}{y^5}$ **43.** $\frac{1}{a^6}$ **45.** $\frac{1}{3x^3}$
47. $\frac{2}{3x^5}$ **49.** $\frac{y^4}{x^2}$ **51.** $\frac{2}{5m^3n^8}$ **53.** $\frac{1}{p^3q}$ **55.** $\frac{1}{2y^3}$ **57.** $\frac{7xz}{8y^3}$ **59.** $\frac{p^2}{2m^3}$ **61.** $-\frac{8x^3}{y^6}$ **63.** $\frac{9}{x^2y^4}$ **65.** $\frac{2}{x^4}$
67. $-\frac{5}{a^8}$ **69.** $-\frac{a^5}{8b^4}$ **71.** $\frac{10y^3}{x^4}$ **73.** $\frac{1}{2x^3}$ **75.** $\frac{3}{x^3}$ **77.** $\frac{1}{2x^2y^6}$ **79.** $\frac{1}{x^6y}$ **81.** $\frac{a^4}{y^{10}}$ **83.** $-\frac{1}{6x^3}$ **85.** $-\frac{a^2b}{6c^2}$
87. $-\frac{7b^6}{a^2}$ **89.** $\frac{s^8t^4}{4r^{12}}$ **91.** $\frac{125p^3}{27m^{15}n^6}$ **93.** 3.24×10^{-9} **95.** 3×10^{-18} **97.** 3.2×10^{16} **99.** 1.22×10^{-19}
101. 5.47×10^8 **103.** 0.000167 **105.** 68,000,000 **107.** 0.0000305 **109.** 0.00000000102 **111.** 6.023×10^{23}
113. 3.7×10^{-6} **115.** 1×10^{-9} **117.** 1.6×10^{-19} **119.** $\frac{1}{4}, \frac{1}{2}, 1, 2, 4$ **121.** $4, 2, 1, \frac{1}{2}, \frac{1}{4}$

123. False. $(2a)^{-3} = \frac{1}{(2a)^3} = \frac{1}{8a^3}$ **125.** False. $(2 + 3)^{-1} = (5)^{-1} = \frac{1}{5}$ **127.** x^{-2}, $x \neq 0$, is always positive. $x^{-2} = \frac{1}{x^2}$; since x^2 is positive, $\frac{1}{x^2}$ is positive.

SECTION 4.5

1. $2a - 5$ **3.** $6y + 4$ **5.** $x - 2$ **7.** $-x + 2$ **9.** $x^2 + 3x - 5$ **11.** $x^4 - 3x^2 - 1$ **13.** $xy + 2$ **15.** $-3y^3 + 5$
17. $3x - 2 + \frac{1}{x}$ **19.** $-3x + 7 - \frac{6}{x}$ **21.** $4a - 5 + 6b$ **23.** $9x + 6 - 3y$ **25.** $b - 7$ **27.** $y - 5$ **29.** $2y - 7$
31. $2y + 6 + \frac{25}{y - 3}$ **33.** $x - 2 + \frac{8}{x + 2}$ **35.** $3y - 5 + \frac{20}{2y + 4}$ **37.** $6x - 12 + \frac{19}{x + 2}$ **39.** $b - 5 - \frac{24}{b - 3}$
41. $3x + 17 + \frac{64}{x - 4}$ **43.** $5y + 3 + \frac{1}{2y + 3}$ **45.** $4a + 1$ **47.** $2a + 9 + \frac{33}{3a - 1}$ **49.** $x^2 - 5x + 2$ **51.** $x^2 + 5$
53. $3ab$

CHAPTER REVIEW

1. $8b^2 - 2b - 15$ [4.3C] **2.** $21y^2 + 4y - 1$ [4.1A] **3.** $b^2 + 5b + 2 + \frac{7}{b - 7}$ [4.5B] **4.** $x^4y^8z^4$ [4.2A]
5. $-8x^3 - 14x^2 + 18x$ [4.3A] **6.** b^2 [4.4A] **7.** 64 [4.2B] **8.** $2x^2 + 3x - 8$ [4.1B] **9.** $\frac{2x^3}{3}$ [4.4A]
10. $25y^2 - 70y + 49$ [4.3D] **11.** $100a^{15}b^{13}$ [4.2B] **12.** $4b^4 + 12b^2 - 1$ [4.5A] **13.** $13y^3 - 12y^2 - 5y - 1$ [4.1B]
14. $10a^2 + 12a - 22$ [4.1A] **15.** $6y^3 + 17y^2 - 2y - 21$ [4.3B] **16.** $-\frac{2a^3}{3b^3}$ [4.4A]
17. $8a^3b^3 - 4a^2b^4 + 6ab^5$ [4.3A] **18.** $-2y^2 + y - 5$ [4.1B] **19.** $9x^4y^6$ [4.2B]
20. $12b^5 - 4b^4 - 6b^3 - 8b^2 + 5$ [4.3B] **21.** $2x^3 + 9x^2 - 3x - 12$ [4.1A] **22.** $-4y + 8$ [4.5A]
23. $a^2 - 49$ [4.3D] **24.** $-54a^{13}b^5c^7$ [4.2A] **25.** $2y - 9$ [4.5B] **26.** $\frac{x^4y^6}{9}$ [4.4A] **27.** $-20x^3y^5$ [4.2A]
28. $10a^2 + 31a - 63$ [4.3C] **29.** 1.27×10^{-7} [4.4B] **30.** 0.0000000000032 [4.4B]
31. The area is $(2w^2 - w)$ ft^2. [4.3E] **32.** The area is $(9x^2 - 12x + 4)$ in^2. [4.3E]

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CHAPTER TEST

1. $4x^3 - 6x^2$ [4.3A] **2.** $4x - 1 + \frac{3}{x^2}$ [4.5A] **3.** $-\frac{4}{x^6}$ [4.4A] **4.** $-6x^3y^6$ [4.2A] **5.** $x - 1 + \frac{2}{x+1}$ [4.5B]

6. $x^3 - 7x^2 + 17x - 15$ [4.3B] **7.** $-8a^6b^3$ [4.2B] **8.** $\frac{9y^{10}}{x^{10}}$ [4.4A] **9.** $a^2 + 3ab - 10b^2$ [4.3C]

10. $4x^4 - 2x^2 + 5$ [4.5A] **11.** $x + 7$ [4.5B] **12.** $6y^4 - 9y^3 + 18y^2$ [4.3A]

13. $-4x^4 + 8x^3 - 3x^2 - 14x + 21$ [4.3B] **14.** $16y^2 - 9$ [4.3D] **15.** a^4b^7 [4.2A] **16.** $8ab^4$ [4.4A]

17. $4a - 7$ [4.5A] **18.** $-5a^3 + 3a^2 - 4a + 3$ [4.1B] **19.** $4x^2 - 20x + 25$ [4.3D] **20.** $2x + 3 + \frac{2}{2x-3}$ [4.5B]

21. $-2x^3$ [4.4A] **22.** $10x^2 - 43xy + 28y^2$ [4.3C] **23.** $3x^3 + 6x^2 - 8x + 3$ [4.1A] **24.** 3.02×10^{-9} [4.4B]

25. The area is $(\pi x^2 - 10\pi x + 25\pi)$ m^2. [4.3E]

CUMULATIVE REVIEW

1. $\frac{5}{144}$ [1.4C] **2.** $\frac{5}{3}$ [1.5A] **3.** $\frac{25}{11}$ [1.5B] **4.** $-\frac{22}{9}$ [2.1A] **5.** $5x - 3xy$ [2.2A] **6.** $-9x$ [2.2B]

7. $-18x + 12$ [2.2D] **8.** -16 [3.1C] **9.** -16 [3.3A] **10.** 15 [3.3B] **11.** 22% [3.1D]

12. $4b^3 - 4b^2 - 8b - 4$ [4.1A] **13.** $3y^3 + 2y^2 - 10y$ [4.1B] **14.** a^9b^{15} [4.2B] **15.** $-8x^3y^6$ [4.2A]

16. $6y^4 + 8y^3 - 16y^2$ [4.3A] **17.** $10a^3 - 39a^2 + 20a - 21$ [4.3B] **18.** $15b^2 - 31b + 14$ [4.3C] **19.** $\frac{1}{2b^2}$ [4.4A]

20. $a - 7$ [4.5B] **21.** 0.0000609 [4.4B] **22.** $8x - 2x = 18$; 3 [3.4B] **23.** The resulting mixture is 28% orange juice. [3.6B] **24.** The car overtakes the cyclist 25 mi from the starting point. [3.6C]

25. The length is 15 m. The width is 6 m. [3.5A]

Answers to Chapter 5 Odd-Numbered Exercises

SECTION 5.1

1. $5(a + 1)$ **3.** $8(2 - a^2)$ **5.** $4(2x + 3)$ **7.** $6(5a - 1)$ **9.** $x(7x - 3)$ **11.** $a^2(3 + 5a^3)$ **13.** $y(14y + 11)$

15. $2x(x^3 - 2)$ **17.** $2x^2(5x^2 - 6)$ **19.** $4a^5(2a^3 - 1)$ **21.** $xy(xy - 1)$ **23.** $3xy(xy^3 - 2)$ **25.** $xy(x - y^2)$

27. $5y(y^2 - 4y + 2)$ **29.** $3y^2(y^2 - 3y - 2)$ **31.** $3y(y^2 - 3y + 8)$ **33.** $a^2(6a^3 - 3a - 2)$ **35.** $ab(2a - 5ab + 7b)$

37. $2b(2b^4 + 3b^2 - 6)$ **39.** $x^2(8y^2 - 4y + 1)$ **41.** $(y + 7)(a + z)$ **43.** $(3r + s)(a - b)$ **45.** $(t - 7)(m - 7)$

47. $(2y + 1)(4a - b)$ **49.** $(x + 2)(x + 2y)$ **51.** $(p - 2)(p - 3r)$ **53.** $(b - 4)(a + 6)$ **55.** $(z + y)(2z - 1)$

57. $(4v + 7)(2v - 3y)$ **59.** $(2x - 5)(x - 3y)$ **61.** $(3y - a)(y - 2)$ **63.** $(y + 1)(3x - y)$ **65.** $(t - 2)(3s + t)$

67a. $(2x + 5)(x + 3)$ **b.** $(x + 3)(2x + 5)$ **69.** 28 is the one perfect number between 20 and 30. **71.** In the equation for the perimeter of a rectangle, when $L + W$ doubles, the perimeter doubles.

SECTION 5.2

1. $(x + 1)(x + 2)$ **3.** $(x - 2)(x + 1)$ **5.** $(a + 4)(a - 3)$ **7.** $(a - 1)(a - 2)$ **9.** $(a + 2)(a - 1)$

11. $(b - 3)(b - 3)$ **13.** $(b + 8)(b - 1)$ **15.** $(y + 11)(y - 5)$ **17.** $(y - 3)(y - 2)$ **19.** $(z - 5)(z - 9)$

21. $(z - 20)(z + 8)$ **23.** $(p + 3)(p + 9)$ **25.** $(x + 10)(x + 10)$ **27.** $(b + 4)(b + 5)$ **29.** $(x + 3)(x - 14)$

31. $(b - 5)(b + 4)$ **33.** $(y - 17)(y + 3)$ **35.** $(p - 7)(p + 3)$ **37.** nonfactorable over the integers

39. $(x - 5)(x - 15)$ **41.** $(p + 3)(p + 21)$ **43.** $(x + 2)(x + 19)$ **45.** $(x + 9)(x - 4)$ **47.** $(a + 4)(a - 11)$

49. $(a - 3)(a - 18)$ **51.** $(z + 21)(z - 7)$ **53.** $(c + 12)(c - 15)$ **55.** $(p + 9)(p + 15)$ **57.** $(c + 2)(c + 9)$

59. $(x + 15)(x - 5)$ **61.** $(x + 25)(x - 4)$ **63.** $(b - 4)(b - 18)$ **65.** $(a + 45)(a - 3)$ **67.** $(b - 7)(b - 18)$

69. $(z + 12)(z + 12)$ **71.** $(x - 4)(x - 25)$ **73.** $(x + 16)(x - 7)$ **75.** $3(x + 2)(x + 3)$ **77.** $-(x + 6)(x - 2)$

79. $a(b + 8)(b - 1)$ **81.** $x(y + 3)(y + 5)$ **83.** $-2a(a + 2)(a + 1)$ **85.** $4y(y + 6)(y - 3)$ **87.** $2x(x^2 - x + 2)$

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89. $6(z+5)(z-3)$ **91.** $3a(a+3)(a-6)$ **93.** $(x+7y)(x-3y)$ **95.** $(a-5b)(a-10b)$ **97.** $(s+8t)(s-6t)$
99. nonfactorable over the integers **101.** $z^2(z+10)(z-8)$ **103.** $b^2(b+2)(b-5)$ **105.** $3y^2(y+3)(y+15)$
107. $-x^2(x-12)(x+1)$ **109.** $3y(x+3)(x-5)$ **111.** $-3x(x-3)(x-9)$ **113.** $(x-3y)(x-5y)$
115. $(a-6b)(a-7b)$ **117.** $(y+z)(y+7z)$ **119.** $3y(x+21)(x-1)$ **121.** $3x(x+4)(x-3)$
123. $4z(z+11)(z-3)$ **125.** $4x(x+3)(x-1)$ **127.** $5(p+12)(p-7)$ **129.** $p^2(p+12)(p-3)$
131. $(t-5s)(t-7s)$ **133.** $(a+3b)(a-11b)$ **135.** $y(x+6)(x-9)$ **137.** $-12, 12, -36, 36$
139. $-22, -10, 10, 22$ **141.** 6, 10, 12 **143.** 6, 10, 12 **145.** 4, 6

SECTION 5.3

1. $(2x+1)(x+1)$ **3.** $(2y+1)(y+3)$ **5.** $(2a-1)(a-1)$ **7.** $(2b-1)(b-5)$ **9.** $(2x-1)(x+1)$
11. $(2x+1)(x-3)$ **13.** $(2t-5)(t+2)$ **15.** $(3p-1)(p-5)$ **17.** $(4y-1)(3y-1)$ **19.** nonfactorable over the integers **21.** $(2t-1)(3t-4)$ **23.** $(8x+1)(x+4)$ **25.** nonfactorable over the integers
27. $(4y+5)(3y+1)$ **29.** $(7a-2)(a+7)$ **31.** $(3b-4)(b-4)$ **33.** $(2z+1)(z-14)$ **35.** $(3p-2)(p+8)$
37. $2(2x+1)(x+1)$ **39.** $5(3y-7)(y-1)$ **41.** $x(2x-1)(x-5)$ **43.** $b(3a-4)(a-4)$ **45.** nonfactorable over the integers **47.** $-3x(x+4)(x-3)$ **49.** $4(4y-1)(5y-1)$ **51.** $z(4z+1)(2z+3)$ **53.** $y(3x+2)(2x-5)$
55. $5(2t-5)(t+2)$ **57.** $p(3p-1)(p-5)$ **59.** $2(13z-3)(z+4)$ **61.** $2y(5y-2)(y-4)$ **63.** $yz(4z-3)(z+2)$
65. $3a(2a+3)(7a-3)$ **67.** $y(3x-5y)(3x-5y)$ **69.** $xy(3x-4y)(3x-4y)$ **71.** $(3x-4)(2x-3)$
73. $(5b-2)(b+7)$ **75.** $(3a+8)(2a-3)$ **77.** $(4z+3)(z+2)$ **79.** $(2p+5)(11p-2)$ **81.** $(8y+9)(y+1)$
83. $(3t+1)(6t-5)$ **85.** $(6b-1)(b+12)$ **87.** $(3x+2)(3x+2)$ **89.** $(3b-2)(2b-3)$ **91.** $(11b-7)(3b+5)$
93. $(3y-4)(6y-5)$ **95.** $(3a+7)(5a-3)$ **97.** $(2y-5)(4y-3)$ **99.** $(4z-5)(2z+3)$ **101.** nonfactorable over the integers **103.** $(2z-5)(5z-2)$ **105.** $(6z+7)(6z+5)$ **107.** $(3x-2y)(x+y)$ **109.** $(3a-b)(a+2b)$
111. $(4y-3z)(y-2z)$ **113.** $-(z-7)(z+4)$ **115.** $-(x-1)(x+8)$ **117.** $3(3x-4)(x+5)$
119. $4(2x-3)(3x-2)$ **121.** $a^2(7a-1)(5a+2)$ **123.** $5(3b-2)(b-7)$ **125.** $(3x-5y)(x-7y)$
127. $3(8y-1)(9y+1)$ **129.** $-(x-1)(x+21)$ **131.** Answers will vary. **133.** $x(x-1)$ **135.** $(2y+1)(y+3)$
137. $(4y-3)(y-3)$ **139.** $-1, 1, -5, 5$ **141.** $-1, 1, -5, 5$ **143.** $-3, 3, -9, 9$

SECTION 5.4

1. $(x-2)(x+2)$ **3.** $(a-9)(a+9)$ **5.** $(y+1)^2$ **7.** $(a-1)^2$ **9.** $(2x+1)(2x-1)$ **11.** $(x^3-3)(x^3+3)$
13. nonfactorable over the integers **15.** $(x+y)^2$ **17.** $(2a+1)^2$ **19.** $(3x-1)(3x+1)$ **21.** $(1+8x)(1-8x)$
23. nonfactorable over the integers **25.** $(3a+1)^2$ **27.** $(b^2-4a)(b^2+4a)$ **29.** $(2a-5)^2$ **31.** $(3a-7)^2$
33. $(5z+y)(5z-y)$ **35.** $(ab-5)(ab+5)$ **37.** $(5x-1)(5x+1)$ **39.** $(2a-3b)^2$ **41.** $(2y-9z)^2$
43. $\left(\frac{1}{x}-2\right)\left(\frac{1}{x}+2\right)$ **45.** $(3ab-1)^2$ **47.** $2(2y-1)(2y+1)$ **49.** $3a(a+1)^2$ **51.** $(m^2+16)(m+4)(m-4)$
53. $(9x+4)(x+1)$ **55.** $4y^2(2y+3)^2$ **57.** $(y^4+9)(y^2-3)(y^2+3)$ **59.** $(5-2p)^2$ **61.** $(4x-3-y)(4x-3+y)$
63. $(x-2-y)(x-2+y)$ **65.** $5(x+1)(x-1)$ **67.** $x(x+2)^2$ **69.** $x^2(x+7)(x-5)$ **71.** $5(b+3)(b+12)$
73. nonfactorable over the integers **75.** $2y(x+11)(x-3)$ **77.** $x(x^2-6x-5)$ **79.** $3(y^2-12)$
81. $(2a+1)(10a+1)$ **83.** $y^2(x-8)(x+1)$ **85.** $5(2a-3b)(a+b)$ **87.** $-2(x-5)(x+5)$ **89.** $b^2(a-5)^2$
91. $ab(4a+b)(3a-b)$ **93.** $3a(2a-1)^2$ **95.** $3(81+a^2)$ **97.** $2a(2a-5)(3a-4)$ **99.** $a(2a+5)^2$
101. $3b(3a-1)^2$ **103.** $-6(x+4)(x-2)$ **105.** $x^2(x+y)(x-y)$ **107.** $2a(3a+2)^2$ **109.** $-b(3a-2)(2a+1)$
111. $2x^2(2x-3)(x-8)$ **113.** $x^2(x+5)(x-5)$ **115.** $(a^2+4)(a+2)(a-2)$ **117.** $-3y^2(2y+5)(4y-3)$
119. $2(x-3)(2a-b)$ **121.** $(y+1)(y-1)(a-b)$ **123.** $(x-y)(a+b)(a-b)$ **125.** $-12, 12$ **127.** $-16, 16$
129. $-10, 10$ **131.** $(2n+1)^2-1=4n(n+1)$. Since n or $n+1$ is an even number, $4n(n+1)$ is divisible by 8.

SECTION 5.5

1. -3 and -2 **3.** 7 and 3 **5.** 0 and 5 **7.** 0 and 9 **9.** 0 and $-\frac{3}{2}$ **11.** 0 and $\frac{2}{3}$ **13.** -2 and 5

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15. 9 and -9 **17.** $\frac{7}{2}$ and $-\frac{7}{2}$ **19.** $\frac{1}{3}$ and $-\frac{1}{3}$ **21.** -2 and -4 **23.** -7 and 2 **25.** $-\frac{1}{2}$ and 5

27. $-\frac{1}{3}$ and $-\frac{1}{2}$ **29.** 0 and 3 **31.** 0 and 7 **33.** -1 and -4 **35.** 2 and 3 **37.** $\frac{1}{2}$ and -4 **39.** $\frac{1}{3}$ and 4

41. 3 and 9 **43.** 9 and -2 **45.** -1 and -2 **47.** -9 and 5 **49.** -7 and 4 **51.** -2 and -3 **53.** -8 and 9

55. 1 and 4 **57.** -5 and 2 **59.** The number is 6. **61.** The numbers are 2 and 4. **63.** The numbers are 4 and 5.

65. The numbers are 3 and 7. **67.** There will be 12 consecutive natural numbers. **69.** There are 6 teams.

71. The object will hit the ground 3 s later. **73.** It will be 3.75 s later. **75.** The length is 15 in. and the width is 5 in.

77. The height of the triangle is 14 m. **79.** The dimensions of the type area are 4 in. by 7 in. **81.** The radius of the original circle is 3.81 in. **83.** Answers will vary. **85.** 0 and 1 **87.** 2 and 32 **89.** -9 and -1 **91.** 1 and 18

CHAPTER REVIEW

1. $(b-3)(b-10)$ [5.2A] **2.** $(x-3)(4x+5)$ [5.1B] **3.** nonfactorable over the integers [5.3A]

4. $5x(x^2+2x+7)$ [5.1A] **5.** $7y^3(2y^6-7y^3+1)$ [5.1A] **6.** $(y-4)(y+9)$ [5.2A] **7.** $(2x-7)(3x-4)$ [5.3A]

8. $3ab(4a+b)$ [5.1A] **9.** $(a^3+10)(a^3-10)$ [5.4A] **10.** $n^2(n+1)(n-3)$ [5.2B] **11.** $(6y-1)(2y+3)$ [5.3A]

12. $2b(3b-4)(2b-7)$ [5.4B] **13.** $(3y^2+5z)(3y^2-5z)$ [5.4A] **14.** $(c+6)(c+2)$ [5.2A]

15. $(6a-5)(3a+2)$ [5.3B] **16.** $\frac{1}{4}$ and -7 [5.5A] **17.** $4x(x-6)(x+1)$ [5.2B] **18.** $3(a+2)(a-7)$ [5.2B]

19. $(2a+5)(a-12)$ [5.3B] **20.** -3 and 7 [5.5A] **21.** $(3a-5b)(7x+2y)$ [5.1B] **22.** $(ab+1)(ab-1)$ [5.4A]

23. $(2x+5)(5x+2y)$ [5.1B] **24.** $5(x+2)(x-3)$ [5.2B] **25.** $3(x+6)^2$ [5.4B] **26.** $(3x-2)(x-5)$ [5.3B]

27. The length is 100 yd. The width is 60 yd. [5.5B] **28.** The distance between the screen and the projector is 20 ft. [5.5B] **29.** The width of the frame is 1.5 in. or $1\frac{1}{2}$ in. [5.5B] **30.** The length of a side is 20 ft. [5.5B]

CHAPTER TEST

1. $(b+6)(a-3)$ [5.1B] **2.** $2y^2(y+1)(y-8)$ [5.2B] **3.** $4(x+4)(2x-3)$ [5.3B] **4.** $(2x+1)(3x+8)$ [5.3A]

5. $(a-3)(a-16)$ [5.2A] **6.** $2x(3x^2-4x+5)$ [5.1A] **7.** $(x+5)(x-3)$ [5.2A] **8.** $\frac{1}{2}$ and $-\frac{1}{2}$ [5.5A]

9. $5(x^2-9x-3)$ [5.1A] **10.** $(p+6)^2$ [5.4A] **11.** 3 and 5 [5.5A] **12.** $3(x+2y)^2$ [5.4B]

13. $(b+4)(b-4)$ [5.4A] **14.** $3y^2(2x+1)(x+1)$ [5.3B] **15.** $(p+2)(p+3)$ [5.2A] **16.** $(x-2)(a+b)$ [5.1B]

17. $(p+1)(x-1)$ [5.1B] **18.** $3(a+5)(a-5)$ [5.4B] **19.** nonfactorable over the integers [5.3A]

20. $(x+3)(x-12)$ [5.2A] **21.** $(2a-3b)^2$ [5.4A] **22.** $(2x+7y)(2x-7y)$ [5.4A] **23.** $\frac{3}{2}$ and -7 [5.5A]

24. The two numbers are 3 and 7. [5.5B] **25.** The length is 15 cm. The width is 6 cm. [5.5B]

CUMULATIVE REVIEW

1. 7 [1.2B] **2.** 4 [1.5B] **3.** -7 [2.1A] **4.** $15x^2$ [2.2B] **5.** 12 [2.2D] **6.** $\frac{2}{3}$ [3.1C] **7.** $\frac{7}{4}$ [3.3A]

8. 3 [3.3B] **9.** 45 [3.1D] **10.** $9a^6b^4$ [4.2B] **11.** x^3-3x^2-6x+8 [4.3B] **12.** $4x+8+\frac{21}{2x-3}$ [4.5B]

13. $\frac{y^6}{x^8}$ [4.4A] **14.** $(a-b)(3-x)$ [5.1B] **15.** $5xy^2(3-4y^2)$ [5.1A] **16.** $(x-7y)(x+2y)$ [5.2A]

17. $(p-10)(p+1)$ [5.2A] **18.** $3a(2a+5)(3a+2)$ [5.3B] **19.** $(6a-7b)(6a+7b)$ [5.4A]

20. $(2x+7y)^2$ [5.4A] **21.** $(3x-2)(3x+7)$ [5.3A] **22.** $2(3x-4y)^2$ [5.4B] **23.** $(x-3)(3y-2)$ [5.1B]

24. $\frac{2}{3}$ and -7 [5.5A] **25.** The pieces measure 4 ft and 6 ft. [3.4B] **26.** The discount rate is 40%. [3.2B]

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27. The measure of angle a is 72° and the measure of angle b is 108°. [3.5B] **28.** The distance to the resort was 168 mi. [3.6C] **29.** The integers are 10, 12, and 14. [3.4A] **30.** The length of the base is 12 in. [5.5B]

Answers to Chapter 6 Odd-Numbered Exercises

SECTION 6.1

1. $\frac{3}{4x}$ **3.** $\frac{1}{x+3}$ **5.** -1 **7.** $\frac{2}{3y}$ **9.** $-\frac{3}{4x}$ **11.** $\frac{a}{b}$ **13.** $-\frac{2}{x}$ **15.** $\frac{y-2}{y-3}$ **17.** $\frac{x+5}{x+4}$ **19.** $\frac{x+4}{x-3}$

21. $-\frac{x+2}{x+5}$ **23.** $\frac{2(x+2)}{x+3}$ **25.** $\frac{2x-1}{2x+3}$ **27.** $-\frac{x+7}{x+6}$ **29.** $\frac{35ab^2}{24x^2y}$ **31.** $\frac{4x^3y^3}{3a^2}$ **33.** $\frac{3}{4}$ **35.** ab^2

37. $\frac{x^2(x-1)}{y(x+3)}$ **39.** $\frac{y(x-1)}{x^2(x+10)}$ **41.** $-ab^2$ **43.** $\frac{x+5}{x+4}$ **45.** 1 **47.** $-\frac{n-10}{n-7}$ **49.** $\frac{x(x+2)}{2(x-1)}$ **51.** $-\frac{x+2}{x-6}$

53. $\frac{x+5}{x-12}$ **55.** $\frac{7a^3y^2}{40bx}$ **57.** $\frac{4}{3}$ **59.** $\frac{3a}{2}$ **61.** $\frac{x^2(x+4)}{y^2(x+2)}$ **63.** $\frac{x(x-2)}{y(x-6)}$ **65.** $-\frac{3by}{ax}$ **67.** $\frac{(x+6)(x-3)}{(x+7)(x-6)}$

69. 1 **71.** $-\frac{x+8}{x-4}$ **73.** $\frac{2n+1}{2n-3}$ **75.** Yes; for example $3.00000001.\ 3 + 10^{\ 8}$ **77.** 5, −5 **79.** $\frac{4x}{3y}$ **81.** $-\frac{1}{y^3}$

83. $\frac{b+2}{b-5}$

SECTION 6.2

1. $24x^3y^2$ **3.** $30x^4y^2$ **5.** $8x^2(x+2)$ **7.** $6x^2y(x+4)$ **9.** $36x(x+2)^2$ **11.** $6(x+1)^2$

13. $(x-1)(x+2)(x+3)$ **15.** $(2x+3)^2(x-5)$ **17.** $(x-1)(x-2)$ **19.** $(x-3)(x+2)(x+4)$

21. $(x+4)(x+1)(x-7)$ **23.** $(x-6)(x+6)(x+4)$ **25.** $(x-10)(x-8)(x+3)$ **27.** $(3x-2)(x-3)(x+2)$

29. $(x+2)(x-3)$ **31.** $(x-5)(x+1)$ **33.** $(x-3)(x-2)(x-1)(x-6)$ **35.** $\frac{5}{ab^2}, \frac{6b}{ab^2}$ **37.** $\frac{15y^2}{18x^2y}, \frac{14x}{18x^2y}$

39. $\frac{ay+5a}{y^2(y+5)}, \frac{6y}{y^2(y+5)}$ **41.** $\frac{a^2y+7a^2}{y(y+7)^2}, \frac{ay}{y(y+7)^2}$ **43.** $\frac{b}{y(y-4)}, -\frac{b^2y}{y(y-4)}$ **45.** $-\frac{3y-21}{(y-7)^2}, \frac{2}{(y-7)^2}$

47. $\frac{2y^2}{y^2(y-3)}, \frac{3}{y^2(y-3)}$ **49.** $\frac{x^3+4x^2}{(2x-1)(x+4)}, \frac{2x^2+x-1}{(2x-1)(x+4)}$ **51.** $\frac{3x^2+15x}{(x-5)(x+5)}, \frac{4}{(x-5)(x+5)}$

53. $\frac{x^2-1}{(x+5)(x-3)(x+1)}, \frac{x^2-3x}{(x+5)(x-3)(x+1)}$ **55.** $\frac{800}{10^5}, \frac{9}{10^5}$ **57.** $\frac{x^3-x}{x^2-1}, \frac{x}{x^2-1}$

59. $\frac{3c^2-3cd}{3(6c+d)(c+d)(c-d)}, \frac{6cd+d^2}{3(6c+d)(c+d)(c-d)}$

SECTION 6.3

1. $\frac{11}{y^2}$ **3.** $-\frac{7}{x+4}$ **5.** $\frac{8x}{2x+3}$ **7.** $\frac{5x+7}{x-3}$ **9.** $\frac{2x-5}{x+9}$ **11.** $\frac{-3x-4}{2x+7}$ **13.** $\frac{1}{x+5}$ **15.** $\frac{1}{x-6}$ **17.** $\frac{3}{2y-1}$

19. $\frac{1}{x-5}$ **21.** $\frac{4y+5x}{xy}$ **23.** $\frac{19}{2x}$ **25.** $\frac{5}{12x}$ **27.** $\frac{19x-12}{6x^2}$ **29.** $\frac{52y-35x}{20xy}$ **31.** $\frac{13x+2}{15x}$ **33.** $\frac{7}{24}$

35. $\frac{x+90}{45x}$ **37.** $\frac{x^2+2x+2}{2x^2}$ **39.** $\frac{2x^2+3x-10}{4x^2}$ **41.** $\frac{-x^2-4x+4}{x+4}$ **43.** $\frac{4x+7}{x+1}$ **45.** $\frac{4x^2+9x+9}{24x^2}$

47. $\frac{3x-1-2xy-3y}{xy^2}$ **49.** $\frac{20x^2+28x-12xy+9y}{24x^2y^2}$ **51.** $\frac{9x^2-3x-2xy-10y}{18xy^2}$ **53.** $\frac{7x-23}{(x-3)(x-4)}$

55. $\frac{-y-33}{(y+6)(y-3)}$ **57.** $\frac{3x^2+20x-8}{(x-4)(x+6)}$ **59.** $\frac{3(4x^2+5x-5)}{(x+5)(2x+3)}$ **61.** $\frac{-4x+5}{x-6}$ **63.** $\frac{2(y+2)}{(y-4)(y+4)}$ **65.** $-\frac{4x}{(x+1)^2}$

67. $\frac{2x-1}{(1+x)(1-x)}$ **69.** $\frac{14}{(x-5)^2}$ **71.** $\frac{-2(x+7)}{(x-7)(x+6)}$ **73.** $\frac{x-4}{x-6}$ **75.** $\frac{2x+1}{x-1}$ **77.** $\frac{-3(x^2+8x+25)}{(x-3)(x+7)}$

79. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{50}{51}, \frac{100}{101}, \frac{1000}{1001}$ **81.** 1

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SECTION 6.4

1. $\frac{x}{x-3}$ **3.** $\frac{2}{3}$ **5.** $\frac{y+3}{y-4}$ **7.** $\frac{2(2x+13)}{5x+36}$ **9.** $\frac{x+2}{x+3}$ **11.** $\frac{x-6}{x+5}$ **13.** $\frac{-x+2}{x+1}$ **15.** $x-1$ **17.** $\frac{1}{2x-1}$

19. $\frac{x-3}{x+5}$ **21.** $\frac{x-7}{x-8}$ **23.** $\frac{2y-1}{2y+1}$ **25.** $\frac{x-2}{2x-5}$ **27.** $\frac{-x-1}{4x-3}$ **29.** $\frac{x+1}{2(5x-2)}$ **31.** $\frac{5}{3}$ **33.** $-\frac{1}{x-1}$

35. $\frac{y+4}{2(y-2)}$ **37.** $\frac{x+1}{x-1}$ **39.** $\frac{y^2+x^2}{xy}$

SECTION 6.5

1. 3 **3.** 1 **5.** 9 **7.** 1 **9.** $\frac{1}{4}$ **11.** 1 **13.** -3 **15.** $\frac{1}{2}$ **17.** 8 **19.** 5 **21.** -1 **23.** 5

25. no solution **27.** 2 and 4 **29.** 4 and $-\frac{3}{2}$ **31.** 3 **33.** 4 **35.** 0 **37.** $-\frac{2}{5}$ **39.** 0 and $-\frac{2}{3}$

SECTION 6.6

1. 9 **3.** 12 **5.** 7 **7.** 6 **9.** 1 **11.** -6 **13.** 4 **15.** $-\frac{2}{3}$ **17.** There will be 20,000 voters voting in favor.

19. The two cities are 175 mi apart. **21.** The sales tax will be \$97.50 higher. **23.** The recommended area of the window is 40 ft^2. **25.** There are an estimated 75 elk in the preserve. **27.** The length of side AC is 6.7 cm. **29.** The height of triangle ABC is 2.9 m. **31.** The perimeter of triangle DEF is 22.5 ft. **33.** The area of triangle ABC is 48 m^2. **35.** The length of BC is 6.25 cm. **37.** The length of DA is 6 in. **39.** The length of OP is 13 cm. **41.** The width of the river is 35 m. **43.** The first person's share of the winnings is \$1.25 million. **45.** The basketball player made 210 shots.

SECTION 6.7

1. $y = -3x + 10$ **3.** $y = 4x - 3$ **5.** $y = -\frac{3}{2}x + 3$ **7.** $y = \frac{2}{5}x - 2$ **9.** $y = -\frac{2}{7}x + 2$ **11.** $y = -\frac{1}{3}x + 2$

13. $y = \frac{2}{9}x - 2$ **15.** $y = 2x + 5$ **17.** $x = -6y + 10$ **19.** $x = \frac{1}{2}y + 3$ **21.** $x = -\frac{3}{4}y + 3$ **23.** $x = 4y + 3$

25. $t = \frac{d}{r}$ **27.** $T = \frac{PV}{nR}$ **29.** $l = \frac{P - 2w}{2}$ **31.** $b_1 = \frac{2A - hb_2}{h}$ **33.** $h = \frac{3V}{A}$ **35.** $S = C - Rt$ **37.** $P = \frac{A}{1 + rt}$

39. $w = \frac{A}{S + 1}$ **41a.** $S = \frac{F + BV}{B}$ **b.** The selling price per unit to break even is \$180. **c.** The selling price per unit to break even is \$75.

SECTION 6.8

1. It would take 2 h with both sprinklers working. **3.** It would take both skiploaders 3 h working together. **5.** It would take 30 h with both computers working. **7.** It would take 30 min with both air conditioners working. **9.** It would take the second pipeline 90 min to fill the tank. **11.** It would take the apprentice 15 h to construct the wall. **13.** It would take the second technician 3 h to complete the wiring. **15.** It would have taken one welder 40 h to complete the welds. **17.** It would take one machine 28 h working alone. **19.** The rate through the congested traffic is 20 mph. **21.** The rate of the jogger is 8 mph, and the rate of the cyclist is 20 mph. **23.** The rate of the jet is 360 mph. **25.** The rate of the motorcycle is 48 mph. **27.** The rate of the car is 48 mph. **29.** The rate of the wind is 20 mph. **31.** The rate of the gulf current is 6 mph. **33.** The rate of the trucker for the first 330 mi is 55 mph. **35.** The less-experienced helper can complete the job in 30 h. **37.** The bus usually travels 60 mph.

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CHAPTER REVIEW

1. $\frac{b^3y}{10ax}$ [6.1C] **2.** $\frac{7x+22}{60x}$ [6.3B] **3.** $\frac{x-2}{3x-10}$ [6.4A] **4.** $-\frac{x+6}{x+3}$ [6.1A] **5.** $\frac{2x^4}{3y^7}$ [6.1A]

6. 62 [6.6A] **7.** $\frac{(3y-2)^2}{(y-1)(y-2)}$ [6.1C] **8.** $\frac{by^3}{6ax^2}$ [6.1B] **9.** $\frac{x}{x-7}$ [6.4A]

10. $\frac{3x^2-x}{(2x+3)(6x-1)(3x-1)}, \frac{24x^3-4x^2}{(2x+3)(6x-1)(3x-1)}$ [6.2B] **11.** $a=\frac{T-2bc}{2b+2c}$ [6.7A] **12.** 2 [6.5A]

13. $c=\frac{100m}{i}$ [6.7A] **14.** The equation has no solution. [6.5A] **15.** $\frac{1}{x^2}$ [6.1C] **16.** $\frac{2y-3}{5y-7}$ [6.3B]

17. $\frac{1}{x+3}$ [6.3A] **18.** $(5x-3)(2x-1)(4x-1)$ [6.2A] **19.** $y=-\frac{4}{9}x+2$ [6.7A] **20.** $\frac{8x+5}{3x-4}$ [6.1B]

21. 5 [6.5A] **22.** $\frac{3x-1}{x-5}$ [6.3B] **23.** 10 [6.6A] **24.** 12 [6.6A] **25.** The length of QO is 15 cm. [6.6C]

26. It would take 6 h to fill the pool using both hoses. [6.8A] **27.** The rate of the car is 45 mph. [6.8B]
28. The rate of the wind is 20 mph. [6.8B] **29.** The pitcher's ERA is 1.35. [6.6B]

CHAPTER TEST

1. $\frac{x^2-4x+5}{(x-2)(x+3)}$ [6.3B] **2.** -1 [6.6A] **3.** $\frac{(x-5)(2x-1)}{(x+3)(2x+5)}$ [6.1B] **4.** $\frac{2x^3}{3y^3}$ [6.1A] **5.** $t=\frac{d-s}{r}$ [6.7A]

6. 2 [6.5A] **7.** $-\frac{x+5}{x+1}$ [6.1A] **8.** $3(2x-1)(x+1)$ [6.2A] **9.** $\frac{5}{(2x-1)(3x+1)}$ [6.3B] **10.** $\frac{x+5}{x+4}$ [6.1C]

11. $\frac{x-3}{x-2}$ [6.4A] **12.** $\frac{3x+6}{x(x-2)(x+2)}, \frac{x^2}{x(x-2)(x+2)}$ [6.2B] **13.** $\frac{2}{x+5}$ [6.3A] **14.** $y=\frac{3}{8}x-2$ [6.7A]

15. The equation has no solution. [6.5A] **16.** $\frac{x+1}{x^3(x-2)}$ [6.1B] **17.** The length of CE is 12.8 ft. [6.6C] **18.** Two additional pounds of salt are required. [6.6B] **19.** It would take 4 h to fill the pool with both pipes turned on. [6.8A]
20. The rate of the wind is 20 mph. [6.8B] **21.** 54 sprinklers are required. [6.6B]

CUMULATIVE REVIEW

1. $\frac{31}{30}$ [1.5B] **2.** 21 [2.1A] **3.** $5x-2y$ [2.2A] **4.** $-8x+26$ [2.2D] **5.** $-\frac{9}{2}$ [3.2A] **6.** -12 [3.3B]

7. 10 [3.1D] **8.** a^3b^7 [4.2A] **9.** $a^2+ab-12b^2$ [4.3C] **10.** $3b^3-b+2$ [4.5A] **11.** x^2+2x+4 [4.5B]
12. $(4x+1)(3x-1)$ [5.3A] **13.** $(y-6)(y-1)$ [5.2A] **14.** $a(2a-3)(a+5)$ [5.3B]

15. $4(b-5)(b+5)$ [5.4B] **16.** -3 and $\frac{5}{2}$ [5.5A] **17.** $\frac{2x^3}{3y^5}$ [6.1A] **18.** $-\frac{x-2}{x+5}$ [6.1A] **19.** 1 [6.1C]

20. $\frac{3}{(2x-1)(x+1)}$ [6.3B] **21.** $\frac{x+3}{x+5}$ [6.4A] **22.** 4 [6.5A] **23.** 3 [6.6A] **24.** $t=\frac{f-v}{a}$ [6.7A]

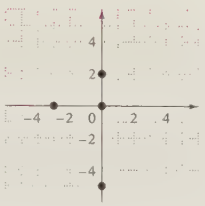
25. $5x-13=-8$; 1 [3.4A] **26.** The 120-gram alloy is 70% silver. [3.6B] **27.** The height is 6 in. and the base is 10 in. [5.5B] **28.** A policy of \$5000 would cost \$80. [6.6B] **29.** Working together, it would take the pipes 6 min to fill the tank. [6.8A] **30.** The rate of the current is 2 mph. [6.8B]

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Answers to Chapter 7 Odd-Numbered Exercises

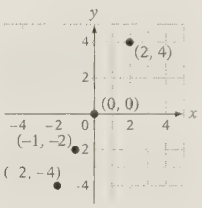
SECTION 7.1

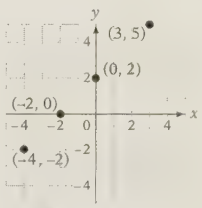
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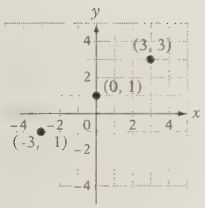
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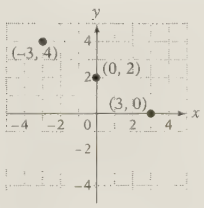
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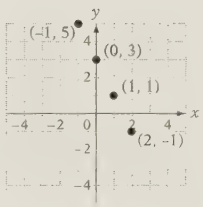
7. A is $(2, 3)$, B is $(4, 0)$, C is $(-4, 1)$, and D is $(-2, -2)$. **9.** A is $(-2, 5)$, B is $(3, 4)$, C is $(0, 0)$, and D is $(-3, -2)$.

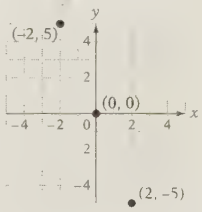
11a. The abscissa of point A is 2. The abscissa of point C is -4. **b.** The ordinate of point B is 1. The ordinate of point D is -3. **13.** yes **15.** no **17.** no **19.** no **21.** 

23. 

25. 

27. 

29. 

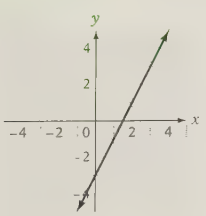
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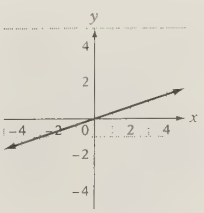
33. {(1, L), (0, L), (5, W), (8, W), (1, W), (3, W)}; no **35.** {(197, 125), (205, 122), (257, 498), (226, 108), (205, 150)}; no

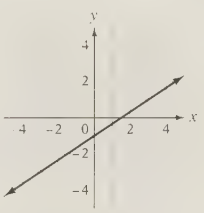
37. yes **39.** no **41.** yes **43.** 8 **45.** 9 **47.** 2 **49.** -1 **51.** 22 **53.** $-\frac{3}{2}$ **55.** -7

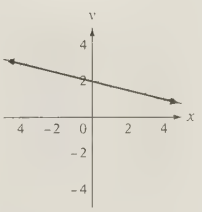
57. The ordered pairs are being graphed in reverse order. **59.** no

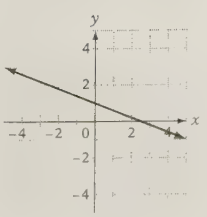
SECTION 7.2

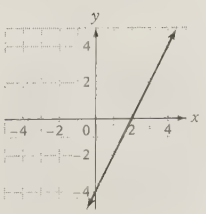
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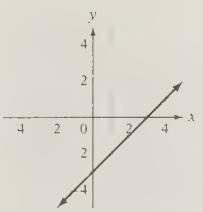
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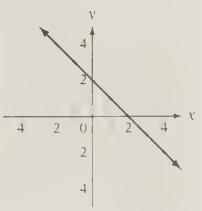
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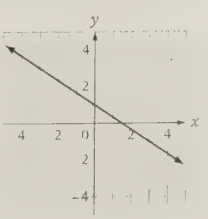
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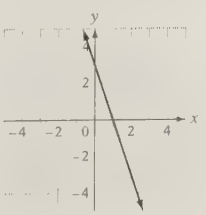
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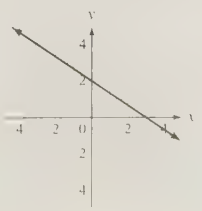
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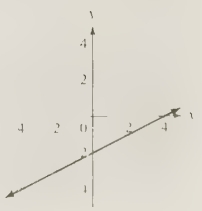
13. 

15. 

17. 

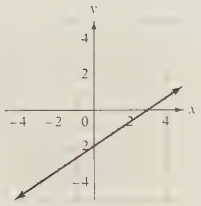
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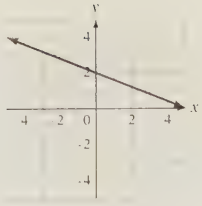
23. 

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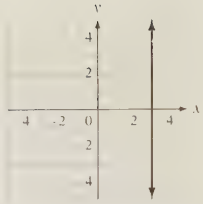
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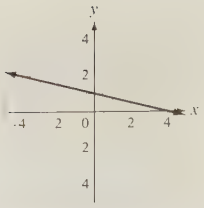
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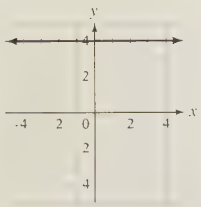
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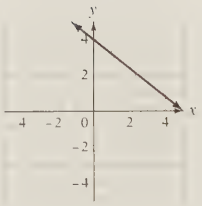
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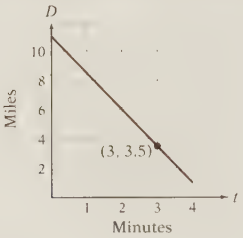
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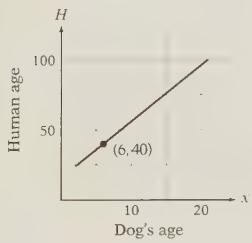
35.



37. After flying for 3 min, the helicopter is 3.5 mi away from the victims.



39. A dog 6 years old is equivalent in age to a human 40 years old. **41.** increases; 3 units; 3 units

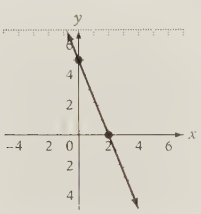


43. They are parallel lines.

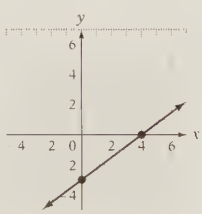
SECTION 7.3

1. (3, 0), (0, −3) **3.** (2, 0), (0, −6) **5.** (10, 0), (0, −2) **7.** (−4, 0), (0, 12) **9.** (0, 0), (0, 0) **11.** (6, 0), (0, 3)

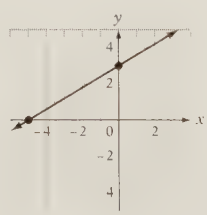
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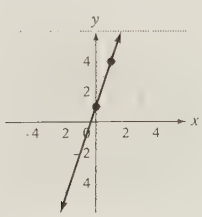
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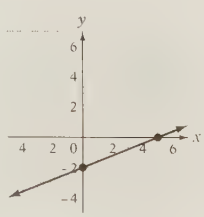
19. −2 **21.** $\frac{1}{3}$ **23.** $-\frac{5}{2}$ **25.** $-\frac{1}{2}$

27. −1 **29.** undefined **31.** zero **33.** $-\frac{1}{3}$ **35.** zero **37.** $m = 0.8$ After being connected, each minute of a transatlantic phone call costs \$.80. **39.** $m = -0.4$ The percent of the population that can afford a median-priced home has decreased by 0.4% per year.

41.



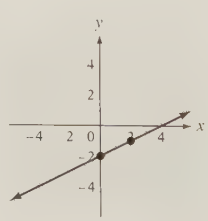
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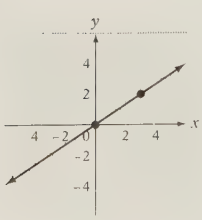
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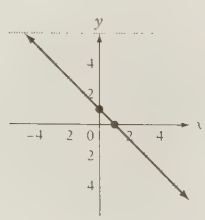
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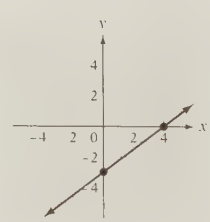
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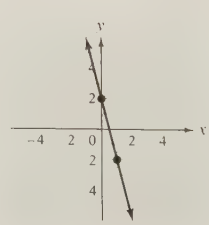
51.



53.



55.



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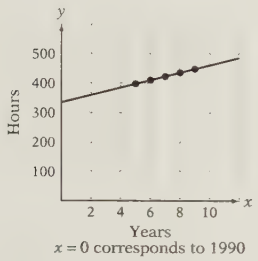
57. yes **59.** $6\% = 0.06 = \frac{6}{100}$. A 6% grade means that the average slope of the road is $\frac{6}{100}$.

SECTION 7.4

1. $y = 2x + 2$ **3.** $y = -3x - 1$ **5.** $y = \frac{1}{3}x$ **7.** $y = \frac{3}{4}x - 5$ **9.** $y = -\frac{3}{5}x$ **11.** $y = \frac{1}{4}x + \frac{5}{2}$ **13.** $y = 2x - 3$

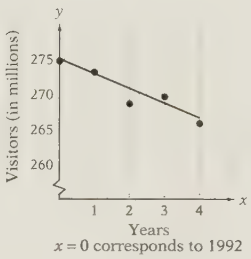
15. $y = -2x - 3$ **17.** $y = \frac{2}{3}x$ **19.** $y = \frac{1}{2}x + 2$ **21.** $y = -\frac{3}{4}x - 2$ **23.** $y = \frac{3}{4}x + \frac{5}{2}$

25. The number of hours of basic cable watched per person increases by 12.9 h per year.



$x = 0$ corresponds to 1990

27. The number of visitors is decreasing by 2.1 million per year.

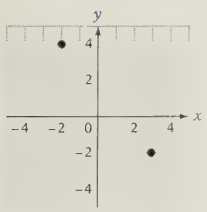


$x = 0$ corresponds to 1992

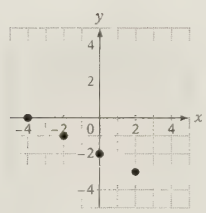
29. no **31.** yes **33.** $-\frac{3}{2}$ **35.** $n = -5$ **37.** $y = -\frac{2}{3}x + \frac{5}{3}$ **39.** The restriction is required to prevent division by 0.

CHAPTER REVIEW

1. a.



2.

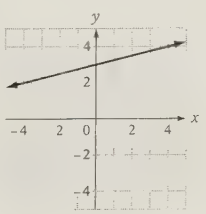


[7.1B]

3. $y = -\frac{8}{3}x + \frac{1}{3}$ [7.4B]

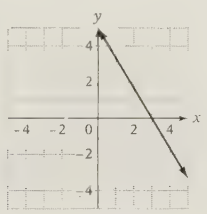
b. The abscissa of point A is -2.
c. The ordinate of point B is -4. [7.1A]

4. $y = -\frac{5}{2}x + 16$ [7.4A] **5.**



[7.2A]

6.

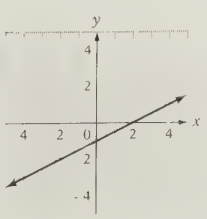


[7.2B]

7. $y = \frac{2}{3}x + 6$ [7.4A]

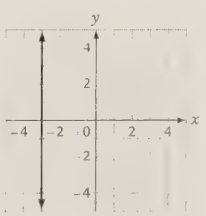
8. -1 [7.1D] **9.** $y = -\frac{2}{3}x + \frac{11}{3}$ [7.4B] **10.** yes [7.1C] **11.** $\frac{7}{11}$ [7.3B] **12.** $(8, 0), (0, -12)$ [7.3A]

13. 0 [7.3B] **14.**



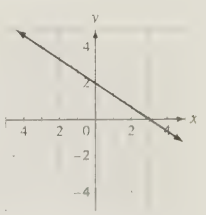
[7.3C]

15.



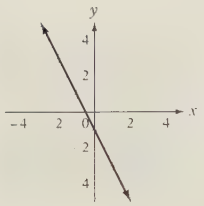
[7.2B]

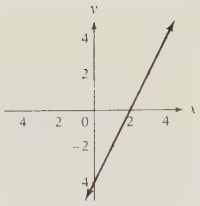
16.

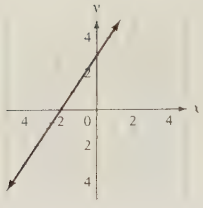


[7.3C]

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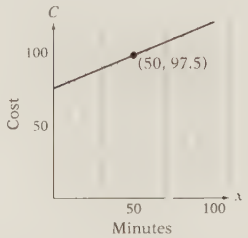
17.  [7.2A]

18.  [7.3C]

19.  [7.2B]

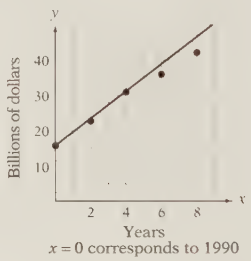
20. {(55, 95), (57,101), (53, 94), (57, 98), (60, 100), (61, 105), (58, 97), (54, 95)}; no [7.1C]

21. The cost of 50 min of access time for one month is \$97.50.



[7.2C]

22. The amount spent on health care is increasing \$3.85 billion per year.

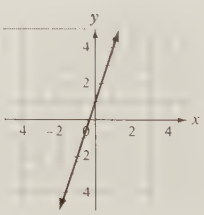


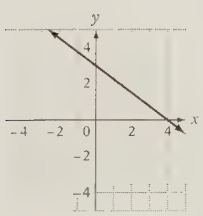
[7.4C]

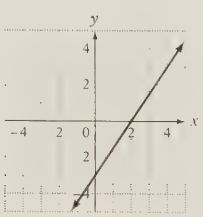
CHAPTER TEST

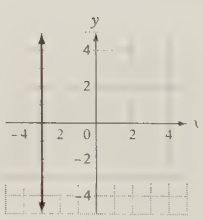
1. (3, −3) [7.1B] **2.**  [7.1B] **3.** yes [7.1C] **4.** $f(2) = 6$ [7.1D] **5.** $f(-1) = 3$ [7.1D]

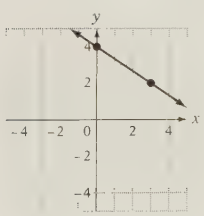
6. {(3.5, 25), (4.0, 30), (5.2, 45), (5.0, 38), (4.0, 42), (6.3, 12), (5.4, 34)}; no [7.1C]

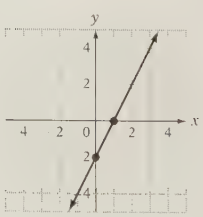
7.  [7.2A]

8.  [7.2A]

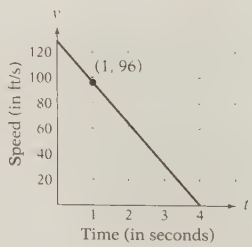
9.  [7.2B]

10.  [7.2B]

11.  [7.3C]

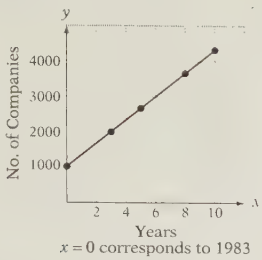
12.  [7.3C]

13. After 1 s, the speed of the ball is 96 ft/s. [7.2C]



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14. The slope of the line is 1.5 thousand. The tuition is increasing by \$1500 per year. [7.3B]

15. 

The number of mutual fund companies is increasing by 330 each year. [7.4C]

16. (2, 0), (0, −3) [7.3A] **17.** (−2, 0), (0, 1) [7.3A] **18.** 2 [7.3B] **19.** zero [7.3B] **20.** undefined [7.3B]

21. $-\frac{2}{3}$ [7.3B] **22.** $y = 3x - 1$ [7.4A] **23.** $y = \frac{2}{3}x + 3$ [7.4A] **24.** $y = -\frac{5}{8}x - \frac{7}{8}$ [7.4B]

25. $y = -\frac{2}{7}x - \frac{4}{7}$ [7.4B]

CUMULATIVE REVIEW

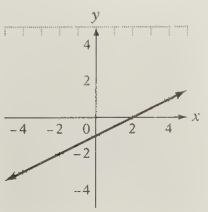
1. −12 [1.5B] **2.** $-\frac{5}{8}$ [2.1A] **3.** $f(-2) = -\frac{2}{3}$ [7.1D] **4.** $\frac{3}{2}$ [3.2A] **5.** $\frac{19}{18}$ [3.3B] **6.** $\frac{1}{15}$ [1.4B]

7. $-32x^8y^7$ [4.2B] **8.** $-3x^2$ [4.4A] **9.** $x + 3$ [4.5B] **10.** $5(x + 2)(x + 1)$ [5.2B] **11.** $(a + 2)(x + y)$ [5.1A]

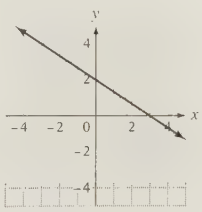
12. 4 and −2 [5.5A] **13.** $\frac{x^3(x + 3)}{y(x + 2)}$ [6.1B] **14.** $\frac{3}{x + 8}$ [6.3A] **15.** 2 [6.5A] **16.** $y = \frac{4}{5}x - 3$ [6.7A]

17. (−2, −5) [7.1B] **18.** zero [7.3B] **19.** $y = \frac{1}{2}x - 2$ [6.7A] **20.** $y = -3x + 2$ [7.4A]

21. $y = 2x + 2$ [7.4A] **22.** $y = \frac{2}{3}x - 3$ [7.4A] **23.** The sale price is \$62.30. [3.2B] **24.** The measure of the first angle is 46°, the measure of the second angle is 43°, and the measure of the third angle is 91°. [3.5C] **25.** The value of the home is \$110,000. [6.6B] **26.** It would take $3\frac{3}{4}$ h with both the electrician and the apprentice working. [6.8A]

27. 

[7.2A]

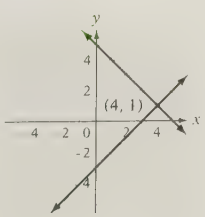
28. 

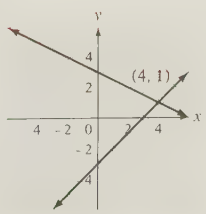
[7.3C]

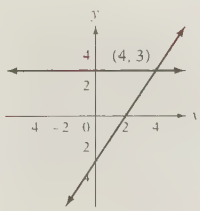
Answers to Chapter 8 Odd-Numbered Exercises

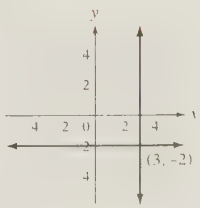
SECTION 8.1

1. yes **3.** yes **5.** no **7.** no **9.** no **11.** yes **13.** yes **15.** yes **17.** no

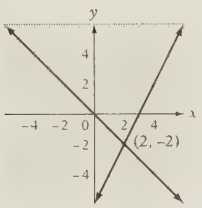
19. 

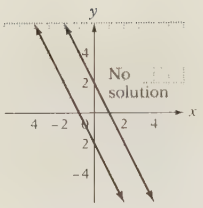
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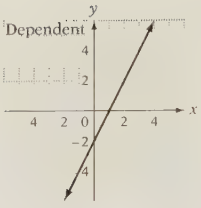
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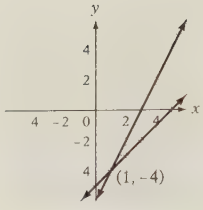
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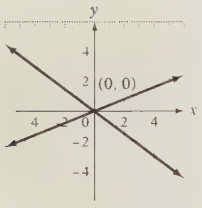
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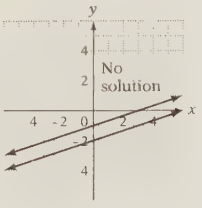
27. 

29. 

31. 

33. 

35. 

37. 

39a. sometimes true **b.** always true **c.** never true **d.** always true

41. The graphs are parallel if the system of equations are inconsistent. **43.** Answers will vary.

SECTION 8.2

1. (2, 1) **3.** (4, 1) **5.** (−1, 1) **7.** The system of equations is inconsistent and has no solution. **9.** The system of equations is inconsistent and has no solution. **11.** $\left(-\frac{3}{4}, -\frac{3}{4}\right)$ **13.** (1, 1) **15.** (2, 0) **17.** (1, −2) **19.** (0, 0) **21.** The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $2x - y = 2$. **23.** (−4, −2) **25.** (10, 31) **27.** (3, −10) **29.** (−22, −5) **31.** $1900 should be invested at 5% and $1600 should be invested at 7.5%. **33.** $3600 must be invested at 6% and $2400 must be invested at 9%. **35.** $4400 should be invested at 8% and $1600 should be invested at 11%. **37.** There was $21,000 invested at 6.5%. **39.** There was $12,000 invested at 8% and $8000 invested at 7%. **41.** There was $3750 invested in the second trust deed. **43.** 1 **45.** Answers will vary. **47.** The assertion is not correct. The solution is (0, 2). **49.** The research consultant invested $45,000. **51.** The simple interest earned was $400. The compounded monthly interest earned was $415. The compounded daily interest earned was $416.39.

SECTION 8.3

1. (5, −1) **3.** (1, 3) **5.** (1, 1) **7.** (3, −2) **9.** The system is dependent. The solutions are the ordered pairs that satisfy the equation $2x - y = 1$. **11.** (3, 1) **13.** The system is dependent. The solutions are the ordered pairs that satisfy the equation $2x - 3y = 1$. **15.** $\left(-\frac{13}{17}, -\frac{24}{17}\right)$ **17.** (2, 0) **19.** (0, 0) **21.** (5, −2) **23.** $\left(\frac{32}{19}, -\frac{9}{19}\right)$ **25.** (3, 4) **27.** (1, −1) **29.** The system is dependent. The solutions are the ordered pairs that satisfy the equation $5x + 15y = 20$. **31.** (3, 1) **33.** (−1, 2) **35.** (1, 1) **37.** Answers will vary. **39.** $A = 3; B = -1$

41a. All real numbers, $k \neq 1$ **b.** All real numbers, $k \neq \frac{3}{2}$ **c.** All real numbers, $k \neq 4$

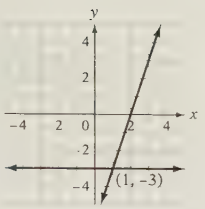
SECTION 8.4

1. The rate of the whale in calm water is 35 mph. The rate of the current is 5 mph. **3.** The rate of the rowing in calm water is 14 km/h. The rate of the current is 6 km/h. **5.** The rate of the jet in calm air is 525 mph. The rate of the wind is 35 mph. **7.** The rate of the helicopter in calm air is 225 mph. The rate of the wind is 45 mph. **9.** The rate of the canoeist in calm water is 6 mph. The rate of the current is 1 mph. **11.** The wheat flour cost $.65 per pound, and the rye flour cost $.70 per pound. **13.** The prime time was $.22 per minute, and the nonprime time was $.15 per minute. **15.** A submarine sandwich costs $4.25, and an order of french fries costs $.90. **17.** There are 1 nickel and 2 dimes or 3 nickels and 1 dime. **19.** There were 12.5 acres of good land and 87.5 acres of bad land.

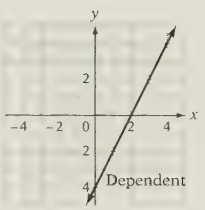
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CHAPTER REVIEW

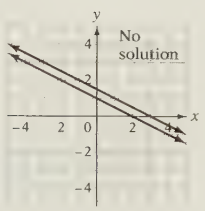
1. yes [8.1A] **2.** no [8.1A] **3.**



[8.1A] **4.**



[8.1A] **5.**

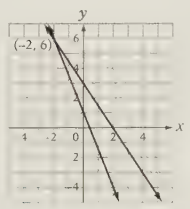


[8.1A]

6. $(-1, 1)$ [8.2A] **7.** $(1, 6)$ [8.2A] **8.** $(-3, 1)$ [8.3A] **9.** $\left(-\frac{5}{6}, \frac{1}{2}\right)$ [8.3A] **10.** no solution [8.2A] **11.** $(1, 6)$ [8.2A] **12.** $(1, -5)$ [8.3A] **13.** no solution [8.3A] **14.** The system is dependent. The solutions are the ordered pairs that satisfy the equation $y = -\frac{4}{3}x + 4$. [8.2A] **15.** $(-1, -3)$ [8.2A] **16.** The system is dependent. The solutions are the ordered pairs that satisfy the equation $3x + y = -2$. [8.3A] **17.** $\left(\frac{2}{3}, -\frac{1}{6}\right)$ [8.3A] **18.** The rate of the sculling team in calm water was 9 mph; the rate of the current was 3 mph. [8.4A] **19.** There were 1300 shares at \$6 per share and 200 shares at \$25 per share. [8.4B] **20.** The rate of the flight crew in calm air was 125 km/h; the rate of the wind was 15 km/h. [8.4A] **21.** The rate of the plane in calm air was 105 mph; the rate of the wind was 15 mph. [8.4A] **22.** The number of ads requiring \$.25 was 130; the number of ads requiring \$.45 was 60. [8.4B] **23.** There was \$7000 invested at 7% and \$5000 invested at 8.5%. [8.2B] **24.** There were 350 bushels of lentils and 200 bushels of corn. [8.4B] **25.** There was \$165,000 invested at 5.4% and \$135,000 invested at 6.6%. [8.2B]

CHAPTER TEST

1. yes [8.1A] **2.** yes [8.1A] **3.**



[8.1A] **4.** $(3, 1)$ [8.2A] **5.** $(1, -1)$ [8.2A]

6. $(2, -1)$ [8.2A] **7.** $\left(\frac{22}{7}, -\frac{5}{7}\right)$ [8.2A] **8.** The system is inconsistent and has no solution. [8.2A] **9.** $(2, 1)$ [8.3A] **10.** $\left(\frac{1}{2}, -1\right)$ [8.3A] **11.** The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $x + 2y = 8$. [8.3A] **12.** $(2, -1)$ [8.3A] **13.** $(1, -2)$ [8.3A] **14.** The rate of the plane in calm air is 100 mph. The rate of the wind is 20 mph. [8.4A] **15.** The price of a reserved-seat ticket is \$10. The price of a general-admission ticket is \$6. [8.4B] **16.** There was \$15,200 invested at 6.4% and \$12,800 invested at 7.6%. [8.2B]

CUMULATIVE REVIEW

1. $\frac{3}{2}$ [2.1A] **2.** $-\frac{3}{2}$ [3.1C] **3.** $f(2) = 7$ [7.1D] **4.** $-6a^3 + 13a^2 - 9a + 2$ [4.3B] **5.** $-2x^5y^2$ [4.4A]

6. $2b - 1 + \frac{1}{2b - 3}$ [4.5B] **7.** $-\frac{4y}{x^3}$ [4.4A] **8.** $4y^2(xy - 4)(xy + 4)$ [5.4B] **9.** 4 and -1 [5.5A]

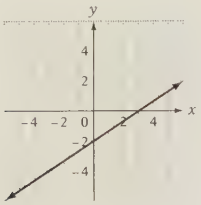
10. $x - 2$ [6.1C] **11.** $\frac{x^2 + 2}{(x - 1)(x + 2)}$ [6.3B] **12.** $\frac{x - 3}{x + 1}$ [6.4A] **13.** $-\frac{1}{5}$ [6.5A] **14.** $r = \frac{A - P}{Pt}$ [6.7A]

15. The x-intercept is $(6, 0)$, and the y-intercept is $(0, -4)$. [7.3A] **16.** $-\frac{7}{5}$ [7.3B] **17.** $y = -\frac{3}{2}x$ [7.4A]

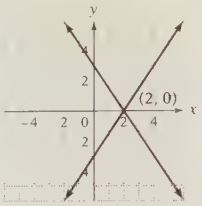
18. yes [8.1A] **19.** $(-6, 1)$ [8.2A] **20.** $(4, -3)$ [8.3A] **21.** The amount invested at 9.6% is \$3750. The

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amount invested at 7.2% is \$5000. [8.2B] **22.** The rate of the freight train is 48 mph. The rate of the passenger train is 56 mph. [3.6C] **23.** A side of the original square measures 8 in. [5.5B] **24.** The rate of the wind is 30 mph. [8.4A]

25. 

[7.2B]

26. 

[8.1A]

27. The rate of the boat in calm water is 14 mph. [8.4A] **28.** 35.3% of the mixture is sugar. [3.6A]

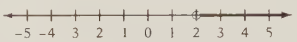
Answers to Chapter 9 Odd-Numbered Exercises

SECTION 9.1

1. A = {16, 17, 18, 19, 20, 21} **3.** A = {9, 11, 13, 15, 17} **5.** A = {b, c} **7.** $A \cup B = \{3, 4, 5, 6\}$

9. $A \cup B = \{-10, -9, -8, 8, 9, 10\}$ **11.** $A \cup B$ = {a, b, c, d, e, f} **13.** $A \cup B = \{1, 3, 7, 9, 11, 13\}$ **15.** $A \cap B = \{4, 5\}$

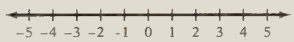
17. $A \cap B = \varnothing$ **19.** $A \cap B$ = {c, d, e} **21.** $\{x \mid x > -5, x \in$ negative integers$\}$ **23.** $\{x \mid x > 30, x \in$ integers$\}$

25. $\{x \mid x > 5, x \in$ even integers$\}$ **27.** $\{x \mid x > 8, x \in$ real numbers$\}$ **29.** 

31. 

33. 

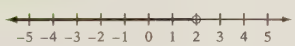
35. 

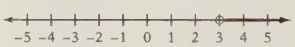
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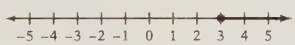
39. Answers will vary. **41a.** never true **b.** always true **c.** always true

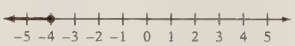
43a. yes **b.** yes

SECTION 9.2

1. $x < 2$ 

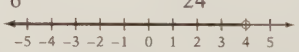
3. $x > 3$ 

5. $n \geq 3$ 

7. $x \leq -4$ 

9. $y \geq -9$ **11.** $x < 12$

13. $x \geq 5$ **15.** $x < -11$ **17.** $x \leq 10$ **19.** $x \geq -6$ **21.** $x > 2$ **23.** $d < -\frac{1}{6}$ **25.** $x \geq -\frac{31}{24}$ **27.** $x < \frac{5}{8}$

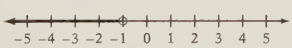
29. $x < \frac{5}{4}$ **31.** $x > \frac{5}{24}$ **33.** $x < -3.8$ **35.** $x \leq -1.2$ **37.** $x < 5.6$ **39.** 

$x < 4$

41. 

$y \geq 3$ **43.** 

$x \leq 1$

45. 

$x < -1$ **47.** 

$b < -4$ **49.** $y \leq 0$ **51.** $x > \frac{2}{7}$

53. $x \leq -\frac{5}{2}$ **55.** $x < 16$ **57.** $x \geq 16$ **59.** $x \geq -14$ **61.** $x \leq 21$ **63.** $x > 0$ **65.** $x \leq -\frac{12}{7}$ **67.** $x > \frac{2}{3}$

69. $x \leq \frac{2}{3}$ **71.** $x \leq 2.3$ **73.** $x < -3.2$ **75.** $x \leq 5$ **77.** The team must win at least 11 more games to qualify for the tournament. **79.** The organization must collect more than 440 lb of aluminum cans to collect the bonus.

81. The person needs at least 50 mg of vitamin C. **83.** The student cannot earn an A grade for the course.

85. $\{c \mid c > 0, c \in$ real numbers$\}$ **87.** $\{c \mid c \in$ real numbers$\}$ **89.** $\{c \mid c > 0, c \in$ real numbers$\}$ **91.** Answers will vary.

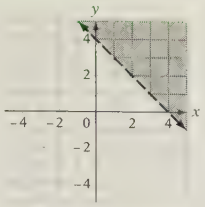
SECTION 9.3

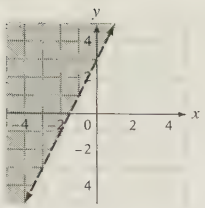
1. $x < 4$ **3.** $x < -4$ **5.** $x \geq 1$ **7.** $x < 5$ **9.** $x < 0$ **11.** $x < 20$ **13.** $y \leq \frac{5}{2}$ **15.** $x < \frac{25}{11}$ **17.** $n \leq \frac{11}{18}$

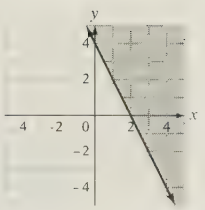
19. $x \geq 6$ **21.** The agent expects to sell at most \$20,000. **23.** To exceed the \$10 fee, a person must use the service more than 60 min. **25.** The maximum amount that can be added is 8 oz. **27.** The distance to the ski area is greater than 38 mi. **29.** {1, 2} **31.** {3, 4, 5} **33.** $\{x \mid x \in$ real numbers$\}$

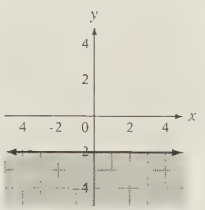
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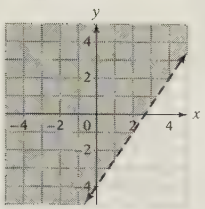
SECTION 9.4

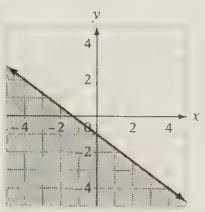
1. 

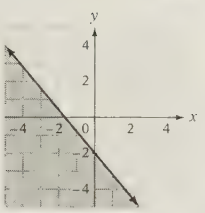
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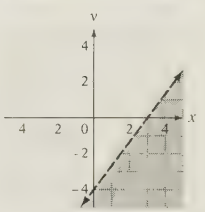
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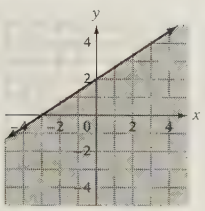
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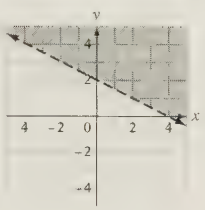
9. 

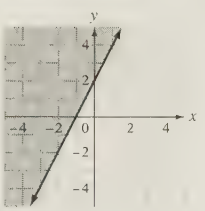
11. 

13. 

15. 

17. 

19. 

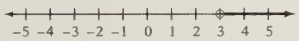
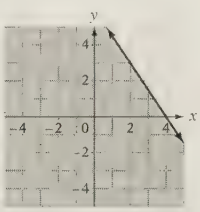
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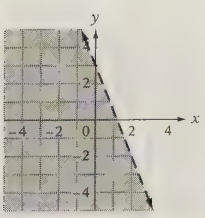
23. $x \le 3$

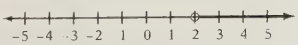
CHAPTER REVIEW

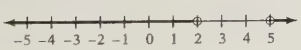
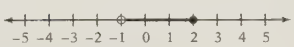
1. $x > 18$ [9.2A] **2.** $A \cap B = \varnothing$ [9.1A] **3.** $\{x \mid x > -8, x \in \text{odd integers}\}$ [9.1B]

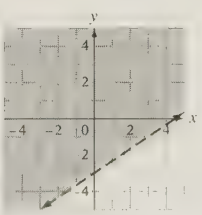
4. $A \cup B = \{2, 4, 6, 8, 10\}$ [9.1A] **5.** $A = \{1, 3, 5, 7\}$ [9.1A] **6.** $x \ge 4$ [9.3A]

7.  [9.1C] **8.** $x \ge -4$ [9.3A] **9.**  [9.4A]

10.  [9.4A] **11.** $\{x \mid x > 3, x \in \text{real numbers}\}$ [9.1B]

12. $x > 2$  [9.2A] **13.** $A \cap B = \{1, 5, 9\}$ [9.1A]

14.  [9.1C] **15.**  [9.1C] **16.** $x \ge -3$ [9.2B]

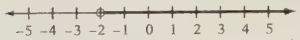
17. $x > -18$ [9.3A] **18.** $x < \frac{1}{2}$ [9.3A] **19.** $x < -\frac{8}{9}$ [9.2B] **20.**  [9.4A]

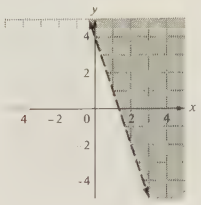
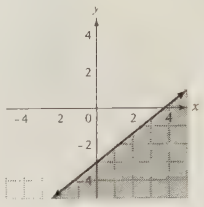
21. $x \ge 4$ [9.3A] **22.** Five or more residents make Florist B the more economical florists. [9.3B] **23.** The minimum length is 24 ft. [9.3B] **24.** The smallest integer that will satisfy the inequality is 32. [9.2C] **25.** The lowest score the student can receive is 72. [9.2C]

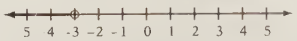
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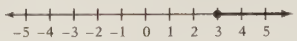
CHAPTER TEST

1.  [9.1C] **2.** $\{x|x < 50, x \in \text{positive integers}\}$ [9.1B] **3.** $A = \{4, 6, 8\}$ [9.1A]

4. $x \le -3$ [9.3A] **5.** $x > \frac{1}{8}$ [9.2A] **6.**  [9.1C] **7.** $x < -1$ [9.3A]

8. $\{x|x > -23, x \in \text{real numbers}\}$ [9.1B] **9.**  [9.4A] **10.**  [9.4A]

11. $A \cap B = \{12\}$ [9.1A] **12.** $x < -3$  [9.2A] **13.** $x \ge -\frac{40}{3}$ [9.2B]

14. $x < -\frac{22}{7}$ [9.3A] **15.** $x \ge 3$  [9.2B] **16.** $x \ge -4$ [9.3A] **17.** The child must grow at least 5 in. [9.2C] **18.** The maximum width is 11 ft. [9.3B] **19.** The allowable diameters are between 0.0389 in. and 0.0395 in. [9.2C] **20.** The broker processed less than $75,000. [9.3B]

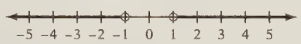
CUMULATIVE REVIEW

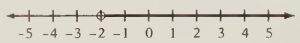
1. $40a - 28$ [2.2D] **2.** $\frac{1}{8}$ [3.2A] **3.** 4 [3.3B] **4.** $-12a^7b^4$ [4.2B] **5.** $-\frac{1}{b^4}$ [4.4A]

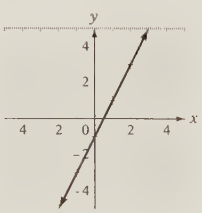
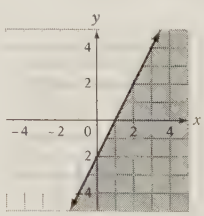
6. $4x - 2 - \frac{4}{4x - 1}$ [4.5B] **7.** $f(-1) = 0$ [7.1D] **8.** $3a^2(3x - 1)(3x + 1)$ [5.4B] **9.** $\frac{1}{x + 2}$ [6.1C]

10. $\frac{18a}{(2a - 3)(a + 3)}$ [6.3B] **11.** $-\frac{5}{9}$ [6.5A] **12.** $C = S + Rt$ [6.7A] **13.** $-\frac{7}{3}$ [7.3B]

14. $y = -\frac{3}{2}x - \frac{3}{2}$ [7.4A] **15.** $(4, 1)$ [8.2A] **16.** $(1, -4)$ [8.3A] **17.** $A \cup B = \{-10, -2, 0, 1, 2\}$ [9.1A]

18. $\{x|x < 48, x \in \text{real numbers}\}$ [9.1B] **19.**  [9.1C]

20. $x > -2$  [9.2B] **21.** $x < -15$ [9.2B] **22.** $x > 2$ [9.3A]

23. $\{x|x \le -26, x \in \text{integers}\}$ [9.2C] **24.** The maximum number of miles is 359. [9.3B] **25.** There are 5000 fish in the lake. [6.6B] **26.** The measure of the first angle is 65°, the measure of the second angle is 35°, and the measure of the third angle is 80°. [3.5C] **27.**  [7.2A] **28.**  [9.4A]

Answers to Chapter 10 Odd-Numbered Exercises

SECTION 10.1

1. 4 **3.** 7 **5.** $4\sqrt{2}$ **7.** $2\sqrt{2}$ **9.** $18\sqrt{2}$ **11.** $10\sqrt{10}$ **13.** $\sqrt{15}$ **15.** $\sqrt{29}$ **17.** $-54\sqrt{2}$ **19.** $3\sqrt{5}$ **21.** 0 **23.** $48\sqrt{2}$ **25.** 15.492 **27.** 16.971 **29.** 16 **31.** x^3 **33.** $y^7\sqrt{y}$ **35.** a^{10} **37.** x^2y^2 **39.** $2x^2$ **41.** $2x\sqrt{6}$ **43.** $2x^2\sqrt{15x}$ **45.** $7a^2b^4$ **47.** $3x^2y^3\sqrt{2xy}$ **49.** $2x^5y^3\sqrt{10xy}$ **51.** $4a^4b^5\sqrt{5a}$ **53.** $8ab\sqrt{b}$ **55.** x^3y **57.** $8a^2b^3\sqrt{5b}$ **59.** $6x^2y^3\sqrt{3y}$ **61.** $4x^3y\sqrt{2y}$ **63.** $5(a + 4)$ or $5a + 20$ **65.** $2(x + 2)^2$ or $2x^2 + 8x + 8$ **67.** $x + 2$ **69.** $y + 1$ **71.** No. For example, $\sqrt{4 + 9} \ne \sqrt{4} + \sqrt{9}$ **73.** Answers will vary. **75.** {3, 4, 5, 6, 7, 8, 9} **77.** No. $\sqrt{18}$ still contains a perfect-square factor. $6\sqrt{2}$ **79a.** 2.9623 **b.** 2.0305

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SECTION 10.2

1. $3\sqrt{2}$ **3.** $-\sqrt{7}$ **5.** $-11\sqrt{11}$ **7.** $10\sqrt{x}$ **9.** $-2\sqrt{y}$ **11.** $-11\sqrt{3b}$ **13.** $2x\sqrt{2}$ **15.** $-3a\sqrt{3a}$
17. $-5\sqrt{xy}$ **19.** $8\sqrt{5}$ **21.** $8\sqrt{2}$ **23.** $15\sqrt{2} - 10\sqrt{3}$ **25.** $\sqrt{x}$ **27.** $-12x\sqrt{3}$ **29.** $2xy\sqrt{x} - 3xy\sqrt{y}$
31. $-9x\sqrt{3x}$ **33.** $-13y^2\sqrt{2y}$ **35.** $4a^2b^2\sqrt{ab}$ **37.** $7\sqrt{2}$ **39.** $6\sqrt{x}$ **41.** $-3\sqrt{y}$ **43.** $-45\sqrt{2}$
45. $13\sqrt{3} - 12\sqrt{5}$ **47.** $32\sqrt{3} - 3\sqrt{11}$ **49.** $6\sqrt{x}$ **51.** $-34\sqrt{3x}$ **53.** $10a\sqrt{3b} + 10a\sqrt{5b}$ **55.** $-2xy\sqrt{3}$
57. $5\sqrt{2}$ **59.** Answers will vary. **61.** $-11\sqrt{2x + y}$ **63.** $13a\sqrt{a + 1}$

SECTION 10.3

1. 5 **3.** 6 **5.** x **7.** x^3y^2 **9.** $3ab^6\sqrt{2a}$ **11.** $12a^4b\sqrt{b}$ **13.** $2 - \sqrt{6}$ **15.** $x - \sqrt{xy}$ **17.** $5\sqrt{2} - \sqrt{5x}$
19. $4 - 2\sqrt{10}$ **21.** $x - 6\sqrt{x} + 9$ **23.** $3a - 3\sqrt{ab}$ **25.** $10abc$ **27.** $-2 + 2\sqrt{5}$ **29.** $16 + 10\sqrt{2}$
31. $6x + 10\sqrt{x} - 4$ **33.** $15x - 22y\sqrt{x} + 8y^2$ **35.** $x - y$ **37.** 3 **39.** 4 **41.** $6x^2$ **43.** $2x^2\sqrt{2y}$
45. $4x\sqrt{y}$ **47.** $\frac{2\sqrt{3x}}{3y}$ **49.** $\frac{-1 + \sqrt{2}}{2}$ **51.** $-\frac{5\sqrt{7} + 15}{2}$ **53.** $-\frac{5\sqrt{3} + 9}{2}$ **55.** $3 + \sqrt{6}$ **57.** $\frac{-12 + 3\sqrt{2}}{7}$
59. $\frac{14 - 9\sqrt{2}}{17}$ **61.** $\sqrt{15} + 2\sqrt{5}$ **63.** $\frac{x\sqrt{y} + y\sqrt{x}}{x - y}$ **65.** $4\sqrt{6} + 12$ **67.** The complete solution is available in the *Solutions Manual.*

SECTION 10.4

1. 25 **3.** 144 **5.** 5 **7.** no solution **9.** -1 **11.** 1 **13.** 15 **15.** $\frac{7}{3}$ **17.** 45 **19.** no solution
21. The pitcher's mound is less than halfway between home plate and second base. **23.** The periscope would have to be 16.67 ft above the surface of the water. **25.** The height of the TV screen is 21.6 in. **27.** The child is 18.75 ft from the center of the merry-go-round. **29.** The perimeter of the triangle is 30 units. **31.** Answers will vary. **33.** The area of the fountain is 244.78 ft^2.

CHAPTER REVIEW

1. 3 [10.3A] **2.** $9a^2\sqrt{2ab}$ [10.1B] **3.** 12 [10.1A] **4.** $3a\sqrt{2} + 2a\sqrt{3}$ [10.3A] **5.** $-x\sqrt{3} - x\sqrt{5}$ [10.3B]
6. $-6\sqrt{30}$ [10.1A] **7.** 20 [10.4A] **8.** $20\sqrt{3}$ [10.1A] **9.** $7x^2y^4$ [10.3B] **10.** no solution [10.4A]
11. $18a\sqrt{5b} + 5a\sqrt{b}$ [10.2A] **12.** $20\sqrt{10}$ [10.1A] **13.** $7x^2y\sqrt{15xy}$ [10.2A] **14.** $8y + 10\sqrt{5y} - 15$ [10.3A]
15. $26\sqrt{3x}$ [10.2A] **16.** no solution [10.4A] **17.** $\frac{8\sqrt{x} + 24}{x - 9}$ [10.3B] **18.** $36x^8y^5\sqrt{3xy}$ [10.1B]
19. $2y^4\sqrt{6}$ [10.1B] **20.** $\frac{3}{5}$ [10.4A] **21.** $-6x^3y^2\sqrt{2y}$ [10.2A] **22.** $\frac{16\sqrt{a}}{a}$ [10.3B] **23.** The explorer would weigh 144 lb on the surface of the earth. [10.4B] **24.** The depth of the water is 100 ft. [10.4B] **25.** The radius of the sharpest corner is 25 ft. [10.4B]

CHAPTER TEST

1. $11x^4y$ [10.1B] **2.** $6x^2y\sqrt{y}$ [10.3A] **3.** $-5\sqrt{2}$ [10.2A] **4.** $3\sqrt{5}$ [10.1A] **5.** 9 [10.3B]
6. 25 [10.4A] **7.** $4a^2b^5\sqrt{2ab}$ [10.1B] **8.** $7ab\sqrt{a}$ [10.3B] **9.** $\sqrt{3} + 1$ [10.3B] **10.** $4x^2y^2\sqrt{5y}$ [10.3A]
11. no solution [10.4A] **12.** $21\sqrt{2y} - 12\sqrt{2x}$ [10.2A] **13.** $6x^3y\sqrt{2x}$ [10.1B] **14.** $y + 2\sqrt{y} - 15$ [10.3A]
15. $-2xy\sqrt{3xy} - 3xy\sqrt{xy}$ [10.2A] **16.** $\frac{17 - 8\sqrt{5}}{31}$ [10.3B] **17.** $a - \sqrt{ab}$ [10.3A] **18.** $5\sqrt{3}$ [10.1A]
19. The length of the pendulum is 7.30 ft. [10.4B] **20.** The larger integer is 51. [10.4B]

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CUMULATIVE REVIEW

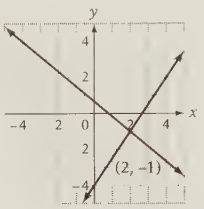
1. $-\frac{1}{12}$ [1.5B] **2.** $2x + 18$ [2.2D] **3.** $\frac{1}{13}$ [3.3B] **4.** $6x^5y^5$ [4.2A] **5.** $-2b^2 + 1 - \frac{1}{3b^2}$ [4.5A]

6. $f(-3) = 1$ [7.1D] **7.** $2a(a - 5)(a - 3)$ [5.2B] **8.** $\frac{1}{4(x + 1)}$ [6.1B] **9.** $\frac{x + 3}{x - 3}$ [6.3B] **10.** $\frac{5}{3}$ [6.5A]

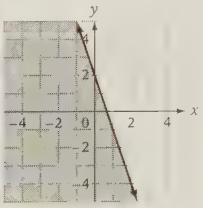
11. $y = \frac{1}{2}x - 2$ [7.4A] **12.** (1, 1) [8.2A] **13.** (3, −2) [8.3A] **14.** $x \le -\frac{9}{2}$ [9.3A] **15.** $6\sqrt{3}$ [10.1A]

16. $-4\sqrt{2}$ [10.2A] **17.** $4ab\sqrt{2ab} - 5ab\sqrt{ab}$ [10.2A] **18.** $14a^5b^2\sqrt{2a}$ [10.3A] **19.** $3\sqrt{2} - x\sqrt{3}$ [10.3A]

20. 8 [10.3B] **21.** $-6 - 3\sqrt{5}$ [10.3B] **22.** 6 [10.4A] **23.** The book costs \$24.50. [3.2B] **24.** 56 oz of pure water must be added. [3.6B] **25.** The numbers are 13 and 8. [5.5B] **26.** Working alone, it would take the small pipe 48 h to fill the tank. [6.8A] **27.** (2, −1)



[8.1A] **28.**



[9.4A]

29. The smaller integer is 40. [10.4B] **30.** The building is 400 ft high. [10.4B]

Answers to Chapter 11 Odd-Numbered Exercises

SECTION 11.1

1. −5 and 3 **3.** 1 and 3 **5.** −1 and −2 **7.** 3 **9.** 0 and $-\frac{2}{3}$ **11.** −2 and 5 **13.** $\frac{2}{3}$ and 1 **15.** −3 and $\frac{1}{3}$

17. $\frac{2}{3}$ **19.** $-\frac{1}{2}$ and $\frac{3}{2}$ **21.** $\frac{1}{2}$ **23.** −3 and 3 **25.** $-\frac{1}{2}$ and $\frac{1}{2}$ **27.** −3 and 5 **29.** 1 and 5 **31.** −6 and 6

33. −1 and 1 **35.** $-\frac{7}{2}$ and $\frac{7}{2}$ **37.** $-\frac{2}{3}$ and $\frac{2}{3}$ **39.** $-\frac{3}{4}$ and $\frac{3}{4}$ **41.** no real number solution **43.** $-2\sqrt{6}$ and $2\sqrt{6}$

45. −5 and 7 **47.** −7 and −3 **49.** $-\frac{1}{3}$ and $\frac{7}{3}$ **51.** $-\frac{2}{7}$ and $-\frac{12}{7}$ **53.** $4 + 2\sqrt{5}$ and $4 - 2\sqrt{5}$

55. no real number solution **57.** $-\frac{3}{4} + 2\sqrt{3}$ and $-\frac{3}{4} - 2\sqrt{3}$ **59.** $\sqrt{2}$ and $-\sqrt{2}$ **61.** 0 and $\frac{b}{a}$ **63.** 0 and 1

65. The annual percentage rate is 8%. **67.** yes

SECTION 11.2

1. 1 and −3 **3.** 8 and −2 **5.** 2 **7.** no real number solution **9.** −1 and −4 **11.** −8 and 1 **13.** $-2 + \sqrt{3}$ and $-2 - \sqrt{3}$ **15.** $-3 + \sqrt{14}$ and $-3 - \sqrt{14}$ **17.** $1 + \sqrt{2}$ and $1 - \sqrt{2}$ **19.** $\frac{-3 + \sqrt{13}}{2}$ and $\frac{-3 - \sqrt{13}}{2}$

21. 2 and 1 **23.** $\frac{-1 + \sqrt{13}}{2}$ and $\frac{-1 - \sqrt{13}}{2}$ **25.** $-5 + 4\sqrt{2}$ and $-5 - 4\sqrt{2}$ **27.** $\frac{3 + \sqrt{29}}{2}$ and $\frac{3 - \sqrt{29}}{2}$

29. $\frac{1 + \sqrt{17}}{2}$ and $\frac{1 - \sqrt{17}}{2}$ **31.** no real number solution **33.** 1 and $\frac{1}{2}$ **35.** −3 and $\frac{1}{2}$ **37.** 2 and $\frac{3}{2}$

39. 1 and $-\frac{1}{2}$ **41.** −2 and $\frac{1}{3}$ **43.** −2 and $-\frac{2}{3}$ **45.** $\frac{1}{2}$ and $-\frac{3}{2}$ **47.** $\frac{1}{3}$ and $-\frac{3}{2}$ **49.** $-\frac{1}{2}$ and $\frac{4}{3}$

51. $\frac{1 + \sqrt{2}}{2}$ and $\frac{1 - \sqrt{2}}{2}$ **53.** −4.193 and 1.193 **55.** 2.766 and −1.266 **57.** −1.651 and 0.151 **59.** Answers will vary. **61.** 7 **63.** $4 + \sqrt{7}$ and $4 - \sqrt{7}$ **65.** $6 + \sqrt{58}$ and $6 - \sqrt{58}$ **67.** 1 and 3 **69.** The ball will hit the basket 1.88 s after it is released.

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SECTION 11.3

1. -1 and 5 **3.** -3 and 5 **5.** -1 and 3 **7.** -5 and 1 **9.** $-\frac{1}{2}$ and 1 **11.** no real number solution

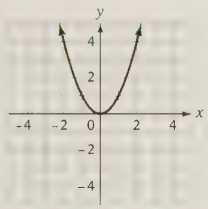
13. 0 and 1 **15.** $-\frac{3}{2}$ and $\frac{3}{2}$ **17.** $-\frac{5}{2}$ and $\frac{3}{2}$ **19.** no real number solution **21.** $1+\sqrt{6}$ and $1-\sqrt{6}$

23. $-3+\sqrt{10}$ and $-3-\sqrt{10}$ **25.** $2+\sqrt{13}$ and $2-\sqrt{13}$ **27.** $\frac{-1+\sqrt{2}}{3}$ and $\frac{-1-\sqrt{2}}{3}$ **29.** $-\frac{1}{2}$ **31.** no real number solution **33.** $\frac{-4+\sqrt{5}}{2}$ and $\frac{-4-\sqrt{5}}{2}$ **35.** $\frac{1+2\sqrt{3}}{2}$ and $\frac{1-2\sqrt{3}}{2}$ **37.** $\frac{-5+\sqrt{2}}{3}$ and $\frac{-5-\sqrt{2}}{3}$

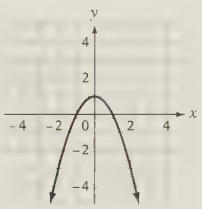
39. 5.690 and -3.690 **41.** 7.690 and -1.690 **43.** 4.589 and -1.089 **45.** -2.118 and 0.118 **47.** 1.105 and -0.905 **49.** Answers will vary. **51a.** false **b.** false **c.** false **d.** true **53.** -3 and -4 **55.** The equation has no solution. **57.** -1 and 11 **59.** After 2 h the distance between the planes is 1000 mi.

SECTION 11.4

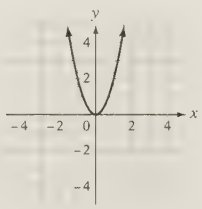
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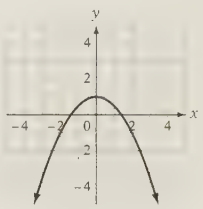
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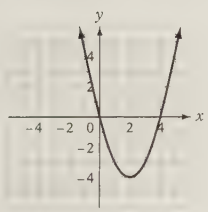
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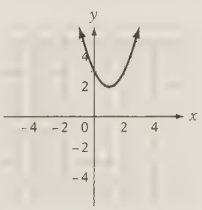
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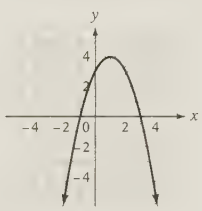
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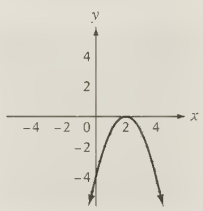
11.



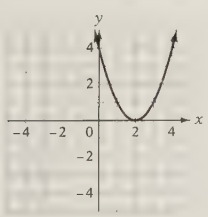
13.



15.



17.



19. down **21.** down **23.** $y = 3x^2 + 6x + 5$ **25.** $(-3, 0)$ and $(3, 0)$

27. $\left(-\frac{1}{2}, 0\right)$ and $(1, 0)$ **29.** $g(2) = -3$ **31.** $P(-3) = 38$

SECTION 11.5

1. The length is 4 m; the height is 10 m. **3.** The length of the batter's box is 6 ft; the width is 4 ft. **5.** The length of the pool is 100 ft; the width is 50 ft. **7.** The first computer alone would take 35 min. The second computer alone would take 14 min. **9.** The first engine alone would take 12 h. The second engine alone would take 6 h. **11.** The rate of the plane is 100 mph. **13.** The four consecutive integers are 3, 4, 5 and 6, or -6, -5, -4, and -3. **15.** The radius of the large pizza is 7 in. **17.** The maximum velocity of the car is 78 ft/s. **19.** The hang time is 5.5 s.

CHAPTER REVIEW

1. $\frac{4}{3}$ and $-\frac{7}{2}$ [11.1A] **2.** $-\frac{5}{7}$ and $\frac{5}{7}$ [11.1B] **3.** -6 and 4 [11.2A] **4.** -6 and 1 [11.3A]

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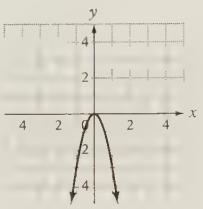
5. -4 and $\frac{3}{2}$ [11.2A] **6.** 2 and $\frac{5}{12}$ [11.1A] **7.** $-2 - 2\sqrt{6}$ and $-2 + 2\sqrt{6}$ [11.1B] **8.** 1 and $\frac{3}{2}$ [11.3A]

9. $-\frac{1}{3}$ and $-\frac{1}{2}$ [11.1A] **10.** no real number solution [11.1B] **11.** $2 - \sqrt{3}$ and $2 + \sqrt{3}$ [11.2A]

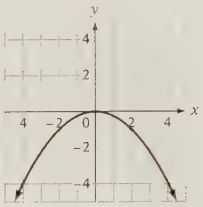
12. $\frac{3 - \sqrt{29}}{2}$ and $\frac{3 + \sqrt{29}}{2}$ [11.3A] **13.** no real number solution [11.2A] **14.** -7 and -10 [11.1A]

15. -1 and 2 [11.1B] **16.** $\frac{-4 - \sqrt{23}}{2}$ and $\frac{-4 + \sqrt{23}}{2}$ [11.2A] **17.** no real number solution [11.3A]

18. -2 and $-\frac{1}{2}$ [11.3A] **19.**

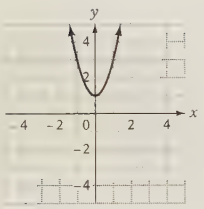


[11.4A] **20.**

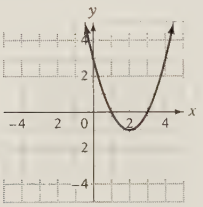


[11.4A]

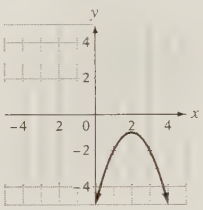
21.



[11.4A] **22.**



[11.4A] **23.**



[11.4A]

24. The rate of the hawk in calm air is 75 mph. [11.5A]

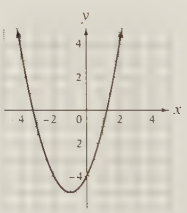
CHAPTER TEST

1. 6 and -1 [11.1A] **2.** -4 and $\frac{5}{3}$ [11.1A] **3.** 0 and 10 [11.1B] **4.** $-4 + 2\sqrt{5}$ and $-4 - 2\sqrt{5}$ [11.1B]

5. $-2 + 2\sqrt{5}$ and $-2 - 2\sqrt{5}$ [11.2A] **6.** $\frac{-3 + \sqrt{41}}{2}$ and $\frac{-3 - \sqrt{41}}{2}$ [11.2A] **7.** $\frac{3 + \sqrt{7}}{2}$ and $\frac{3 - \sqrt{7}}{2}$ [11.2A]

8. $\frac{-4 + \sqrt{22}}{2}$ and $\frac{-4 - \sqrt{22}}{2}$ [11.2A] **9.** $-2 + \sqrt{2}$ and $-2 - \sqrt{2}$ [11.3A] **10.** $\frac{3 + \sqrt{33}}{2}$ and $\frac{3 - \sqrt{33}}{2}$ [11.3A]

11. $-\frac{1}{2}$ and 3 [11.3A] **12.** $\frac{1 + \sqrt{13}}{6}$ and $\frac{1 - \sqrt{13}}{6}$ [11.3A] **13.**



[11.4A]

14. The length is 8 ft. The width is 5 ft. [11.5A] **15.** The rate of the boat in calm water is 11 mph. [11.5A]

CUMULATIVE REVIEW

1. $-28x + 27$ [2.2D] **2.** $\frac{3}{2}$ [3.1C] **3.** 3 [3.3B] **4.** $-12a^8b^4$ [4.2B] **5.** $x + 2 - \frac{4}{x - 2}$ [4.5B]

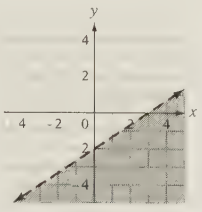
6. $x(3x - 4)(x + 2)$ [5.3B] **7.** $\frac{9x^2(x - 2)^2}{(2x - 3)^2}$ [6.1C] **8.** $\frac{x + 2}{2(x + 1)}$ [6.3B] **9.** $\frac{x - 4}{2x + 5}$ [6.4A] **10.** The x-intercept is $(3, 0)$, and the y-intercept is $(0, -4)$. [7.3A] **11.** $y = -\frac{4}{3}x - 2$ [7.4A] **12.** $(2, 1)$ [8.2A] **13.** $(2, -2)$ [8.3A]

14. $x > \frac{1}{9}$ [9.3A] **15.** $a - 2$ [10.3A] **16.** $6ab\sqrt{a}$ [10.3B] **17.** $\frac{-6 + 5\sqrt{3}}{13}$ [10.3B] **18.** 5 [10.4A]

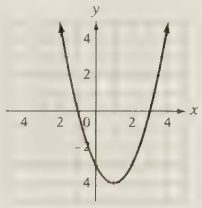
19. $\frac{5}{2}$ and $\frac{1}{3}$ [11.1A] **20.** $5 + 3\sqrt{2}$ and $5 - 3\sqrt{2}$ [11.1B] **21.** $\frac{-7 + \sqrt{13}}{6}$ and $\frac{-7 - \sqrt{13}}{6}$ [11.2A]

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22. 2 and $-\frac{1}{2}$ [11.3A] **23.** The cost of the mixture is \$2.25 per pound. [3.6A] **24.** 250 additional shares are required. [6.6B] **25.** The rate of the plane in still air is 200 mph. The rate of the wind is 40 mph. [8.4A] **26.** The student must receive a score of 77 or above. [9.2C] **27.** The integer is -5 or 5. [11.5A] **28.** The rate for the last 8 mi was 4 mph. [11.5A] **29.**



[9.4A] **30.**



[11.4A]

FINAL EXAM

1. -3 [1.1B] **2.** -6 [1.2B] **3.** -256 [1.5A] **4.** -11 [1.5B] **5.** $-\frac{15}{2}$ [2.1A] **6.** $9x + 6y$ [2.2A]

7. $6z$ [2.2B] **8.** $16x - 52$ [2.2D] **9.** -50 [3.1C] **10.** -3 [3.3B] **11.** 12.5% [1.4B] **12.** 15.2 [3.1D]

13. $-3x^2 - 3x + 8$ [4.1B] **14.** $81x^4y^{12}$ [4.2B] **15.** $6x^3 + 7x^2 - 7x - 6$ [4.3B] **16.** $-\frac{x^4y}{2}$ [4.4A]

17. $\frac{3x}{y} - 4x^2 - \frac{5}{x}$ [4.5A] **18.** $5x - 12 + \frac{23}{x + 2}$ [4.5B] **19.** $\frac{4y^6}{x^6}$ [4.4A] **20.** $\frac{3}{4}$ [7.1D]

21. $(x - 6)(x + 1)$ [5.2A] **22.** $(3x + 2)(2x - 3)$ [5.3A] **23.** $4x(2x - 1)(x - 3)$ [5.3B]

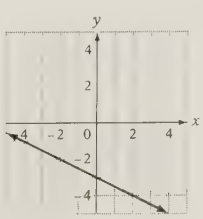
24. $(5x - 4)(5x + 4)$ [5.4A] **25.** $2(a + 3)(4 - x)$ [5.1B] **26.** $3y(5 - 2x)(5 + 2x)$ [5.4B] **27.** $\frac{1}{2}$ and 3 [5.5A]

28. $\frac{2(x + 1)}{x - 1}$ [6.1B] **29.** $\frac{-3x^2 + x - 25}{(2x - 5)(x + 3)}$ [6.3B] **30.** $\frac{x^2 - 2x}{x - 1}$ [6.4A] **31.** 2 [6.5A] **32.** $a = b$ [6.7A]

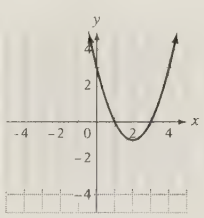
33. $\frac{2}{3}$ [7.3B] **34.** $y = -\frac{2}{3}x - 2$ [7.4A] **35.** (6, 17) [8.2A] **36.** (2, -1) [8.3A] **37.** $x \leq -3$ [9.2A]

38. $y \geq \frac{5}{2}$ [9.3A] **39.** $7x^3$ [10.1B] **40.** $38\sqrt{3a}$ [10.2A] **41.** $\sqrt{15} + 2\sqrt{3}$ [10.3B] **42.** 2 [10.4A]

43. -1 and $\frac{4}{3}$ [11.1A] **44.** $\frac{1 + \sqrt{5}}{4}$ and $\frac{1 - \sqrt{5}}{4}$ [11.3A] **45.** $2x + 3(x - 2)$, $5x - 6$ [2.3B] **46.** The original value is \$3000. [3.1D] **47.** The markup rate is 65%. [3.2B] **48.** An additional \$6000 must be invested. [8.2B]
49. The mixture costs \$3 per pound. [3.6A] **50.** The percent concentration of the acid in the mixture is 36%. [3.6B]
51. In the first hour, the plane flew 215 km. [3.6C] **52.** The measures of the angles are 50°, 60°, and 70°. [3.5C]
53. The integer is 4 or -4. [11.5A] **54.** The length is 10 m. The width is 5 m. [5.5B] **55.** Sixteen ounces of dye are required for 120 oz of base paint. [6.6B] **56.** Working together, it would take 0.6 h to prepare the dinner. [6.8A]
57. The rate of the boat in calm water is 15 mph. The rate of the current is 5 mph. [8.4A] **58.** The rate of the wind is 25 mph. [11.5A] **59.**



[7.3C] **60.**



[11.4A]

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Glossary

abscissa The first number in an ordered pair. It measures a horizontal distance and is also called the first coordinate. (Sec. 7.1)

absolute value of a number The distance of the number from zero on the number line. (Sec. 1.1)

acute angle An angle whose measure is between 0° and 90°. (Sec. 3.5)

addend In addition, a number being added. (Sec. 1.2)

addition The process of finding the total of two numbers. (Sec. 1.2)

addition method An algebraic method of finding an exact solution of a system of linear equations. (Sec. 8.3)

additive inverses Numbers that are the same distance from zero on the number line, but on opposite sides; also called opposites. (Sec. 1.1, 2.2)

adjacent angles Two angles that share a common side. (Sec. 3.5)

alternate exterior angles Two angles that are on opposite sides of the transversal and outside the parallel lines. (Sec. 3.5)

alternate interior angles Two angles that are on opposite sides of the transversal and between the parallel lines. (Sec. 3.5)

analytic geometry Geometry in which a coordinate system is used to study the relationships between variables. (Sec. 7.1)

arithmetic mean of values Average determined by calculating the sum of the values and then dividing that result by the number of values. (Sec. 1.3)

axes The two number lines that form a rectangular coordinate system; also called coordinate axes. (Sec. 7.1)

base In exponential notation, the factor that is multiplied the number of times shown by the exponent. (Sec. 1.5)

basic percent equation Percent times base equals amount. (Sec. 3.1)

binomial A polynomial of two terms. (Sec. 4.1)

clearing denominators Removing denominators from an equation that contains fractions by multiplying each side of the equation by the LCM of the denominators. (Sec. 6.5)

coefficient The number part of a variable term. (Sec. 2.1)

combining like terms Using the Distributive Property to add the coefficients of like variable terms; adding like terms of a variable expression. (Sec. 2.2)

complementary angles Two angles whose sum is 90°. (Sec. 3.5)

completing the square Adding to a binomial the constant term that makes it a perfect-square trinomial. (Sec. 11.2)

complex fraction A fraction whose numerator or denominator contains one or more fractions. (Sec. 6.4)

conjugates Binomial expressions that differ only in the sign of a term. The expressions $a + b$ and $a - b$ are conjugates. (Sec. 10.3)

consecutive even integers Even integers that follow one another in order. (Sec. 3.4)

consecutive integers Integers that follow one another in order. (Sec. 3.4)

consecutive odd integers Odd integers that follow one another in order. (Sec. 3.4)

constant term A term that includes no variable part; also called a constant. (Sec. 2.1)

coordinate axes The two number lines that form a rectangular coordinate system; also simply called axes. (Sec. 7.1)

coordinates of a point The numbers in an ordered pair that is associated with a point. (Sec. 7.1)

corresponding angles Two angles that are on the same side of the transversal and are both acute angles or are both obtuse angles. (Sec. 3.5)

cost The price that a business pays for a product. (Sec. 3.2)

decimal notation Notation in which a number consists of a whole-number part, a decimal point, and a decimal part. (Sec. 1.4)

degree A unit used to measure angles. (Sec. 3.5)

degree of a polynomial in one variable The largest exponent that appears on the variable. (Sec. 4.1)

dependent system of equations A system of equations that has an infinite number of solutions. (Sec. 8.1)

dependent variable In a function, the variable whose value depends on the value of another variable known as the independent variable. (Sec. 7.1)

descending order The terms of a polynomial in one variable arranged so that the exponents on the variable decrease from left to right. The polynomial $9x^5 - 2x^4 + 7x^3 + x^2 - 8x + 1$ is in descending order. (Sec. 4.1)

discount The amount by which a retailer reduces the regular price of a product for a promotional sale. (Sec. 3.2)

discount rate The percent of the regular price that the discount represents. (Sec. 3.2)

domain The set of first coordinates of the ordered pairs in a relation. (Sec. 7.1)

element of a set One of the objects in a set. (Sec. 1.1, 9.1)

empty set The set that contains no elements; also called the null set. (Sec. 9.1)

equation A statement of the equality of two mathematical expressions. (Sec. 3.1)

equilateral triangle A triangle in which all three sides are of equal length. (Sec. 3.5)

equivalent equations Equations that have the same solution. (Sec. 3.1)

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evaluating a function Replacing x in $f(x)$ with some value and then simplifying the numerical expression that results. (Sec. 7.1)

evaluating a variable expression Replacing each variable by its value and then simplifying the resulting numerical expression. (Sec. 2.1)

even integer An integer that is divisible by 2. (Sec. 3.4)

exponent In exponential notation, the elevated number that indicates how many times the base occurs in the multiplication. (Sec. 1.5)

exponential form The expression 2^5 is in exponential form. Compare *factored form*. (Sec. 1.5)

factor In multiplication, a number being multiplied. (Sec. 1.3)

factor a polynomial To write the polynomial as a product of other polynomials. (Sec. 5.1)

factor a trinomial of the form $x^2 + bx + c$ To express the trinomial as the product of two binomials. (Sec. 5.1)

factored form The expression $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is in factored form. Compare *exponential form*. (Sec. 1.5)

first coordinate The first number in an ordered pair. It measures a horizontal distance and is also called the abscissa. (Sec. 7.1)

FOIL A method of finding the product of two binomials; the letters stand for First, Outer, Inner, and Last. (Sec. 4.3)

formula A literal equation that states rules about measurements. (Sec. 6.7)

function A relation in which no two ordered pairs that have the same first coordinate have different second coordinates. (Sec. 7.1)

functional notation A function designated by $f(x)$, which is the value of the function at x. (Sec. 7.1)

graph of a relation The graph of the ordered pairs that belong to the relation. (Sec. 7.1)

graph of an equation in two variables A graph of the ordered-pair solutions of the equation. (Sec. 7.2)

graph of an integer A heavy dot directly above that number on the number line. (Sec. 1.1)

graph of an ordered pair The dot drawn at the coordinates of the point in the plane. (Sec. 7.1)

graphing a point in the plane Placing a dot at the location given by the ordered pair; also called plotting a point in the plane. (Sec. 7.1)

greater than A number a is greater than another number b, written $a > b$, if a is to the right of b on the number line. (Sec. 1.1)

greater than or equal to The symbol $\geq$ means "is greater than or equal to." (Sec. 1.1)

greatest common factor The greatest common factor (GCF) of two or more integers is the greatest integer that is a factor of all the integers. The greatest common factor of two or more monomials is the product of the GCF of the coefficients and the common variable factors. (Sec. 5.1)

half-plane The solution set of an inequality in two variables. (Sec. 9.4)

hypotenuse In a right triangle, the side opposite the 90° angle. (Sec. 10.4)

inconsistent system of equations A system of equations that has no solution. (Sec. 8.1)

independent system of equations A system of equations that has one solution. (Sec. 8.1)

independent variable In a function, the variable that varies independently and whose value determines the value of the dependent variable. (Sec. 7.1)

inequality An expression that contains the symbol $>$, $<$, $\geq$ (is greater than or equal to), or $\leq$ (is less than or equal to). (Sec. 9.1)

integers The numbers $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$. (Sec 1.1)

intersection of sets A and B The set that contains the elements that are common to both A and B. (Sec. 9.1)

irrational number The decimal representation of an irrational number never repeats or terminates and can only be approximated. (Sec. 1.4, 10.1)

isosceles triangle A triangle that has two equal angles and two equal sides. (Sec. 3.5)

least common denominator The smallest number that is a multiple of each denominator in question. (Sec. 1.4)

least common multiple (LCM) The LCM of two or more numbers is the smallest number that contains the prime factorization of each number. The LCM of two or more polynomials is the polynomial of least degree that contains the factors of each polynomial. (Sec. 1.4, 6.2)

less than A number a is less than another number b, written $a < b$, if a is to the left of b on the number line. (Sec. 1.1)

less than or equal to The symbol $\leq$ means "is less than or equal to". (Sec. 1.1)

like terms Terms of a variable expression that have the same variable part. (Sec. 2.2)

line of best fit A line drawn to approximate data that are graphed as points in a coordinate system. (Sec. 7.4)

linear equation in two variables An equation of the form $y = mx + b$, where m and b are constants; also called a linear function. (Sec. 7.2)

linear function An equation of the form $y = mx + b$, where m and b are constants; also called a linear equation in two variables. (Sec. 7.2)

linear model A first-degree equation that is used to describe a relationship between quantities. (Sec. 7.4)

literal equation An equation that contains more than one variable. (Sec. 6.7)

markdown The amount by which a retailer reduces the regular price of a product for a promotional sale. (Sec. 3.2)

markup The difference between selling price and cost. (Sec. 3.2)

markup rate The percent of retailer's cost that the markup represents. (Sec. 3.2)

monomial A number, a variable, or a product of numbers and variables; a polynomial of one term. (Sec. 4.1)

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multiplicative inverse of a number The reciprocal of a number. (Sec. 2.2)

natural numbers The numbers 1, 2, 3, ... (Sec. 1.1)

negative integers The numbers ..., −4, −3, −2, −1. (Sec. 1.1)

negative slope A property of a line that slants downward to the right. (Sec. 7.3)

nonfactorable over the integers A polynomial that does not factor using only integers. (Sec. 5.2)

null set The set that contains no elements; also called the empty set. (Sec. 9.1)

numerical coefficient The number part of a variable term. When the numerical coefficient is 1 or −1, the 1 is usually not written. (Sec. 2.1)

obtuse angle An angle whose measure is between 90° and 180°. (Sec. 3.5)

odd integer An integer that is not divisible by 2. (Sec. 3.4)

opposite of a polynomial The polynomial created when the sign of each term of the original polynomial is changed. (Sec. 4.1)

opposites Numbers that are the same distance from zero on the number line, but on opposite sides; also called additive inverses. (Sec. 1.1)

ordered pair Pair of numbers, such as (a, b) that can be used to identify a point in the plane determined by the axes of a rectangular coordinate system. (Sec. 7.1)

Order of Operations Agreement A set of rules that tell us in what order to perform the operations that occur in a numerical expression. (Sec. 1.6)

ordinate The second number in an ordered pair. It measures a vertical distance and is also called the second coordinate. (Sec. 7.1)

origin The point of intersection of the two coordinate axes that form a rectangular coordinate system. (Sec. 7.1)

parabola The graph of a quadratic equation in two variables. (Sec. 11.4)

parallel lines Lines that never meet; the distance between them is always the same. (Sec. 3.5)

percent Parts of 100. (Sec. 1.4)

perfect square The square of an integer (Sec. 10.1)

perfect-square trinomial A trinomial that is a product of a binomial and itself. (Sec. 5.4)

perimeter The distance around a plane geometric figure. (Sec. 3.5)

perpendicular lines Intersecting lines that form right angles. (Sec. 3.5)

plane Flat surface determined by the intersection of two lines. (Sec. 7.1)

plotting a point in the plane Placing a dot at the location given by the ordered pair; also called graphing a point in the plane. (Sec. 7.1)

point-slope formula If (x_1, y_1) is a point on a line with slope m, then $y - y_1 = m(x - x_1)$. (Sec. 7.4)

polynomial A variable expression in which the terms are monomials. (Sec. 4.1)

positive integers The integers, 1, 2, 3, 4, ... (Sec. 1.1)

positive slope A property of a line that slants upward to the right. (Sec. 7.3)

prime polynomial A polynomial that is nonfactorable over the integers. (Sec. 5.2)

principal square root The positive square root of a number. (Sec. 10.1)

product In multiplication, the result of multiplying two numbers. (Sec. 1.3)

proportion An equation that states the equality of two ratios or rates. (Sec. 6.6)

Pythagorean Theorem The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. (Sec. 10.4)

quadrant One of the four regions into which the two axes of a rectangular coordinate system divide the plane. (Sec. 7.1)

quadratic equation An equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants and a is not equal to zero; also called a second-degree equation. (Sec. 5.5, 11.1)

quadratic equation in two variables An equation of the form $y = ax^2 + bx + c$, where a is not equal to zero. (Sec. 11.4)

quadratic function A quadratic function is given by $f(x) = ax^2 + bx + c$, where a is not equal to zero. (Sec. 11.4)

radical The symbol $\sqrt{\ }$, which is used to indicate the positive, or principal, square root of a number. (Sec. 10.1)

radical equation An equation that contains a variable expression in a radicand. (Sec. 10.4)

radicand In a radical expression, the expression under the radical sign. (Sec. 10.1)

range The set of second coordinates of the ordered pairs in a relation. (Sec. 7.1)

rate The quotient of two quantities that have different units. (Sec. 6.6)

rate of work That part of a task that is completed in one unit of time. (Sec. 6.8)

ratio The quotient of two quantities that have the same unit. (Sec. 6.6)

rational expression A fraction in which the numerator or denominator is a polynomial. (Sec. 6.1)

rational number A number that can be written in the form a/b where a and b are integers and b is not equal to zero. (Sec. 1.4)

rationalizing the denominator The procedure used to remove a radical from the denominator of a fraction. (Sec. 10.3)

real numbers The rational numbers and the irrational numbers. (Sec. 1.4)

reciprocal Interchanging the numerator and denominator of a rational number yields that number's reciprocal. (Sec. 2.2, 6.1)

rectangular coordinate system System formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. (Sec. 7.1)

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relation Any set of ordered pairs. (Sec. 7.1)

repeating decimal Decimal that is formed when dividing the numerator of its fractional counterpart by the denominator results in a decimal part wherein a block of digits repeats infinitely. (Sec. 1.4)

right angle An angle whose measure is 90 degrees. (Sec. 3.5)

roster method Method of writing a set by enclosing a list of the elements in braces. (Sec. 1.1, 9.1)

scatter diagram A graph of collected data as points in a coordinate system. (Sec. 7.4)

scientific notation Notation in which each number is expressed as the product of two factors, one a number between 1 and 10 and the other a power of ten. (Sec. 4.4)

second coordinate The second number in an ordered pair. It measures a vertical distance and is also called the ordinate. (Sec. 7.1)

second-degree equation An equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants and a is not equal to zero; also called a quadratic equation. (Sec. 11.1)

selling price The price for which a business sells a product to a customer. (Sec. 3.2)

set A collection of objects. (Sec. 1.1, 9.1)

set-builder notation A method of designating a set that makes use of a variable and a certain property that only elements of that set possess. (Sec. 9.1)

similar objects Similar objects have the same shape but not necessarily the same size. (Sec. 6.6)

simplest form of a fraction A fraction is in simplest form when there are no common factors in the numerator and the denominator. (Sec. 1.4)

simplest form of a rational expression A rational expression is in simplest form when the numerator and denominator have no common factors. (Sec. 6.1)

slope The measure of the slant of a line. The symbol for slope is m. (Sec. 7.3)

slope-intercept form The slope-intercept form of an equation of a straight line is $y = mx + b$. (Sec. 7.3)

solution of a system of equations in two variables An ordered pair that is a solution of each equation of the system. (Sec. 8.1)

solution of an equation A number that, when substituted for the variable, results in a true equation. (Sec. 3.1)

solution of an equation in two variables An ordered pair whose coordinates make the equation a true statement. (Sec. 7.1)

solution set of an inequality A set of numbers, each element of which, when substituted for the variable, results in a true inequality. (Sec. 9.2)

solving an equation Finding a solution of the equation. (Sec. 3.1)

square root A square root of a positive number x is a number a for which $a^2 = x$. (Sec. 10.1)

standard form A quadratic equation is in standard form when the polynomial is in descending order and equal to zero. $ax^2 + bx + c = 0$ is in standard form. (Sec. 5.5, 11.1)

straight angle An angle whose measure is 180 degrees. (Sec. 3.5)

substitution method An algebraic method of finding an exact solution of a system of equations. (Sec. 8.2)

sum In addition, the total of two or more numbers. (Sec. 1.2)

supplementary angles Two angles whose sum is 180°. (Sec. 3.5)

system of equations Equations that are considered together. (Sec. 8.1)

terminating decimal Decimal that is formed when dividing the numerator of its fractional counterpart by the denominator results in a remainder of zero. (Sec. 1.4)

terms of a variable expression The addends of the expression. (Sec. 2.1)

transversal A line intersecting two other lines at two different points. (Sec. 3.5)

triangle A three-sided closed figure. (Sec. 3.5)

trinomial A polynomial of three terms. (Sec. 4.1)

undefined slope A property of a vertical line. (Sec. 7.3)

uniform motion The motion of a moving object whose speed and direction do not change. (Sec. 3.6, 6.8)

union of sets *A* and *B* The set that contains all the elements of A and all the elements of B. (Sec. 9.1)

value of the variable The number assigned to the variable. (Sec. 2.1)

variable A letter of the alphabet used to stand for a number that is unknown or that can change. (Sec. 1.1)

variable expression An expression that contains one or more variables. (Sec. 2.1)

variable part In a variable term, the variable or variables and their exponents. (Sec. 2.1)

variable term A term composed of a numerical coefficient and a variable part. (Sec. 2.1)

vertical angles Two angles that are on opposite sides of the intersection of two lines. (Sec. 3.5)

***x*-coordinate** The abscissa in an xy-coordinate system. (Sec. 7.1)

***x*-intercept** The point at which a graph crosses the x-axis. (Sec. 7.3)

***xy*-coordinate system** A rectangular coordinate system in which the horizontal axis is labeled x and the vertical axis is labeled y. (Sec. 7.1)

***y*-coordinate** The ordinate in an xy-coordinate system. (Sec. 7.1)

***y*-intercept** The point at which a graph crosses the y-axis. (Sec. 7.3)

zero slope A property of a horizontal line. (Sec. 7.3)

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MATH STUDY SKILLS WORKBOOK

Your Guide to
Reducing Test Anxiety
and Improving Study Strategies

Paul D. Nolting, Ph.D.
Learning Specialist

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Preface

Many students have difficulty in mathematics; in fact, math is considered one of the most difficult subjects in college. Although other students may not find it as difficult, it still takes them many homework hours to understand it. Most students do not know that 25 percent of their math grade is based on affective characteristics, such as math study skills. However, most students are never taught math study skills. In any case, these students need to pass their math course to graduate.

What kind of assistance do most students want? Students want tips and procedures they can use to help them improve their math grades. The math study suggestions, however, have to be based on research and be statistically proven to improve student learning and grades. ***Math Study Skills Workbook*** is based on ***Winning at Math: Your Guide to Learning Mathematics Through Successful Study Skills*** (1997), which is the only math study skills text that can boast statistical evidence demonstrating an improvement in students' ability to learn math and make better grades. Learning and using these study skills techniques will improve your grades.

Math Study Skills Workbook is designed to supplement your math course or study skills course or to be used as part of a math anxiety workshop. The workbook is designed for independent work or to be supplemented by lectures. To gain the most benefit, the workbook needs to be completed by midterm.

This workbook is designed to enhance learning by teaching math learning skills in small "chunks." After each chapter section, you are required to recall the most important information by writing it down. The writing exercises are especially designed for you to personalize learning techniques, such as the ten steps to better test taking. Each chapter review is designed to reinforce your learning and to

give you the opportunity to immediately select and use the best learning strategy from that chapter. Lecture Notes pages have been added, where possible, to allow you to include your notes in this workbook.

> **Remember:** To get the most benefit, you must complete this workbook and be using these learning techniques by midterm.

Chapter 1

What You Need to Know to Study Math

Mathematics (math) courses are not like other courses. Because they are different, they require different study procedures. Passing most of your other courses requires only that you read, understand, and recall the subject material. To pass math, however, an extra step is required: You must use the information you have learned to correctly solve math problems.

Learning general study skills can help you pass most of your courses. But special math study skills are needed to help you learn more and get better grades in math. In this chapter, you will find out

- why learning math is different from learning other subjects,
- what the differences are between high school and college math, and
- why your first math test is very important.

Why learning math is different from learning other subjects

In a math course, you must be able to do three things:

1. *Understand* the material
2. *Process* the material
3. *Apply* what you have learned to correctly solve a problem.

Of these three tasks, applying what you have learned to correctly solve a problem is the hardest.

Examples: Political science courses require that you learn about politics and public service. But your instructor will not make you run for governor to pass the course. Psychology courses require you to understand the concepts of different psychology theories. But you will not have to help a patient overcome depression to pass the course. In math, however, you must be able to correctly solve problems to pass the course.

Sequential Learning Pattern

Another reason learning math is different from learning other subjects is that it follows a *sequential learning pattern*, which simply means that the material learned on one day is used the next day and the next day, and so forth. This building-block approach to learning math is the reason it is difficult to catch up when you fall behind. *All* building blocks must be included to be successful in learning math.

You can compare learning math to building a house. Like a house, which must be built foundation first, walls second, and roof last, math must be learned in a specific order. Just as you cannot build a house roof first, you cannot learn to solve complex problems without first learning to solve simple ones.

Example: In a history class, if you study Chapters 1 and 2, do not understand Chapter 3, and end up studying and having a test on Chapter 4, you *could* pass. Understanding Chapter 4 in history is not totally based on comprehending Chapters 1, 2, and 3. To succeed in math, however, each previous chapter has to be completely understood before you can continue to the next chapter.

Sequential learning affects your ability to study for math tests, as well. If you study Chapter 1 and understand it, study Chapter 2 and understand it, and study Chapter 3 and *do not* understand it, then when you study for a test on Chapter 4, you are not going to understand it either, and you probably will not do well on the test.

Remember: To learn the new math material for the test on Chapter 5, you must first go back and learn the material in Chapter 4. This means you will have to go back and learn Chapter 4 while learning Chapter 5. (The best of us can fall behind under these circumstances.) However, if you do not understand the material in Chapter 4, you will not understand the material in Chapter 5 either, and you will fail the test on Chapter 5. This is why the sequential learning of math concepts if so important.

The sequential learning pattern is also affected by

- your previous math course grade,
- your math placement tests scores, and
- the time elapsed since your last math course.

Sequential learning is influenced by how much math knowledge you have at the beginning of your course. Students who forgot or never acquired the necessary skills from their previous math course will have difficulty with their current math course. If you do not remember what you learned in your last math course, you will have to relearn the math concepts from the previous course as well as the new material for the current course. In most other courses, such as the humanities, previous course knowledge is not required. However, in math you must remember what the last course taught you so that you are prepared for the current course. Measuring previous course knowledge will be explained in Chapter 2, "How to Discover Your Math-Learning Strengths and Weaknesses."

Sequential learning is also affected by your math placement test scores. If you barely scored high enough to be placed into a math course, then you will have math-learning gaps. Learning problems will occur when new math material is based on one of your learning gaps. The age of the placement test score also affects sequential learning. Placement test scores are designed to measure your *current* math knowledge and are to be used immediately.

Sequential learning is interrupted if math courses are taken irregularly. Math courses are designed to be taken one after another. By taking math courses each semester, without semester breaks in between courses, you are less likely to forget the concepts required for the next course.

Math as a Foreign Language

Another helpful technique for studying math is to consider it a foreign language. Looking at math as a foreign language can improve your study procedures. If you do not practice a foreign language, what happens? You forget it. If you do not practice math, what happens? You are likely to forget it, too. Students who excel in a foreign language must practice it *at least* every other day. The same study skills apply to math, because it is considered a foreign language.

Like a foreign language, math has unfamiliar vocabulary words or terms that must be put in sentences called *equations*. Understanding and solving a math equation is similar to speaking and understanding a sentence in a foreign language.

Example: Math sentences use symbols (which stand for spoken words), such as

= (for which you *say*, "equal"),
- (for which you *say*, "less"), and
a (for which you *say*, "unknown").

Learning *how* to speak math as a language is the key to math success. Currently, most universities consider computer and statistics (a form of math) courses as foreign languages. Some universities have now gone so far as to actually classify math as a foreign language.

Math as a Skill Subject

Math is a *skill subject*, which means you have to practice actively the skills involved to master it. Learning math is similar to learning to play a sport, learning to play a musical instrument, or learning auto mechanics skills. You can listen and watch your coach or instructor all day, but unless you *practice* those skills yourself, you will not learn.

Examples: In basketball, the way to improve your free throw is to *see and understand* the correct shooting form and then *practice* the shots yourself. Practicing the shots improves your free-throwing percentage. However, if you simply listen to your coach describe the correct form and see him demonstrate it, but you do not practice the correct form yourself, you will not increase your shooting percentage.

Suppose you want to play the piano, and you hired the best available piano instructor. You sit next to your instructor on the piano bench and watch the instructor demonstrate beginning piano-playing techniques. You see and understand how to place your hand on the keys and play. But what does it take to learn to play the piano? You have to place your hands on the keys and *practice*.

continued

Math works the same way. You can go to class, listen to your instructor, watch the instructor demonstrate skills, and understand everything that is said (and feel that you are quite capable of solving the problems). However, if you leave the class *and do not practice* – by working and successfully solving the problems – you will not learn math.

Many of your other courses can be learned by methods other than practicing. In social studies, for example, information can be learned by listening to your instructor, taking good notes, and participating in class discussions. Many students mistakenly believe that math can be learned the same way.

Remember: Math is different. If you want to learn math, *you must practice.* Practice not only means doing your homework but also means spending the time it takes to understand the reasons for each step in each problem.

A Bad Math "Attitude"

Students' attitudes about learning math are different from their attitudes about learning their other subjects. Many students who have had bad past experiences with math do not like math and have a bad attitude about learning it. In fact, some students actually *hate* math, even though these same students have positive experiences with their other subjects and look forward to going to class.

Society, as a whole, reinforces students' negative attitudes about math. It has become socially acceptable not to do well in math. This negative attitude has become evident even in popular comic strips, such as *Peanuts*. The underlying message is that math should be feared and hated and that it is all right not to learn math.

This "popular" attitude toward math may reinforce your belief that it is all right to fail math. Such a belief is constantly being reinforced by others.

The bad math attitude is not a major problem,

however. Many students who hate math pass it anyway, just as many students who hate history still pass it. The major problem concerning the bad math attitude is how you use this attitude. If a bad math attitude leads to poor class attendance, poor concentration, and poor study skills, then you have a bad math attitude *problem*.

Remember: Passing math is your goal, regardless of your attitude.



1. How does a sequential learning pattern affect math learning?

Give two examples:

Example 1:

Example 2:

2. List two examples of how learning math is similar to learning a foreign language.

Example 1:

Example 2:

3. How is math similar to a skills subject?

4. List three ways a bad math attitude could affect your mathematics learning.

Way 1:

Way 2:

Way 3:

The differences between high school and college math

Math, as a college-level course, is almost two to three times as difficult as high school-level math courses. There are many reasons for the increased difficulty: course class time allowance, the amount of material covered in a course, the length of a course, and the college grading system.

The first important difference between high school and college math courses is the length of time devoted to instruction each week. Most college math instruction, for the fall and spring semesters, has been cut to three hours per week; high school math instruction is provided five hours per week. Additionally, college courses cover twice the material in the same time frame as do high school courses. What is learned in one year of high school math is learned in one semester (four months) of college math.

Simply put, in college math courses you are receiving less instructional time per week and covering twice the ground per course as you were in high school math courses. The responsibility for learning in college is the student's. As a result, most of your learning (and *practicing*) will have to occur outside of the college classroom.

College Summer Semester Versus Fall or Spring Semester and the Difference Between Night and Day

College math courses taught during summer semesters are more difficult than those taught during fall or spring. Further, math taught during night courses is more difficult than math taught during day courses.

Students attending a six-week summer math session must learn the information – and master the skills – two and a half times as fast as students attending regular, full-semester math sessions. Though you receive the same amount of instructional classroom time, there is less time to understand and *practice the skills* between class sessions.

Summer classes are usually two hours per day, four days per week (nighttime summer classes are four hours per night, two nights per week).

> **Example:** If you do not understand the lecture on Monday, then you have only Monday night to learn the material before progressing to more difficult material on Tuesday. During a night course, you have to learn and understand the material before the break; after the break, you will move on to the more difficult material – *that night.*

Because math is a sequential learning experience, where every building block must be understood before proceeding to the next block, you can quickly fall behind, and you may never catch up. In fact, some students become lost during the first half of a math lecture and never understand the rest of the lecture (this can happen during just one session of night class). This is called "kamikaze" math because most students do not survive summer courses.

If you *must* take a summer math course, take a 10- or 12-week *daytime* session so that you will have more time to process the material between classes.

Course Grading System

The grading system for math is different in college than in high school.

> **Example:** While in high school, if you make a D or borderline D/F, the teacher more than likely will give you a D, and you may continue to the next course. However, in some college math courses, a D is not considered a passing grade, or if a D is made, the course will not count toward graduation.

College instructors are more likely to give the grade of N (no grade), W (withdrawal from class), or F for barely knowing the material. This is because the instructor knows that you will be unable to pass the next course if you barely know the current one.

Most colleges require students to pass two col-

lege-level algebra courses to graduate. In most high schools, you may graduate by passing one to three math courses. In some college-degree programs, you may even have to take four math courses and make a C in all of them to graduate.

The grading systems for math courses are very precise compared to the grading systems for humanities courses.

> **Example:** In a math course, if you have a 79 percent average and you need 80 percent to get a B, you will get a C in the course. But if you make a 79 percent in English class, you may be able to talk to your instructor and do extra credit work to earn a B.

Because math is an exact science and not as subjective as English, do not expect your math instructor to let you do extra work to earn a better grade. In college, there usually is not a grade given for "daily work," as is often offered in high school. In fact, *your test scores may be the only grades that will count toward your final grade.* Therefore, you should not assume that you will be able to "make up" for a bad test score.

The Ordering of College Math Courses

College math courses should be taken, *in order,* from the fall semester to the spring semester. If at all possible, avoid taking math courses from the spring to fall semesters. There is less time between the fall and spring semester for you to forget the information. During the summer break, you are more likely to forget important concepts required for the next course and therefore experience greater difficulty.



1. Compare the amount of college class time to high school class time.

_____

_____

_____

_____

2. How do college summer math courses differ from courses offered in the spring or fall?

_____

_____

_____

_____

_____

_____

3. How can the order of taking math courses affect your learning?

_____

_____

_____

_____

_____

_____

Why your first math test is very important

Making a high grade on the first major math test is more important than making a high grade on the first major test in other subjects. The first major math test is the easiest and, most often, the one the student is least prepared for.

Students often feel that the first major math test is mainly a review and that they can make a B or C without much study. These students are overlooking an excellent opportunity to make an A on the easiest major math test of the semester. (Do not forget that this test counts the same as the more difficult remaining math tests.)

At the end of the semester, these students sometimes do not pass the math course or do not make an A because of their first major test grade. In other words, the first math test score was not high enough to "pull up" a low test score on one of the remaining major tests.

Studying hard for the first major math test and obtaining an A offers you several advantages:

- A high score on the first test can compensate for a low score on a more difficult fourth or fifth math test. All major tests have equal value in the final grade calculations.

- A high score on the first test can provide assurance that you have learned the basic math skills required to pass the course. This means you will not have to spend time relearning the misunderstood material covered on the first major test while learning new material for the next test.

- A high score on the first test can motivate you to do well. Improved motivation can cause you to increase your math study time, which will allow you to master the material.

- A high score on the first test can improve your confidence for higher test scores. With more confidence, you are more likely to work harder on the difficult math homework assignments, which will increase your chances of doing well in the course.



1. Why is your first math test supposed to be the easiest?

_____
_____
_____
_____
_____
_____

2. Give three reasons why your first math test is so important.

Reason 1: _____
_____
_____
_____
_____
_____
_____

Reason 2: _____
_____
_____
_____
_____
_____
_____

Reason 3: _____
_____
_____
_____
_____
_____
_____

Lecture Notes

Name: _____ Date: _____



Chapter 1 Review



1. In a math course, you must be able to do three things. You must _____ the material, _____ the material, and _____ what you have learned to solve the problem.

2. Math requires _____ _____, which means one concept builds on the next concept.

3. Placement test scores are designed to measure your _____ math knowledge and are to be used _____.

4. Like a foreign language, math has unfamiliar vocabulary words or terms that must be put in sentences called _____.

5. Learning how to speak math as a _____ _____ is one key to math success.

6. Keeping a _____ attitude about math will help you study more efficiently.

7. College math courses are _____ to _____ times as difficult as high school math courses.

8. Math course are even more difficult than other courses because a grade of _____ or better is usually required to take the next course.

9. The math grading is exact; in many cases, you cannot do _____ _____ to improve your grade.

10. College courses should be taken _____ _____, from the fall semester to the spring semester.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

How can you immediately use it?

_____
_____
_____
_____
_____
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_____
_____

Chapter 2

How to Discover Your Math-Learning Strengths and Weaknesses

Math-learning strengths and weaknesses affect students' grades. You need to understand your strengths and weaknesses to improve your math-learning skills.

Just as a mechanic does a diagnostic test on a car before repairing it, you need to do your own testing to learn what you need to improve upon. You do not want the mechanic to charge you for something that does not need repairing, nor do you want to work on learning areas that do not need improvement. You want to identify learning areas you need to improve or better understand.

In this chapter, you will learn

- how what you know about math affects your grades,
- how quality of math instruction affects your grades,
- how affective student characteristics affect your math grades,
- how to determine your learning style,
- how to develop a math-learning profile of your strengths and weaknesses, and
- how to improve your math knowledge.

Areas of math strengths and weaknesses include math knowledge, level of test anxiety, study skills, study attitudes, motivation, and test-taking skills. Before we start identifying your math strengths and weaknesses, you need to understand what contributes to math academic success.

Dr. Benjamin Bloom, a famous researcher in the field of educational learning, discovered that your IQ (intelligence) and your cognitive entry skills account for 50 percent of your course grade (see Figure 1 on the next page). Quality of instruction represents 25 percent of your course grade, while affective student characteristics reflect the remaining 25 percent of your grade.

- *Intelligence* may be considered, for our purpose, how fast you can learn or relearn math concepts.

- *Cognitive entry skills* refer to how much math you knew before entering your current math course.

- *Quality of instruction* is concerned with the effectiveness of math instructors when presenting material to students in the classroom and math lab. This effectiveness depends on the course textbook, curriculum, teaching style, extra teaching aids (videos, audiocassettes), and other assistance.

- *Affective student characteristics* are characteristics you possess that affect your course grades – excluding how much math you knew before entering your math course. Some of these affective characteristics are anxiety, study skills, study attitudes, self-concept, motivation, and test-taking skills.

How what you know about math affects your grades

Poor math knowledge can cause low grades. A student placed in a math course that requires a more extensive math background than the student possesses will probably fail that course. Without the correct math background, you will fall behind and never catch up.

Placement Tests and Previous Course Grades

The math you need to know to enroll in a particular math course can be measured by a placement test (ACT, SAT) or by the grade received in the prerequisite math course. Some students are incorrectly placed in math courses by placement tests.

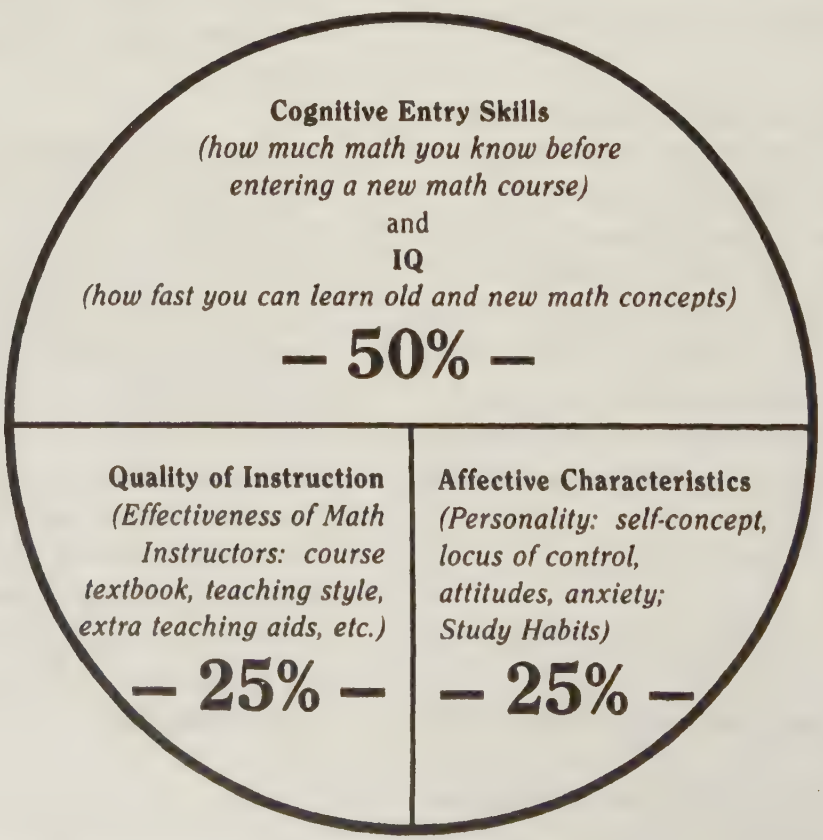
If, by the second class meeting, everything looks like Greek and you do not understand what is being explained, move to a lower-level course. In the lower-level math course you will have a better chance to understand the material and to pass the course. A false evaluation of math ability and knowledge can only lead to frustration, anxiety, and failure.

Requests by Students for Higher Placement

Some students try to encourage their instructors to move them to a higher-level math course because they believe they have received an inaccurate placement score. Many students try to avoid non-credit math courses, while other students do not want to repeat a course that they have previously failed. These moves can also lead to failure.

Some older students imagine that their math skills are just as good as when they completed their last math course, which was five to ten years ago. If they have not been practicing their math skills, they

Figure 1 – Variables Contributing to Student Academic Achievement



are just fooling themselves. Still other students believe they do not need the math skills obtained in a prerequisite math course to pass the next course. This is also incorrect thinking. Research indicates that students who were placed correctly in their prerequisite math course, and who subsequently failed it, will not pass the next math course.

What My Research Shows

I have conducted research on thousands of students who have either convinced their instructors to place them in higher-level math courses or have placed themselves in higher-level math courses. The results? These students failed their math courses many times before realizing they did not possess the prerequisite math knowledge needed to pass the course. Students who, without good reason, talk their instructors into moving them up a course level are setting themselves up to fail.

To be successful in a math course, you must have the appropriate math knowledge. If you think you may have difficulty passing a higher-level math course, you probably do not have an adequate math background. Even if you do pass the math course with a D or C, research indicates that you will most likely fail the next higher math course.

It is better to be conservative and pass a lower-level math course with an A or B instead of making a C or D in a higher-level math course and failing the next course at a higher level.

This is evident when many students repeat a higher-level math course up to five times before repeating the lower-level math course that was barely passed. After repeating the lower-level math course with an A or B, these students passed their higher-level math course.

How quality of math instruction affects your grades

Quality of instruction accounts for 25 percent of your grade. Quality of instruction includes such things as classroom atmosphere, instructor's teaching style, lab instruction, and textbook content and format. All of these "quality" factors can affect your ability to learn in the math classroom.

Interestingly enough, probably the most important "quality" variable is the compatibility of an instructor's teaching style with your learning style. You need to discover your learning style and compare it to the instructional style. Noncompatibility can be best solved by finding an instructor who better matches your learning style. However, if you cannot find an instructor to match your learning style, improving your math study skills and using the math lab or learning resource center (LRC) can compensate for most of the mismatch.

Use of the math lab or learning resource center (LRC) can dramatically improve the quality of instruction. With today's new technologies, students are able to select their best learning aids. These learning aids could be videotapes, CD-ROMs, computer programs, math study skills computer programs, and math texts.

The quality of tutors is also a major factor in the math lab or learning resource center. A low student-to-tutor ratio and trained tutors are essential for good tutorial instruction. Otherwise, the result is just a math study hall with a few helpers.

The math textbook should be up to date with good examples and a solutions manual. This increases the amount of learning from the textbook compared to older, poorly designed texts.

The curriculum design affects the sequence of math courses, which could cause learning problems. Some math courses have gaps between them that cause learning problems for all students.

How affective student characteristics affect your math grades

Affective student characteristics account for about 25 percent of your grade. These affective characteristics include math study skills, test anxiety, motivation, locus of control, learning style, and other variables that affect your personal ability to learn math.

Most students do not have this 25 percent of the grade in their favor. In fact, most students have never been taught *any* study skills, let alone how to study math, specifically. Students also do not know their best learning style, which means they may study ineffectively by using their least effective learning style. However, it is not your fault that you were not taught math study skills or made aware of your learning style.

By working to improve your affective characteristics, you will reap the benefits by learning math more effectively and receiving higher grades. Thousands of students have improved their math affective characteristics and thereby their grades by using this workbook.



1. List and explain the three variables that contribute to your academic success.

   Variable 1: _____

   Variable 2: _____

   Variable 3: _____

2. How does your placement test or previous math course grade affect your math learning?

3. How does the quality of math instructors affect your grade?

4. How do your affective characteristics affect your grade?

How to determine your learning style

Research has shown that matching a student's best learning style with the instructional style improves learning. Research has also shown that students who understand their learning style can improve their learning effectiveness. A learning disadvantage will occur for students who do not know or do not understand their learning style. Students should talk to their instructor or counselor about taking one or more learning style inventories.

Taking Stock of Your Learning Style

There are different types of learning styles assessments. One type of learning style assessment focuses on learning modalities, whereas other assessments focus on cognitive or environmental learning styles. We will focus on learning modalities and cognitive learning styles.

Learning modalities
(using your senses)

If it is available to you, take a learning style inventory that measures *learning modalities*. Learning modalities focus on the best way your brain receives information – that is, learning *visually* (seeing), *auditorially* (hearing), or *kinesthetically* (touching).

If possible, take an inventory that offers math-learning modalities, such as *Learning Styles Inventory* (Brown and Cooper, 1978). Other learning style inventories can measure learning modalities, but they focus on English- or reading-learning modes. Sometimes students' math-learning modes may be different from their language-learning modes.

If you cannot locate a modality-type inventory, then you may want to do a self-assessment on your math-learning modality. Do you learn math best by seeing it, by hearing it, or by having hands-on learning experiences? Hands-on learning means you need to touch or feel things to best learn about them. This is not the most accurate way to determine your learning modality; however, you may use this informal assessment until you have an opportunity to take a more formal modality learning style inventory.

Cognitive learning styles
(processing what you sense)

The second type of learning style inventory focuses on *cognitive learning styles*. Cognitive learning styles are based on how you process math information once it is received though a learning modality. In other words, once you have heard, seen, or felt the information, how does your brain process it? Based on McCarthy (1981), you can process the information four different ways: innovatively, analytically, through common sense, or dynamically. Results of either the *Kolb Learning Styles Inventory* (1985) or the *McCarthy Inventory* (1981) will determine your cognitive learning style.

If you do not have the opportunity to take a cognitive learning style inventory, you may want to do a self-assessment of your cognitive learning style. Which of the following best describes you?

Innovative learners solve problems or make decisions by personally relating to them and using their feelings. They learn best by listening and discussing their ideas with others while searching for the meaning. They are students with the good ideas, and they are innovative. They usually major in social sciences and the psychological helping professions.

Analytic learners look for facts and ask experts for advice when solving problems. They learn by thinking about the concepts and solving problems by using logic. They have more interest in ideas and concepts than in people. They generally major in math and the hard sciences; they enjoy doing research.

Common sense learners solve problems by knowing how things work. They learn by testing concepts in a practical way and make decisions by applying theories. They want to know practical ways to use theories and concepts. They major in applied sciences, health fields, engineering, and technologies.

Dynamic learners solve problems by looking at hidden possibilities and processing information concretely. They learn best by independent self-instruction and trial-and-error practice. They are flexible and will take risks. They major in business and sales occupations.

Learning modalities and cognitive learning styles are neither good nor bad. Learning modalities and cognitive styles are concerned with how you best take in information and process it. Different subject areas are best understood through certain learning modalities and cognitive styles.

Most math instructors are visual/abstract learners, and they tend to teach the way they best learn. Visual/abstract learners teach through a visual mode, and they have students process information abstractly. The second most frequent learning modality/style of math instructors is visual/common sense.

> **Remember:** It is important to recognize that students who do not have a math-learning modality/style can still learn math. A mismatch of modality/learning styles between student and instructor can be compensated by good study skills.

– Section Review –

1. My learning modality is _____ because I learn math best by _____
   _____
   _____
   _____

2. My cognitive learning style is _____ because I process math information by _____
   _____
   _____
   _____
   _____

How to develop a math-learning profile of your strengths and weaknesses

Students have learning strengths and weaknesses that affect math learning and their course grade. They must identify and understand these strengths and weaknesses to know which strengths support them and which weaknesses to improve upon.

> **Examples**: Having good math study skills is a *positive* math-learning characteristic, whereas having high test anxiety is a *negative* math-learning characteristic.

Understanding Math-Learning Profiles

By developing a math-learning profile, you will be able to identify your learning strengths and weaknesses.

Figure 2, on the facing page, allows you to rate yourself as having excellent, good, or poor math-learning skills. It helps you measure your previous math study skills, test anxiety, math knowledge, learning styles, and math attitude. These measurements are based on both the results of certain surveys and your previous math background. Doing all or some of these personal assessments will provide you with a good learning profile, and they may predict your success in math.

Now complete question 1 of the Math-Learning Profile Sheet (Figure 2). Check excellent or good if you were taught math study skills or poor if you were not taught math study skills.

Measuring Your Math Anxiety Level

The second part of the assessment measures your math anxiety level. You need to determine if you have high, medium, or low test anxiety. If you never get anxious on a math test, then select low anxiety. If you sometimes get anxious on a math test, select medium. If you always get anxious on a math test, select high.

Figure 2
Math-Learning Profile Sheet

Student _____ Math Course _____ Math Instructor _____

1. **My math study skills are**

   Excellent _____ Good _____ Poor _____
   (5 points) (3 points) (1 point)

2. **My math test anxiety level is**

   High _____ Medium _____ Low _____
   (1 point) (3 points) (5 points)

3. **Math knowledge (choose either placement score or last course grade):**

   (Placement score)
   High _____ Middle _____ Low _____
   (5 points) (3 points) (1 point)

   OR

   (Last course and grade) _____
   A ____ B ____ C, D, F, W, N, I ____
   (5 points) (3 points) (1 point)

4. **Math attitude:**

   Good _____ Neutral _____ Negative _____
   (5 points) (3 points) (1 point)

5. **Years since last math course:**

   1 Year ____ 2-3 Years ____ 4 or more years ____
   (5 points) (3 points) (1 point)

6. **Previous study skills course or training:**

   Course ____ Study-skills training ____ None ____
   (5 points) (3 points) (1 point)

7. **Previous/current required reading course:**

   No required college-level reading course _____
   (5 points)
   First developmental course _____
   (3 points)
   Second developmental course _____
   (1 point)

8. **How much control do you have over your math learning?**

   A lot _____ Some _____ Very little _____
   (5 points) (3 points) (1 point)

9. **My learning modality is**

   Visual _____ Auditory _____ Kinesthetic _____
   (5 points) (3 points) (1 point)

10. **My cognitive learning style is**

    Analytic _____
    (5 points)
    Commonsense _____
    (3 points)
    Innovative/dynamic _____
    (1 point)

TOTAL POINTS: _____

SCORING

45+ points — an excellent math-learning profile

40-45 points — a good math-learning profile

40 or below — a poor math-learning profile

Complete question 2 of the Math-Learning Profile Sheet.

Measuring Previous Math Knowledge

The third part of the assessment measures your previous math knowledge. Your previous math knowledge is an excellent predictor of future math success. In fact, the best predictor of math success is your previous math grade (if taken within one year of your current course).

The measurement of your math knowledge is based on your previous math grade or your math placement test. If your college does not require a placement test, contact the chairperson of the math department to help you with your course placement. Selecting a math course yourself is like playing Russian roulette with five bullets in the chamber. Without help, you will probably commit mathematical suicide.

Visit your counselor or instructor to discuss your math placement test score. Ask your counselor or instructor how your placement score relates to the scores for the math course below and above your course.

Find out if your score barely placed you into your course, if your score was in the middle, or if your score almost put you into the next course. Mark one of these three outcomes in question 3. If your placement score barely put you into your current math course, you will have difficulty learning the math required in that course.

A barely passing placement score means that you are missing the math knowledge you require to be successful, and you have a poor chance for passing the course. A middle placement score indicates that you probably have most of the math knowledge required for the course; however, you still may have difficulty passing the course. A high placement score means you have most of the math knowledge needed to pass and may have a good chance of passing the course. (Still, if you have poor math study skills and high test anxiety, you may not pass the course.)

Students who have low placement test scores need to improve their math knowledge as soon as possible. Students with middle placement scores should also consider improving their math knowledge.

You need to improve your math skills within the first few weeks of the semester. If you wait too long, you will become hopelessly lost. Suggestions for improving your math knowledge are included in a later section in this chapter.

If you are taking your second math course, use your previous math course grade to determine your math knowledge. Your previous course grade is the best predictor of success in your next math course. (However, this is only true if your last math course was taken within the last year.)

Record the name and grade of your previous math course in question 3. Students who made an A or B in their previous math course have a good chance of passing their next math course. However, students who made a C in their previous math course have a poor chance of passing their next math course. Students who made below a C or withdrew from the previous math course have virtually no chance of passing their next math course.

Determining Your Math Attitude

Math attitude can play a major part in being successful in your course. Some students who have a poor attitude toward math may not attend class as often as they should or may procrastinate in doing their homework. Other students avoid math until they *have* to take it. Mark the type of math attitude you have in question 4 of the Math-Learning Profile Sheet.

The Amount of Time Since Your Last Math Course

Another indicator of math success can be the length of time since your last math course. The longer the time since your last math course, the less likely you will be to pass the current course. The exception to this rule is if you were practicing math while you were not taking math courses. Math is very

easily forgotten if not practiced. Indicate in question 5 of the Math-Learning Profile Sheet the number of years since your last math course.

Previous Study Skills Courses or Training

General study skills training that does not focus on math may help some students in time management, reading techniques, learning styles, and overcoming procrastination. If you had a course in study skills or some study skills training, mark that area in question 6 of the Math-Learning Profile Sheet. If you had no study skills training, indicate that on your profile sheet.

Previous/Current Required Reading Courses

College reading skills are also needed for success in math courses. This is especially true when you are trying to solve story or word problems.

At community colleges and some universities, there are three levels of reading courses. Usually, the first two levels are noncredit reading courses, and the third level is for college credit.

If you are enrolled in the lowest reading level, you will have difficulty reading and understanding the math text. Students who are enrolled in the second level may also have difficulty reading the text. Students who are not required to take a reading class or who are reading at grade level may still have some difficulty reading the math text.

> **Remember:** Math texts are not written like English or history texts. Even students with college-level reading skills may experience difficulty understanding their math text.

If you do not know your reading level, ask to take a reading test. If in doubt about your reading level, take a reading class. Indicate the type of reading skills you have in question 7 of the Math-Learning Profile Sheet.

Your Locus of Control

"Locus of control" is a concept that describes how much control you feel you have over both your life and your course grades. Some students feel they have a lot of control over their life and learning, while other students feel they have very little control over their life or grades. Indicate in question 8 of the Math-Learning Profile Sheet how much control you feel you have over your math learning and grades.

Your Best Learning Modality

Learning modalities are best described by your preference in how you receive lecture information. As discussed, some of these modalities match your math instructor and some do not. Most math instructors teach through a visual mode, with the distant second modality being auditory. Mark your best learning modality in question 9 of the Math-Learning Profile Sheet.

Your Cognitive Learning Style

Cognitive learning styles are best described as how information is processed once it is received by your brain. Most math instructors process information *analytically*, and they therefore teach math the same way. Using *common sense* is the second most popular way math instructors cognitively process information.

If you do not process information analytically or through common sense, then you may be at a learning disadvantage. However, improved math study skills can compensate for a mismatch in learning modes/styles. Mark your cognitive learning style in question 10 of the Math-Learning Profile Sheet.

Evaluating Your Findings

Your profile sheet is an informal way to mea-

sure your math strengths, weaknesses, and learning modalities/styles. Any area with 5 points is a strength, and an area with 1 point is a weakness. Review your Math-Learning Profile Sheet and locate your strengths, weaknesses, or mismatches.

Learning modalities/styles are counted the same way but are considered matches and mismatches instead of strengths and weaknesses. Total up your score to determine if you have an overall strong or weak math-learning profile.

Some of the weak areas can be compensated for, while other areas, such as learning styles, can be made more effective. Strategies for improving math knowledge and finding the best instructor will be discussed in the last part of this chapter. Other profile weaknesses will be improved by using the suggestions in this workbook.



1. Complete the Math-Learning Profile Sheet if it has not been completed.

2. See your instructor about improving your weaknesses.

How to improve your math knowledge

Instructors always operate on the premise that you finished your previous math course just last week; they do not wait for you to catch up on current material. It does not matter if your previous math course was a month ago or five years ago. Instructors expect you to know the previous course material – period.

Review Your Previous Math Course Material and Tests

There are several ways to improve your math knowledge. Review your previous math course material before attending your present math course. Look closely at your final exam to determine your weak areas. Work on your weak areas as soon as possible so they can become building blocks (instead of stumbling blocks) for your current course.

If it has been some time since your last math course, visit the math lab or learning resource center to locate review material. Ask the instructor if there are any computer programs that will assess your math skills to determine your strengths and weaknesses for your course. Review math videotapes on the math course below your level. Also review any computer software designed for the previous math course.

Another way to enhance learning is to review the previous math course text by taking all of the chapter review tests. If you score above 80 percent on one chapter review test, move on to the next chapter. A score below 80 percent means you need to work on that chapter before moving on to the next chapter. Get a tutor to help you with those chapters if you have trouble. Make sure you review all the chapters required in the previous course as soon as possible. If you wait more than two weeks to conclude this exercise, it may be too late to catch up (while learning new material at the same time).

Employ a Tutor

One last way to improve your cognitive entry skills is to employ a private tutor. If you have a history of not doing well in math courses, you may need to start tutorial sessions *the same week class begins.* This will give the tutor a better chance of helping you regain those old math skills.

You still need to work hard to relearn old math skills while continuing to learn the new material. If you wait four to five weeks to employ a tutor, it will probably be too late to catch up and do well or even pass the course.

Remember: Tutorial sessions work best when the sessions begin during the first two weeks of a math course.

Schedule Math Courses "Back to Back"

Another way to maintain appropriate math knowledge is to take your math courses "back to back." It is better to take math courses every semester – even if you do not like math – so that you can maintain sequential (linear) learning.

I have known students who have made B's or C's in a math class and who then waited six months to a year to take the next math course. Inevitably, many failed. These students did not complete any preparatory math work before the math course and were lost after the second chapter. This is similar to having one semester of Spanish, not speaking it for a year, then visiting Spain and not understanding what is being said.

The only exception to taking math courses "back to back" is taking a six-week "kamikaze" math course (an ultracondensed version of a regular course), which should be avoided.

If you are one of the unfortunate many who are currently failing a math course, you need to ask yourself, "Am I currently learning any math or just becoming more confused?" If you are learning some math, stay in the course. If you are getting more confused, withdraw from the course. Improve your math knowledge prior to enrolling in a math course during the next semester.

Example: You have withdrawn from a math course after midterm due to low grades. Instead of waiting until next semester, attend a math lab or seek a tutor and learn Chapters 1, 2, and 3 *to perfection*. Also use this time to improve your math study skills. You will enter the same math course next semester with excellent math knowledge and study skills. In fact, you can make an A on the first test and complete the course with a high grade. Does this sound farfetched? It may, but I know hundreds of students who have used this learning procedure and passed their math course instead of failing it again and again.

Finding Your Best Instructor

Finding an instructor who best matches your learning style can be a difficult task. Your learning style is important; your learning style is how you best acquire information.

Example: Auditory learners do better when hearing the information over and over again instead of carefully reading the information. If an auditory learner is taught by a visual-style instructor who prefers that students read materials on their own and who prefers working problems instead of describing them, the mismatch could cause the student to do worse than if the student were taught by an auditory instructor.

Most students are placed in their first math course by an academic advisor. Usually, academic advisors know who are the most popular and least popular math instructors. However, advisors can be reluctant to discuss teacher popularity. And unfortunately, students may want the counselor to devise a course schedule based on the student's time limits instead of teacher selection.

To learn who are the best math instructors, ask the academic advisor which math instructor's classes fill up first. This does not place the academic advisor in the position of making a value judgment; neither does it guarantee the best instructor. But it will increase the odds in your favor.

Another way to acquire a good math instructor is to ask your friends about their current and previous math instructors. However, if another student says that an instructor is excellent, make sure your learning style matches your friend's learning style. Ask you friend, "Exactly what makes the instructor so good?" Then compare the answer with how you learn best. If you have a different learning style than your friend, look for another instructor, or ask another friend whose learning style more closely matches your own.

To obtain the most from an instructor, discover your best learning style and match it to the instructor's teaching style. Most learning centers or student personnel offices will have counselors who

can measure and explain your learning style. Interview or observe the instructor while the instructor is teaching. This process is time consuming, but it is well worth the effort!

Once you have found your best instructor, *do not* change. Remain with the same instructor for every math class whenever possible.

Remember: The first step in becoming a better math student is knowing your learning strengths and weaknesses. Now you can focus on what you need to improve.

– Section Review –

1. Give four examples of how you can review math course materials.

Example 1: _____

Example 2: _____

Example 3: _____

Example 4: _____

2. What is the advantage of taking math courses back to back?

3. Give two examples of how to find your best math instructor.

Example 1: _____

Example 2: _____



Chapter 2 Review



1. _____ _____ _____ refers to how much math you knew before entering your current math course.

2. _____ _____ _____ are the learning skills you possess that affect your grades.

3. Students who talk their instructors into moving up a course level usually _____ that course.

4. The _____ of tutors is a major part of the math lab or learning resource center.

5. Affective student characteristics account for about _____ percent of your grade.

6. _____ learners look for facts and ask experts for their advice.

7. Hands-on learners are called _____ learners.

8. Developing your math-learning profile will help you identify your _____ and _____ to help you improve your learning.

9. Students having previous difficulty in mathematics should start tutorial sessions the _____ week class begins.

10. Once you find your best math instructor _____ _____ change instructors for your next math course.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

How can you immediately use it?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

Chapter 3

How to Reduce Math Test Anxiety

Math test anxiety is a common problem for many college students, especially for students who are in developmental courses. Unfortunately, it is also common for students to experience test anxiety *only* in math and not in their other courses.

Mild test anxiety can be a motivational factor, but high test anxiety can cause major problems both in learning and in taking tests. For many students, reducing test anxiety is the key to becoming successful in math. Such students need to learn the causes of test anxiety and how to reduce the test anxiety that affects their learning and grades.

Several techniques have proved helpful in reducing both math anxiety and math test anxiety. However, reducing math test anxiety does not guarantee good math grades. It must be coupled with effective study skills and a desire to do well in math.

In this chapter, you will learn

- how to recognize test anxiety,
- the causes of test anxiety,
- the different types of test anxiety, and
- how to reduce test anxiety.

How to recognize test anxiety

Test anxiety has existed for as long as tests have been used to evaluate student performance. Because it is so common and because it has survived the test of time, test anxiety has been carefully studied over the last 50 years. Pioneering studies indicate that test anxiety generally leads to low test scores.

At the University of South Florida (Tampa), Dr. Charles Spielberger investigated the relationship between test anxiety and intellectual ability. The study results suggested that anxiety coupled with high ability can improve academic performance; but anxiety coupled with low or average ability can interfere with academic performance. That is:

Anxiety + high ability = improvement
Anxiety + low or average ability = no improvement

Example: Students with average ability and low test anxiety had better performance and higher grades than did students with average ability and high test anxiety. However, there are students who make good grades, take calculus, and still have test anxiety.

Test anxiety is a *learned* response; a person is not born with it. An environmental situation brings about test anxiety. The good news is that because it is a learned response, it can be *unlearned.* Test anxiety is a special kind of general stress. General stress is considered "strained exertion," which can lead to physical and psychological problems.

Defining Test Anxiety

There are several definitions of test anxiety. One definition states, "Test anxiety is a conditioned emotional habit to either a single terrifying experience, recurring experience of high anxiety, or a continuous condition of anxiety" (Wolpe, 1958).

Another definition of test anxiety relates to the educational system. The educational system develops evaluations that measure one's mental performance, and this creates test anxiety. This definition suggests that test anxiety is the *anticipation* of some realistic or nonrealistic situational threat (Cattell, 1966). The "test" can be a research paper, an oral report, work at the chalkboard, a multiple-choice exam, a written essay, or a math test.

Math test anxiety is a relatively new concept in education. *Ms.* magazine published "Math Anxiety: Why Is a Smart Girl Like You Counting on Your Fingers?" (Tobias, 1976) and coined the phrase *math anxiety*. During the 1970s, other educators began referring to *mathophobia* and *mathemaphobia* as a possible cause for children's unwillingness to learn math. Additional studies on the graduate level discovered that math anxiety was common among adults as well as children. Previously, educators had regarded math anxiety as a skill deficiency rather than a state of mind.

There are several other definitions of math anxiety:

> Math anxiety is the extreme reaction to a very negative attitude toward math. (There is a strong relationship between low math confidence and high math test anxiety; this could be the same concept.) (Fox, 1977)

> Math anxiety is the feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of math problems during school tests. (Richardson and Suinn, 1972)

> Math anxiety is a state of panic, helplessness, paralysis and mental disorganization that occurs in some students when required to solve math problems. This discomfort varies in intensity and is the outcome of numerous previous situations. (Tobias, 1986)

One of my students once described math test anxiety as "being in a burning house with no way out." No matter how you define it, math test anxiety is real, and it affects millions of students.

Why Math Tests Create Anxiety

Math anxiety can be divided into two separate anxieties: *math test* anxiety and *numerical* anxiety. Math test anxiety involves anticipation, completion, and feedback of math tests. Numerical anxiety refers to everyday situations requiring working with numbers.

It has been shown that math anxiety exists among many students who usually do not suffer from other tensions. Counselors at a major university reported that one-third of the students who enrolled in behavior therapy programs offered through counseling centers had problems with math anxiety (Suinn, 1988).

Educators know that math anxiety is common among college students and is more prevalent in women than in men. They also know that math anxiety frequently occurs in students with a poor high school math background. These students were found to have the greatest amount of anxiety.

Approximately half of the students in college prep math courses (designed for students with inadequate high school math background or low placement scores) could be considered to have math anxiety. However, math anxiety also occurs in students in high-level math courses, such as college algebra and calculus.

Educators investigating the relationship between anxiety and math have indicated that anxiety contributes to poor grades in math. They also found that simply *reducing* math test anxiety does not guarantee higher math grades. Students often have other problems that affect their math grades, such as poor study skills, poor test-taking skills, or poor class attendance.

 **– Section Review –** 

1. What is your personal definition of test anxiety?

_____
_____
_____
_____
_____
_____

2. What type of student does math anxiety affect most?

_____
_____
_____
_____
_____
_____

The causes of test anxiety

The causes of test anxiety can be different for each student, but they can be explained by seven basic concepts (described on the next page).

The most common situation students have reported as a known cause of their math test anxiety is their elementary school math experiences. When asked, many students indicated that they were made fun of when trying to solve math problems at the chalkboard. When they could not solve the problem, the teacher and/or students would call them "stupid."

Teacher and peer embarrassment and humiliation become the conditioning experience that causes some students' test anxiety. Over the years, this test anxiety is reinforced and still exists. In fact, many math-anxious students – now 30 and 40 years old – *still* have extreme fear about working math problems on the board. Some students said that they absolutely refuse to go to the board.

Math Autobiography

The math autobiography is a way to find out where some of your fears about math began. In the math autobiography, you write down some of your positive and negative math experiences. A better understanding of your math history can lead to more control over math. Use the answers to the following questions to help you write your own math autobiography.

My first positive math experience was when _____
_____
_____
_____.

My first negative math experience was when _____
_____
_____
_____.

One of my best math teachers helped me in math by
_____
_____
_____.

One math teacher didn't help me learn math because he/she _____
_____
_____
_____.

I did well in math until _____
_____
_____
_____
_____. OR

I am still doing well in math because _____
_____
_____
_____.

When I hear the word *math* I first think of _____
_____
_____
_____

Because _____
_____
_____
_____.

Math is only for people who _____
_____
_____
_____.

Doing math successfully means I can _____
_____
_____
_____.

For me, math is like _____
_____
_____
_____.

After answering these questions, you now have a better understanding of how math affects you. Write a three- to five-page math autobiography about your math history from elementary school to your current math course. Write down how you felt about math and how you feel about math now. Your conclusion should include what you want to do NOW about your math success. Turn the autobiography in to your instructor.

Remember: For some students, just writing about their previous math history helps them.

– Section Review –

1. List and describe five causes of test anxiety.

   Cause 1: _____

The Causes of Test Anxiety

Test anxiety can be a learned behavior resulting from the expectations of parents, teachers, or other significant people in the student's life.

Test anxiety can be caused by the association between grades and a student's personal worth.

Test anxiety develops from fear of alienating parents, family, or friends due to poor grades.

Test anxiety can stem from a feeling of lack of control and an inability to change one's life situation.

Test anxiety can be caused by a student's being embarrassed by the teacher or other students when trying to do math problems.

Test anxiety can be caused by timed tests and the fear of not finishing the test, even if one can do all the problems.

Test anxiety can be caused by being put in math courses above one's level of competence.

What are the cause(s) of *your* test anxiety?

Cause 2: _____

Cause 3: _____

Cause 4: _____

Cause 5: _____

2. Put into your own words your cause(s) of test anxiety. If you don't have test anxiety, list what you think is the major cause of test anxiety.

The different types of test anxiety

The two basic types of test anxiety are emotional (educators call this *somatic*) and worry (educators call this *cognitive*). Students with high test anxiety have *both* emotional and worry anxiety.

Signs of emotional anxiety are upset stomach, nausea, sweaty palms, pain in the neck, stiff shoulders, high blood pressure, rapid shallow breathing, rapid heartbeat, or general feelings of nervousness. As anxiety increases, these feelings intensify. Some students even run to the bathroom to throw up or have diarrhea.

Even though these *feelings* are caused by anxiety, the physical response is real. These feelings and physical inconveniences can affect your concentration and your testing speed, and they can cause you to completely "draw a blank."

Worry anxiety causes the student to think about failing the test. These negative thoughts can happen either before or during the test. This negative "self-talk" causes students to focus on their anxiety instead of recalling math concepts.

The effects of test anxiety range from a "mental block" on a test to avoiding homework. One of the most common side effects of test anxiety is getting the test and immediately forgetting information that you know. Some students describe this event as having a "mental block," "going blank," or finding that the test looks like Greek.

After five or ten minutes into the test, some of these students can refocus on the test and start working the problems. They have, however, lost valuable time. For other students, anxiety persists throughout the test and they cannot recall the needed math information. It is only after they walk out the door that they can remember how to work the problems.

Sometimes math anxiety does not cause students to "go blank" but slows down their mental processing speed. This means it takes longer to recall formulas and concepts and to work problems. The result is frustration and loss of time, leading to *more* anxiety. Since,

The 12 Myths About Test Anxiety

Students are born with test anxiety.

Test anxiety is a mental illness.

Test anxiety cannot be reduced.

Any level of test anxiety is bad.

All students who are not prepared have test anxiety.

Students with test anxiety cannot learn math.

Students who are well prepared will not have test anxiety.

Very intelligent students and students taking high-level courses, such as calculus, do not have test anxiety.

Attending class and doing homework should reduce all test anxiety.

Being told to relax during a test will make a student relaxed.

Doing nothing about test anxiety will make it go away.

Reducing test anxiety will guarantee better grades.

in most cases, math tests are *speed* tests (those in which you have a certain amount of time to complete the test), you may not have enough time to work all the problems or to check the answers if you have mentally slowed down. The result is a lower test score, because even though you know the material, you do not complete all of the questions before test time runs out.

Not using all of the time allotted for the test is another problem caused by test anxiety. Students know that they should use all of the test time to check their answers. In fact, math is one of the few subjects in which you can check test problems to find out if your work is correct. However, most students do not use all of the test time, and this results in lower test scores. Why does this happen?

Students with high test anxiety do not want to stay in the classroom. This is especially true of students whose test anxiety increases as the test progresses. The test anxiety gets so bad that they would rather leave early and receive a lower grade than stay in that "burning house."

Students have another reason for leaving the test early: the fear of what the instructor and other students will think about them for being the last one to hand in the test. These students refuse to be the last ones to finish the test because they think that the instructor or other students will think they are dumb. This is middle-school thinking, but the feelings are still real – no matter what the age of the student. These students do not realize that some students who turn in their tests first fail, whereas many students who turn in their tests last make A's and B's.

Another effect of test anxiety relates to completing homework assignments. Students who have high test anxiety may have difficulty starting or completing their math homework. Doing homework reminds some students of their learning problems in math. More specifically, it reminds them of their previous math failures, which causes further anxiety. This anxiety can lead to total homework avoidance or "approach-avoidance" behavior.

Total homework avoidance is called procrastination. The very thought of doing their homework

causes these students anxiety, which causes them to put off tackling their homework. This makes them feel better for a short amount of time – *until test day.*

Example: Some students begin their homework and work some problems successfully. They then get stuck on an problem that causes them anxiety, so they take a break. During their break the anxiety disappears until they start doing their homework again. Doing their homework causes more anxiety, which leads to another break. The breaks become more frequent. Finally, the student ends up taking one long break and not doing the homework. Quitting, to them, means *no more anxiety* until the next homework assignment.

The effects of math test anxiety can be different for each student. Students can have several of the mentioned characteristics that can interfere with math learning and test taking. However, there are certain myths about math that each student needs to know. Review The 12 Myths About Test Anxiety on the facing page to see which ones you believe. If you have test anxiety, which of the mentioned characteristics are true for you?



1. List and describe the two basic types of test anxiety.

   Type 1: _____

   _____

   _____

   _____

   Type 2: _____

   _____

   _____

   _____

2. List two reasons students leave the test room early instead of checking their answers.

   Reason One: _____

   _____

   _____

   _____

   Reason Two: _____

   _____

   _____

   _____

3. List 6 of the 12 myths about test anxiety that you most believed.

Myth 1: _____

_____

_____

Myth 2: _____

_____

_____

Myth 3: _____

_____

_____

Myth 4: _____

_____

_____

Myth 5: _____

_____

_____

Myth 6: _____

_____

_____

How to reduce test anxiety

To reduce math test anxiety, you need to understand both the relaxation response and how negative self-talk undermines your abilities.

Relaxation Techniques

The relaxation response is any technique or procedure that helps you to become relaxed. It will take the place of an anxiety response. Someone simply telling you to relax or even telling yourself to relax, however, does little to reduce your test anxiety. There are both short-term and long-term relaxation response techniques that help control emotional (somatic) math test anxiety. These techniques will also help reduce worry (cognitive) anxiety. Effective *short-term* techniques include the tensing and differential relaxation method, the palming method, and deep breathing.

Short-Term Relaxation Techniques

The Tensing and Differential Relaxation Method

The tensing and differential relaxation method helps you relax by tensing and relaxing your muscles all at once. Follow these procedures while you are sitting at your desk before taking a test:

1. Put your feet flat on the floor.
2. With your hands, grab underneath the chair.
3. Push down with your feet and pull up on your chair at the same time for about five seconds.
4. Relax for five to ten seconds.
5. Repeat the procedure two or three times.
6. Relax all your muscles except the ones that are actually used to take the test.

The Palming Method

The palming method is a visualization procedure used to reduce test anxiety. While you are at your desk before or during a test, follow these procedures:

1. Close and cover your eyes using the center of the palms of your hands.
2. Prevent your hands from touching your eyes by resting the lower parts of your palms on your cheekbones and placing your fingers on your forehead. Your eyeballs must not be touched, rubbed, or handled in any way.
3. Think of some real or imaginary relaxing scene. Mentally visualize this scene. Picture the scene as if you were actually there, looking through your own eyes.
4. Visualize this relaxing scene for one to two minutes.

Practice visualizing this scene several days before taking a test and the effectiveness of this relaxation procedure will improve.

Deep Breathing

Deep breathing is another short-term relaxation technique that can help you relax. Proper breathing is a way to reduce stress and decrease test anxiety. When breathing properly, enough oxygen gets into your bloodstream to nourish your body and mind. A lack of oxygen in your blood contributes to an anxiety state that makes it more difficult to react to stress. Proper deep breathing can help you control your test anxiety.

Deep breathing can replace the rapid, shallow breathing that sometimes accompanies test anxiety, or it can prevent test anxiety. Here are the steps to deep breathing:

1. Sit straight up in your chair in a good posture position.
2. Slowly inhale through your nose.
3. As you inhale, first fill the lower section of your lungs and work your way up to the upper part of your lungs.
4. Hold your breath for a few seconds.
5. Exhale slowly through your mouth.
6. Wait a few seconds and repeat the cycle.

You need to practice deep breathing exercises before using them during a test.

Side one of the audiocassette *How to Reduce Test Anxiety* (Nolting, 1986) further explains test anxiety and discusses these and other short-term relaxation response techniques. Short-term relaxation techniques can be learned quickly but are not as successful as the long-term relaxation technique. Short-term techniques are intended to be used while learning the long-term technique.

Long-Term Relaxation Techniques

The cue-controlled relaxation response technique is the best long-term relaxation technique. It is presented on side two of the audiocassette *How To Reduce Test Anxiety* (Nolting, 1986). Cue-controlled relaxation means you can induce your own relaxation based on repeating certain cue words to yourself. In essence, you are taught to relax and then silently repeat cue words, such as "I am relaxed."

After enough practice, you can relax during math tests. The cue-controlled relaxation technique has worked with thousands of students. For a better understanding of test anxiety and how to reduce it, listen to *How to Reduce Test Anxiety* (Nolting, 1986).

Negative Self-Talk

Negative self-talk is a form of worry (cognitive) anxiety. This type of worrying can interfere with your test preparation and can keep you from concentrating on the test. Worrying can motivate you to study, but too much worrying may prevent you from studying at all.

Negative self-talk is defined as the negative statements you tell yourself before and during tests. Negative self-talk causes students to lose confidence and to give up on tests. Further, it can give you an inappropriate excuse for failing math and cause you to give up on learning math.

Students need to change their negative self-talk to positive self-talk without making unrealistic statements.

Positive self-statements can improve your studying and test preparation. During tests, positive self-talk can build confidence and decrease your test anxiety. These positive statements (see examples on the next page) can help reduce your test anxiety and improve your grades. Some more examples of positive self-statements are on the cassette *How to Reduce Test Anxiety* (Nolting, 1986). Before the test, make up some positive statements to tell yourself.

Thought-Stopping Techniques

Some students have difficulty stopping their negative self-talk. These students cannot just tell themselves to eliminate those thoughts. These students need to use a thought-stopping technique to overcome their worry and become relaxed.

Thought stopping involves focusing on the unwanted thoughts and, after a few seconds, suddenly stopping those thoughts by emptying your mind. Using the command "Stop!" or a loud noise like clapping your hands can effectively interrupt the negative self-talk. In a homework situation, you may be able to use a loud noise to stop your thoughts, but don't use it during a test.

To stop your thoughts in the classroom or during a test, silently shout to yourself "Stop!" or "Stop

thinking about that." After your silent shout, either relax yourself or repeat one of your positive self-talk statements. You may have to shout to yourself several times during a test or while doing homework to control negative self-talk. After every shout, use a different relaxation technique/scene or positive self-talk statement.

Examples of Negative Self-Talk:

"No matter what I do, I will not pass this course."
"I failed this course last semester, and I will fail it again."
"I am no good at math, so why should I try?"
"I cannot do it; I cannot do the problems, and I am going to fail this test."
"I have forgotten how to do the problems, and I am going to fail."
"I am going to fail this test and never graduate."
"If I can't pass this test, I am too dumb to learn math and will flunk out."

Examples of Positive Self-Talk:

"I failed the course last semester, but I can now use my math study skills to pass this course."
"I went blank on the last test, but I now know how to reduce my test anxiety."
"I know that my poor math skills are due to poor study skills, not my own ability, and since I am working on my study skills, my math skills will improve."
"I know that with hard work, I will pass math."
"I prepared for this test and will do the best I can. I will reduce my test anxiety and use the best test-taking procedures. I expect some problems will be difficult, but I will not get discouraged."
"I am solving problems and feel good about myself. I am not going to worry about that difficult problem; I am going to work the problems that I can do. I am going to use all the test time and check for careless errors. Even if I do not get the grade I want on this test, it is not the end of the world."

Thought stopping works because it interrupts the worry response before it can cause high negative emotions. During that interruption, you can replace the negative self-talk with positive statements or relaxation. However, students with high worry anxiety should practice this technique three days to one week before taking a test. Contact your counselor if you have additional questions about the thought-stopping techniques.

– Section Review –

1. Describe your best short-term relaxation technique.

2. You can use the palming method by closing your eyes and visualizing a scene without putting your hands to your face. Describe a very relaxing scene that you could visualize. Make sure to include some sounds and visual images in your scene.

3. Practice your relaxation scene for three to five minutes for the next five days. List the times and dates you practiced your scene.

Date: _____ Time: _____
Date: _____ Time: _____
Date: _____ Time: _____
Date: _____ Time: _____
Date: _____ Time: _____

4. How does negative self-talk cause you to have text anxiety?

5. Make up three positive self-talk statements that are not listed in this text.

Statement 1: _____

Statement 2: _____

Statement 3: _____

6. Describe the thought-stopping technique in your own words.

7. What words will you use as your silent shout?

8. What will you do after your silent shout?

9. How does the thought-stopping technique work?

Lecture Notes

Name: _____ Date: _____



Chapter 3 Review



1. Reducing math test anxiety _____ _____ guarantee good math grades.

2. Test anxiety is a _____ response; a person is not _____ with it.

3. Math test anxiety involves _____, _____,and _____ of math tests.

4. One cause of test anxiety is that a student goes to the board to work a problem but is called _____ when he or she cannot work the problem.

5. The two basic types of test anxiety are _____and _____.

6. The effects of test anxiety range from a "_____ _____" on a test to _____ homework.

7. The _____and _____ _____ method helps you relax by tensing and relaxing your muscles all at once..

8. The palming method is a _____ procedure used to reduce text anxiety.

9. Negative self-talk is a form of _____anxiety.

10. Thought stopping works because it interrupts the _____ response before it can cause _____emotions.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

How can you immediately use it?

_____
_____
_____
_____
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Chapter 4

How to Improve Your Listening and Note-Taking Skills

Listening and note-taking skills in a math class are very important, since most students either do not read the math text or have difficulty understanding it. In most of your other classes, if you do not understand the lecture, you can read the text and get almost all the information. In math class, however, the instructor can usually explain the textbook better than the students can read and understand it.

Students who do not have good listening skills or note-taking skills will be at a disadvantage in learning math. Most math understanding takes place in the classroom. Students must learn how to take advantage of learning math in the classroom by becoming effective listeners, calculator users, and notetakers.

In this chapter you will learn

- how to become an effective listener,
- how to become a good notetaker,
- when to take notes,
- the seven steps to math note-taking, and
- how to rework your notes.

How to become an effective listener

Becoming an effective listener is the foundation for good note-taking. You can become an effective listener using a set of skills that you can learn and practice. To become an effective listener, you must prepare yourself both physically and mentally.

Sitting in the Golden Triangle

The physical preparation for becoming an effective listener involves *where you sit* in the classroom. Sit in the best area to obtain high grades, that is, in "the golden triangle of success." The golden triangle of success begins with seats in the front row facing the instructor's desk (see Figure 3 on the next page).

Students seated in this area (especially in the front row) directly face the teacher and are most likely to pay attention to the lecture. This is a great seating location for visual learners. There is also less tendency for them to be distracted by activities outside the classroom or by students making noise within the classroom.

The middle seat in the back row is another point in the golden triangle for students to sit, especially those who are auditory (hearing) learners. You can hear the instructor better because the instructor's voice is projected to that point. This means that there is less chance of misunderstanding the instructor, and you can hear well enough to ask appropriate questions.

By sitting in the golden triangle of success, you can force yourself to pay more attention during class and be less distracted by other students. This is

very important for math students because math instructors usually go over a point once and continue on to the next point. If you miss that point in the lesson, then you could be lost for the remainder of the class.

Warming Up for Math Class

The mental preparation for note-taking involves "warming up" before class begins and becoming an active listener. Just as an athlete must warm up before a game begins, you must warm up before taking notes. Warm up by

- reviewing the previous day's notes,
- reviewing the reading material,
- reviewing the homework,
- preparing questions, and
- working one or two unassigned homework problems.

This mental warm up before the lecture allows you to refresh your memory and prepare pertinent questions, making it easier to learn the new lecture material.

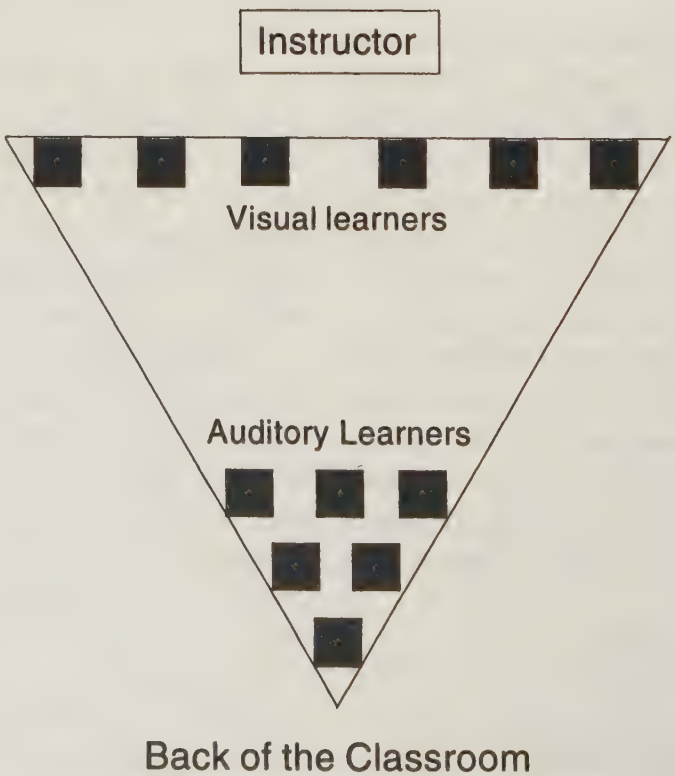
How to Become an Active Listener

Becoming an active listener is the second part of the mental preparation for note-taking.

Do not anticipate what the instructor is going to say or immediately judge the instructor's information before the point is made. This will distract you from learning the information.

Figure 3

The Golden Triangle of Success



Examples: Watch the speaker, listen for main ideas, and nod your head or say to yourself, "I understand," when agreeing with the instructor.

Expend energy looking for interesting topics in the lecture. When the instructor discusses information that you need to know, immediately repeat it to yourself to begin the learning process.

You can practice this exercise by viewing math videotapes and repeating important information. This is an especially good learning technique for auditory learners.

Remember: Class time is an intense study period that should not be wasted.

Listening and Learning

Some students think that listening to the instructor and taking notes is a waste of valuable time. Students too often sit in class and use only a fraction of their learning ability. Class time should be considered a valuable study period where you can listen, take notes, and learn at the same time. One way to do this is by memorizing important facts when the instructor is talking about material you already know. Another technique is to repeat back to yourself the important concepts right after the instructor says them in class. Using class time to learn math is an efficient learning system.



1. Explain how sitting in the golden triangle of success makes you a better listener.

2. List five ways you can warm up before math class begins.

Way 1: _____

Way 2: _____

Way 3: _____

Way 4: _____

Way 5: _____

3. Select your best warm-up process and try it before your next math class. What was your warm-up process, and how did it help you?

4. How can you listen and learn at the same time?

How to become a good notetaker

Becoming a good notetaker requires two basic strategies. One strategy is to be specific in detail. In other words, *copy* the problems down, step by step. The second strategy is to *understand* the general principles, general concepts, and general ideas.

Copying from the Board

While taking math notes, you need to copy each and every step of the problem even though you may already know every step of the problem. While in the classroom, you might understand each step, but a week later you might not remember how to do the problem unless all the steps were written down. In addition, as you write down each step, you are memorizing it. Make sure to copy every step for each problem written on the board.

There will be times when you will get lost while listening to the lecture. Nevertheless, you should keep taking notes, even though you do not understand the problem. This will provide you with a reference point for further study. Put a question mark (?) by those steps that you do not understand; then, after class, review the steps you did not understand with the instructor, your tutor, or another student.

Taking Notes

The goal of note-taking is to take the least amount of notes and get the greatest amount of information on your paper. This could be the opposite of what most instructors have told you. Some instructors tell you to take down everything. This is not necessarily a good note-taking system, since it is very difficult to take precise, specific notes while trying to understand the instructor.

What you need to develop is a note-taking system in which you write the least amount possible and get the most information down while still understanding what the instructor is saying.

Develop an Abbreviation List

To reduce the amount of written notes, an abbreviation system is needed. An abbreviation system is your own system of reducing long words to shorter versions that you still can understand. By writing less, you can listen more and have a better understanding of the material.

> **Example:** When the instructor starts explaining the commutative property, you need to write "commutative property" out the first time. After that, use "COM." You should develop abbreviations for all the most commonly used words in math.

Figure 4 on the next page provides a list of abbreviations. Add your own abbreviations to this list. By using abbreviations as much as possible, you can obtain the same meaning from your notes and have more time to listen to the instructor.

When to take notes

To become a better notetaker, you must know when to take notes and when not to take notes. The instructor will give cues that indicate what material is important. These cues include

- presenting usual facts or ideas,
- writing on the board,
- summarizing,
- pausing,
- repeating statements,
- enumerating, such as "1, 2, 3" or "A, B, C,"
- working several examples of the same type of problem on the chalkboard,

Figure 4
Abbreviations

E.G. (for example)

CF. (compare, remember in context)

N.B. (note well, this is important)

∴ (therefore)

(because)

⊃ (implies, it follows from this)

> (greater than)

< (less than)

= (equals, is the same)

≠ (does not equal, is not the same)

( ) (parentheses in the margin, around a sentence or group of sentences, indicates an important idea)

? (used to indicate you do not understand the material)

0 (a circle around a word may indicate that you are not familiar with it; look it up)

E (marks important materials likely to be used in an exam)

1, 2, 3, 4 (to indicate a series of facts)

D (shows disagreement with statement or passage)

REF (reference)

et al. (and others)

bk (book)

p (page)

etc. (and so forth)

V (see)

VS (see above)

SC (namely)

SQ (the following)

Com. (commutative)

Dis. (distributive)

APA (associative property of addition)

AI (additive inverse)

IPM (identity property of multiplication)

– saying, "This is a tricky problem." Most students will miss it (for example, 5/0 is "undefined" instead of "zero").
– saying, "This is the most difficult step in the problem,"
– indicating that certain types of problems will be on the test, such as coin or age problems, and
– explaining bold-face words.

You must learn the cues your instructor gives to indicate important material. If you are in doubt about the importance of the class material, do not hesitate to ask the instructor about its importance.

While taking notes, you may become confused about math material. At that point, take as many notes as possible, and do not give up on note-taking.

As you take notes on confusing problem steps, leave extra space; then go back and fill in information that clarifies your misunderstanding of the steps in question. Ask your tutor or instructor for help with the uncompleted problem steps, and write down the reasons for each step in the space provided.

Another procedure to save time while taking notes is to stop writing complete sentences. Write your main thoughts in phrases. Phrases are easier to jot down and easier to memorize.

– Section Review –

1. Note-taking requires two basic strategies:

_____and
_____.

2. List five abbreviations (and their meanings) that you use in your math class (one abbreviation and meaning per line).

_____
_____
_____
_____

_____

3. List five cues that your instructor gives that indicate what material is important (one cue per line).

_____
_____
_____
_____
_____

The seven steps to math note-taking

The key to effective note-taking is to record the fewest words while retaining the greatest information. As you know, it is very difficult to record notes and at the same time fully understand the instructor. The seven steps to math note-taking were developed to decrease the amount of note-taking while at the same time improving math learning.

The seven steps to math note-taking consists of three major areas. The first area, steps 1 through 3, focuses on *recording* your notes. Steps 4 through 6 focus on *checking* yourself to see how much information is retained. This is done by recalling key words and concepts and putting a check mark by misunderstood information. Recalling information is one of the best learning techniques. Step 7 focuses on *understanding* key words and concepts that are frequently used.

Note-Taking Memory Cues

One of the best math note-taking methods is demonstrated in Figure 5 (Modified Three-Column Note-Taking Sample) on the facing page. To use this effective note-taking system, you need to record a few memory cues as reminders. Label the top space between the notebook ring and the red line "Key Words." Label the other side of the red line "Examples." Next, label "Explanations/ Rules" about 4 inches from the red line. Draw a vertical line between the "Examples" and "Explanations/Rules" sections. Record the same information on the next ten pages. After using this system for ten pages, you may not need to label each page.

Follow these steps to improve your note-taking:

Step 1 Record each problem step in the "Examples" section.

Step 2 Record the reasons for each step in the "Explanations/Rules" section by using

- abbreviations,
- short phrases, not sentences, and
- key words, properties, principles, or formulas.

Step 3 Record key words and concepts in the left 2-inch margin either during or immediately after lecture by reworking your notes.

Step 4 Cover up the "Examples" and "Explanations/ Rules" sections, and recite out loud the meaning of the key words or concepts.

Step 5 Place a check mark by the key words and concepts that you *did not* know.

Step 6 Review the information that you checked until it is understood.

Step 7 Develop a math glossary for difficult-to-remember key words and concepts.

A Math Glossary

The third area of focus in the seven steps to math note-taking is devoted to developing a math glossary. The math glossary is created to define a math vocabulary in your own words. Since math is consid-

Figure 5
Modified Three-Column Note-Taking Sample

Key Words	Examples	Explanations/Rules
Natural numbers	1, 2, 3, 4, 5, . . .	You can count them.
Whole numbers	0, 1, 2, 3, 4, . . .	Natural numbers and zero
Integers	. . . -2, -1, 0, 1, 2, . . .	Negative numbers and whole numbers
$-n$	$-n$ If $n = s$, then $-n = -s$ $-(-10) = 10$ $-(-15)(-15) = -15$ Opposite of $-x$ is x	Opposite of any number Count the number of signs; even means +, odd means -
Rational	Rational numbers are fractions (¼, ½, ¾).	A/B, $B/0$ is rational Division by 0 undefined
Numerator Denominator	numerator/denominator N/D	Numerator on top Denominator on bottom (D = Down)
Terminating decimal	41.5, 45.0, 7, 1576, 0.39, are terminating decimals	Decimal numbers Some are rational numbers, some are not rational numbers.
Repeating decimals	0.333, 0.51515151 = 0.51	A repeating decimal = rational number
Pi, irrational number	$\pi = 3.14159$ but does not repeat and is not a rational number. It is irrational.	Irrational numbers do not repeat.

Figure 5
Modified Three-Column Note-Taking System
(Blank)

Key Words	Examples	Explanations/Rules

ered a foreign language, understanding the math vocabulary becomes the key to comprehending math. Creating a glossary for each chapter of your textbook will help you understand math.

Your glossary should include all words printed in bold face in the text and any words you do not understand. If you cannot explain the math vocabulary in your own words, ask your instructor or tutor for help. You may want to use the last pages in your notebook to develop a math glossary for each chapter in your textbook. Review your math glossary before each test.



1. List and describe the seven steps to note-taking.

   Step 1: _____
   _____
   _____
   Step 2: _____
   _____
   _____
   Step 3: _____
   _____
   _____
   Step 4: _____
   _____
   _____
   Step 5: _____
   _____
   _____
   Step 6: _____
   _____
   _____
   Step 7: _____
   _____
   _____

2. Make several copies of the facing page (the modified three-column note-taking system, and take notes using this system in your next math class. Or develop the modified three-column note-taking system in your own notebook.

3. List two benefits of a math glossary.

   Benefit 1: _____
   _____
   _____
   Benefit 2: _____
   _____
   _____

4. From your current math chapter, list and define five words that you can put in your math glossary.

   Word 1: _____
   _____
   Word 2: _____
   _____
   Word 3: _____
   _____
   Word 4: _____
   _____
   Word 5: _____
   _____

How to rework your notes

The note-taking system does not stop when you leave the classroom. As soon as possible after class, rework your notes. You can rework the notes between classes or as soon as you get home. By reworking your notes as soon as possible, you can decrease the amount of forgetting. This is an excellent procedure to transfer math information from short-term memory to long-term memory.

Remember: Most forgetting occurs right after learning the material. You need to rework the notes as soon as possible. Waiting means that you probably will not understand what was written.

Here are important steps in reworking your notes:

Step 1 *Rewrite the material you cannot read or will not be able to understand a few weeks later.* If you do not rework your notes, you will be frustrated when studying for a test if you come across notes you cannot read. Another benefit of rewriting the notes is that you immediately learn the new material. Waiting means it will take more time to learn the material.

Step 2 *Fill in the gaps.* Most of the time, when you are listening to the lecture, you cannot write down everything. It is almost impossible to write down everything, even if you know shorthand. Locate the portions of your notes that are incomplete. Fill in the concepts that were left out. In the future, skip two or three lines in your notebook page for anticipated lecture gaps.

Step 3 *Add additional key words and ideas in the left-hand column.* These key words or ideas were the ones not recorded during the lecture.

Example: You did not know that you should add the *opposite* of 18 to solve a particular problem, and you incorrectly added 18. Put additional important key words and ideas (such as "opposite" and "negative of") in the notes; these are the words that will improve your understanding of math.

Step 4 *Make a problem log on those problems that the teacher worked in class.* The problem log is a separate section of your notebook that contains a listing of the problems (without explanations – just problems) that your teacher worked in class. If your teacher chose those problems to work in class, you can bet that they are considered important. The problems in this log can be used as a practice test for the next exam. Your regular class notes will not only contain the solutions but also all the steps involved in arriving at those solutions and can be used as a reference when you take your practice test.

Step 5 *Make a calculator handbook and put in it your keystroke sequences.* The calculator handbook can be a spiral-bound set of note cards or a separate section of your notebook that holds only calculator-related information. Your handbook should also include an explanation of when that particular set of keystrokes is to be used.

Step 6 *Reflection and synthesis.* Once you have finished going over your notes, review the major points in your mind. Combine your new notes with your previous knowledge to have a better understanding of what you have learned today.

Use a Tape Recorder

If you have a math class during which you cannot get all the information down while listening to the lecture, ask your instructor about using a tape recorder. To ensure success, the tape recorder must have a tape counter and must be voice activated.

The tape counter displays a number indicating the amount of tape to which you have listened. When you find you are in an area of confusing information, write the beginning and ending tape counter number in the left margin of your notes. When reviewing your notes, the tape count number will be a reference point for obtaining information to work the problem. You can also reduce the time it takes to listen to the tape by using the pause button to stop the recording of unnecessary material.

Ask Questions

To obtain the most from a lecture, you must ask questions in class. By asking questions, you improve your understanding of the material and decrease your homework time. By *not* asking questions, you create for yourself unnecessary confusion during the remainder of the class period. Also, it is much easier to ask questions in class about potential homework problems than it is to spend hours trying to figure out the problems on your own at a later time.

If you are shy about asking questions in class, write down the questions and read them to your instructor. If the instructor seems confused about the questions, tell him or her that you will discuss the problem after class. To encourage yourself to ask questions, remember that

- you have paid for the instructor's help,
- five other students probably have the same question,
- the instructor needs feedback on his or her teaching to help the class learn the material, and
- there is no such thing as a "stupid" question.

Record Each Problem Step

The final suggestion in note-taking is to record each step of every problem written or verbally explained. By recording each problem step, you begin *overlearning* how to work the problems. This will increase your problem-solving speed during future tests. If you get stuck on the homework, you will also have complete examples to review.

The major reason for recording every step of a problem is to understand how to do the problems while the instructor is explaining them instead of trying to remember unwritten steps. Although it may seem time consuming, it pays off during homework and test time.



1. List and describe the six steps to reworking your notes.

   Step 1: _____
   _____
   _____
   Step 2: _____
   _____
   _____
   Step 3: _____
   _____
   _____
   Step 4: _____
   _____
   _____
   Step 5: _____
   _____
   _____
   Step 6: _____
   _____
   _____

2. How can using a tape recorder in class improve your learning?

   _____
   _____
   _____
   _____

3. List three reasons for asking questions in class.

   Reason 1: _____
   _____
   _____
   Reason 2: _____
   _____
   _____

Reason 3: _____

_____

_____

4. What is the major reason for recording each problem step?

_____

_____

_____

_____

Lecture Notes

Name: _____ Date: _____



Chapter 4 Review



1. Sitting in the golden triangle of success can help you pay more _____during class.

2. Class time is an intense _____ _____ that should not be wasted.

3. Auditory learners should review the math videotape and _____back the important information.

4. The goal of note-taking is to take the _____amount of notes and get the _____amount of information.

5. Using abbreviations can help you write _____ and listen _____.

6. Your _____ should include any words you don't know and their explanations.

7. Rework your notes to transfer information from _____-_____ memory to _____-_____memory.

8. A _____ _____is a separate section in your notebook that contains problems that your teacher worked in class.

9. Asking _____ in class will improve your understanding and decrease your homework time.

10. _____how to work problems will increase your problem-solving speed and memory.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
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_____
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_____

How can you immediately use it?

_____
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_____

Chapter 5

How to Improve Your Reading, Homework, and Study Techniques

Reading a math textbook is more difficult than reading other textbooks. Math textbooks are written differently than your English or social science textbooks. Math textbooks contain condensed material and therefore take longer to read.

Mathematicians can reduce a page of writing to one paragraph using math formulas and symbols. To make sure you understood that same information, an English instructor would take that original page of writing and expand it into two pages. Mathematicians pride themselves on how little they can write and still cover the concept. This is one reason it may take you two to three times as long to read your math text as it would any other text.

Remember: Reading your math text will take longer than reading your other texts.

Math students are expected to know how to do their homework; however, most math students do not have a homework system. Most students begin their homework by going directly to the problems and trying to work them. When they get stuck, they usually quit. This is not a good homework system. A good homework system will improve your homework success and math learning at the same time.

In this chapter, you will learn

- how to read a math textbook,
- how to do your homework,
- how to solve word problems,
- how to work with a study buddy.

How to read a math textbook

The way you read a math textbook is different from the traditional way students are taught to read textbooks in high school or college. Students are taught to read quickly or skim the material. If you do not understand a word, you are supposed to keep on reading. Instructors of other courses want students to continue to read so they can pick up the unknown words and their meanings from context.

This reading technique may work with your other classes, but using it in your math course will be totally confusing. By skipping some major concept words or bold-face words, you will not understand the math textbook or be able to do the homework. Reading a math textbook takes more time and concentration than reading your other textbooks.

If you have a reading problem, it would be wise to take a developmental reading course before taking math. This is especially true with reformed math courses, where reading and writing are emphasized.

Reform math classes deal more with word problems than do traditional math courses. If you cannot take the developmental reading course before taking math, then take it during the same semester as the math course.

Eight Steps to Understanding Reading Materials

There are several appropriate steps in reading a math textbook:

Step 1 *Skim the assigned reading material.* Skim the material to get the general idea about the major topics. Read the chapter introduction and each section summary. You do not want to learn the material at this time; you simply want to get an overview of the assignment. Then think about similar math topics that you already know.

Example: Skimming will allow you to see if problems presented in one chapter section are further explained in later chapter sections.

Step 2 *As you skim the chapter, circle (using pencil) the new words that you do not understand.* If you do not understand these new words after reading the assignment, then ask the instructor for help. Skimming the reading assignments should take only five to ten minutes.

Step 3 *Put all your concentration into reading.* While reading the textbook, highlight the material that is important to you. However, do not highlight more than 50 percent of a page because the material is not being narrowed down enough for future study. Especially highlight the material that is also discussed in the lecture. Material discussed in both the textbook and lecture usually appears on the test. The purpose of highlighting is to emphasize the important material for future study. Do not skip reading assignments.

Remember: Reading a math textbook is very difficult. It might take you half an hour to read and understand just *one* page.

Step 4 *When you get to the examples, go through each step.* If the example skips any steps, make sure you write down each one of those skipped steps in the textbook for better understanding. Later on, when you go back and review, the steps are already filled in. You will understand how each step was completed. Also, by filling in the extra steps, you are starting to overlearn the material for better recall on future tests.

Step 5 *Mark the concepts and words that you do not know.* Maybe you marked them the first time while skimming. If you understand them now, erase the marks. If you do not understand the words or concepts, then reread the page or look them up in the glossary. Try not to read any further until you understand all the words and concepts.

Step 6 *If you do not clearly understand some words or concepts, add these words to the note-taking glossary in the back of your notebook.* Your glossary will contain the bold-face words that you do not understand. If you have difficulty understanding the bold-face words, ask the instructor for a better explanation. You should know all the words and concepts in your notebook's glossary before taking the test.

Step 7 *If you do not understand the material, follow these eight points, one after the other, until you do understand the material:*

Point 1 – Go back to the previous page and reread the information to maintain a train of thought.

Point 2 – Read ahead to the next page to discover if any additional information better explains the misunderstood material.

Point 3 – Locate and review any diagrams, examples, or rules that explain the misunderstood material.

Point 4 – Read the misunderstood paragraph(s) several times aloud to better understand their meaning.

Point 5 – Refer to your math notes for a better explanation of the misunderstood material.

Point 6 – Refer to another math textbook, computer software program, or videotape that expands the explanation of the misunderstood material.

Point 7 – Define exactly what you do not understand and call your study buddy for help.

Point 8 – Contact your math tutor or math instructor for help in understanding the material.

Step 8 *Reflect on what you have read.* Combine what you already know with the new information that you just read. Think about how this new information enhances your math knowledge. Prepare questions for your instructor on the confusing information. Ask those questions at the next class meeting.

By using this reading technique, you have narrowed down the important material to be learned. You have skimmed the textbook to get an overview of the assignment. You have carefully read the material and highlighted the important parts. You then added to your note-taking glossary unknown words or concepts.

Remember: The highlighted material should be reviewed before doing the homework problems, and the glossary has to be learned 100 percent before taking the test.

How Reading Ahead Can Help

Reading ahead is another way to improve learning. If you read ahead, do not expect to understand everything. Read ahead two or three sections and put question marks (in pencil) by the material you do not understand.

When the instructor starts discussing that material, have your questions prepared and take good notes. Also, if the lecture is about to end, ask the instructor to explain the confusing material in the textbook. Reading ahead will take more time and effort, but it will better prepare you for the lectures.

How to Establish Study Period Goals

Before beginning your homework, establish goals for the study period. Do not just do the homework problems.

Ask yourself this question: "What am I going to do tonight to become more successful in math?"

By setting up short-term homework goals and reaching them, you will feel more confident about math. This also improves your self-esteem and helps you become a more internally motivated student. Set up homework tasks that you can complete. Be realistic.

Study period goals are set up either on a time-line basis or an item-line basis. Studying on a time-line basis is studying math for a certain amount of time.

Example: You may want to study math for an hour, then switch to another subject. You will study on a time-line basis.

Studying on an item-line basis means you will study your math until you have completed a certain number of homework problems.

Example: You might set a goal to study math until you have completed all the odd-numbered problems in the chapter review. The odd-numbered problems are the most important problems to work. These, in most texts, are answered in the answer section in the back of the book. Such problems provide the opportunity to recheck your work if you do not get the answer correct. Once you have completed these problems, do the even-numbered problems.

No matter what homework system you use, remember this important rule: Always finish a homework session by understanding a concept or doing a homework problem correctly.

Do not end a homework session with a problem you cannot complete. You will lose confidence, since all you will think about is the last problem you could not solve instead of the 50 problems you correctly solved. If you did quit on a problem you could not solve, return and rework problems you have done correctly.

Remember: Do not end your study period with a problem you could not complete.

– Section Review –

1. List and describe the eight steps to reading a math textbook.

Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

Step 5: _____

Step 6: _____

Step 7: _____

Step 8: _____

2. After trying each of the eight steps to reading your math text, make up, list, and try your own condensed version.

Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

Step 5: _____

Step 6: _____

Step 7: _____

Step 8: _____

3. List two reasons why reading ahead can improve your learning.

Reason 1: _____

Reason 2: _____

4. Describe the two types of study period goals. Which one do you use?

Type 1: _____

Type 2: _____

How to do your homework

Doing your homework can be frustrating or rewarding. Most students jump right into their homework, become frustrated, and stop studying. These students usually go directly to the math problems and start working them without any preparation. When they get stuck on one problem, they flip to the back of the text for the answer. Then they either try to work the problem backward to understand the problem steps or they just copy down the answer.

Other students go to the solution guide and just copy the steps. After getting stuck several times, these students will inevitably quit doing their homework assignment. Their homework becomes a frustrating experience, and they may even quit doing their math homework altogether.

10 Steps to Doing Your Homework

To improve your homework success and learning, refer to these ten steps:

Step 1 *Review the textbook material that relates to the homework.* A proper review will increase the chances of successfully completing your homework. If you get stuck on a problem, you will have a better chance of remembering the location of similar problems. If you do not review prior to doing your homework, you could get stuck and not know where to find help in the textbook.

Remember: To be successful in learning the material and in completing homework assignments, you must first review your textbook.

Step 2 *Review your lecture notes that relate to the homework.* If you could not understand the explanation in the textbook on how to complete the homework assignment, then review your notes.

Remember: Reviewing your notes will give you a better idea about how to complete your homework assignment.

Step 3 *Do your homework as neatly as possible.* Doing your homework neatly has several benefits. When approaching your instructor about problems with your homework, he or she will be able to understand your previous attempts to solve the problem. The instructor will easily locate the mistakes and show you how to correct the steps without having to decipher your handwriting. Another benefit is that when you review for midterm or final exams, you can quickly relearn the homework material without having to decipher your own writing.

Remember: Neatly prepared homework can help you now and in the future.

Step 4 *When doing your homework, write down every step of the problem.* Even if you can do the step in your head, write it down anyway. This will increase the amount of homework time, but you are overlearning how to solve problems, which improves your memory. Doing every step is an easy way to memorize and understand the material. Another advantage is that when you rework the problems you did wrong, it is easy to review each step to find the mistake.

Remember: In the long run, doing every step of the homework will save you time and frustration.

Step 5 *Understand the reasons for each problem step and check your answers.* Do not get into the bad habit of memorizing how to do problems without knowing the reasons for each step. Many students are smart enough to memorize procedures required to complete a set of homework problems. However, when similar home-

work problems are presented on a test, the student cannot solve the problems. To avoid this dilemma, keep reminding yourself about the rules, laws, or properties used to solve problems.

Example: *Problem:* $2(a + 5) = 0$. What property allows you to change the equation to $2a + 10 = 0$? *Answer:* The distributive property.

Once you know the correct reason for going from one step to another in solving a math problem, you can solve any problem requiring that property. Students who simply memorize how to do problems instead of understanding the reasons for correctly working the steps will eventually fail their math course.

How to Check Your Answers

Checking your homework answers should be part of your homework duties. Checking your answers can improve your learning and help you prepare for tests.

Check the answers of the problems for which solutions are not given. This may be the even-numbered or odd-numbered problems or the problems not answered in the solutions manual.

First, check your answer by estimating the correct answer.

Example: If you are multiplying 2.234 by 5.102, Remember that 2 times 5 is 10. The answer should by a little over 10.

You can also check your answers by substituting the answer back into the equation or doing the opposite function required to answer the question. The more answers you check, the faster you will become. This is very important because increasing your answer-checking speed can help you catch more careless errors on future tests.

Step 6 *If you do not understand how to do a problem, refer to the following points:*

Point 1 – Review the textbook material that relates to the problem.

Point 2 – Review the lecture notes that relate to the problem.

Point 3 – Review any similar problems, diagrams, examples, or rules that explain the misunderstood material.

Point 4 – Refer to another math textbook, solutions guide, math computer program software, or videotape to obtain a better understanding of the material.

Point 5 – Call your study buddy.

Point 6 – Skip the problem and contact your tutor or math instructor as soon as possible for help.

Step 7 *Always finish your homework by successfully completing problems.* Even if you get stuck, go back and successfully complete previous problems before quitting. You want to end your homework assignment with feelings of success.

Step 8 *After finishing your homework assignment, recall to yourself or write down the most important learned concepts.* Recalling this information will increase your ability to learn these new concepts. Additional information about step 8 will be presented later in this chapter.

Step 9 *Make up note cards containing hard-to-remember problems or concepts.* Note cards are an excellent way to review material for a test. More information on the use of note cards as learning tools is presented later in this chapter.

Step 10 *Do not fall behind.* As mentioned in Chapter 1, math is a sequential learning process. If you get behind, it is difficult to catch up because each topic builds on the next. It would

be like going to Spanish class without learning the last set of vocabulary words. The teacher would be talking to you using the new vocabulary, but you would not understand what was being said.

Do Not Fall Behind

To keep up with your homework, it is necessary to complete the homework every school day and even on weekends. Doing your homework one-half hour each day for two days in a row is better than one hour every other day.

If you have to get behind in one of your courses, *make sure it is not math.* Fall behind in a course that does not have a sequential learning process, such as psychology or history. After using the ten steps to doing your homework, you may be able to combine two steps into one. Find your best combination of homework steps and use them.

Remember: Getting behind in math homework is the fastest way to fail the course.



1. List the ten steps to doing your math homework.

Step 1: _____
_____
_____
Step 2: _____
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Step 3: _____
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Step 4: _____
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Step 5: _____
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Step 6: _____
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Step 7: _____
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Step 8: _____
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Step 9: _____
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_____
Step 10: _____
_____
_____

2. After trying the ten steps to doing your math homework, make up, list, and try your own condensed version.

Step 1: _____
_____
Step 2: _____
_____
Step 3: _____
_____
Step 4: _____
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Step 5: _____
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Step 6: _____
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Step 7: _____
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Step 8: _____
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Step 9: _____
_____
Step 10: _____
_____

continued on the next page

3. How will reviewing your notes or math text help you complete your homework more successfully?

_____
_____
_____
_____

4. List two ways you can check your homework answers.

Way 1: _____
_____
_____
_____
Way 2: _____
_____
_____
_____

5. What happens to students who fall behind in their homework?

_____
_____
_____
_____
_____
_____

How to solve word problems

The most difficult homework assignment for most math students is working story/word problems. Solving word problems requires excellent reading comprehension and translating skills.

Students often have difficulty substituting algebraic symbols and equations for English terms. But once an equation is written, it is usually easily solved. To help you solve word problems, follow these ten steps:

Step 1 *Read the problem three times.* Read the problem quickly the first time as a scanning procedure. As you are reading the problem the second time, answer these three questions:

1. *What is the problem asking me?* (Usually at the end of the problem)

2. *What is the problem telling me that is useful?* (Cross out unneeded information.)

3. *What is the problem implying?* (Usually something you have been told to remember)

Read the problem a third time to check that you fully understand its meaning.

Step 2 *Draw a simple picture of the problem to make it more real to you* (e.g., a circle with an arrow can represent travel in any form – by train, by boat, by plane, by car, or by foot).

Step 3 *Make a table of information and leave a blank space for the information you are not told.*

Step 4 *Use as few unknowns in your table as possible.* If you can represent all the unknown information in terms of a single letter, do so! When using more than one unknown, use a letter that reminds you of that unknown. Then write down what your unknowns represent. This eliminates the problem of assigning the right answer to the wrong unknown. Remember that you have to create as many separate equations as you have unknowns.

Step 5 *Translate the English terms into an algebraic equation using the list of terms in Figures 6 and 7 on page 69.* Remember, the English terms are sometimes stated in a different order than the algebraic terms.

Figure 6

Translating English Terms into Algebraic Symbols

Sum	+
Add	+
In addition	+
More than	+
Increased	+
In excess	+
Greater	+
Decreased by	-
Less than	-
Subtract	-
Difference	-
Diminished	-
Reduce	-
Remainder	-
Times as much	x
Percent of	x
Product	x
Interest on	x
Per	/
Divide	/
Quotient	/
Quantity	( )
Is	=
Was	=
Equal	=
Will be	=
Results	=
Greater than	>
Greater than or equal to	≥
Less than	<
Less than or equal to	≤

Figure 7

Translating English Words into Algebraic Expressions

English Words	Algebraic Expressions
Ten more than x	$x + 10$
A number added to 5	$5 + x$
A number increased by 13	$x + 13$
5 less than 10	$10 - 5$
A number decreased by 7	$x - 7$
Difference between x and 3	$x - 3$
Difference between 3 and x	$3 - x$
Twice a number	$2x$
Ten percent of x	$0.10x$
Ten times x	$10x$
Quotient of x and 3	$x/3$
Quotient of 3 and x	$3/x$
Five is three more than a number	$5 = x + 3$
The product of 2 and a number is 10	$2x = 10$
One-half a number is 10	$x/2 = 10$
Five times the sum of x and 2	$5(x + 2)$
Seven is greater than x	$7 > x$
Five times the difference of a number and 4	$5(x - 4)$
Ten subtracted from 10 times a number is that number plus 5	$10x - 10 = x + 5$
The sum of $5x$ and 10 is equal to the product of x and 15	$5x + 10 = 15x$
The sum of two consecutive integers	$(x) + (x + 1)$
The sum of two consecutive even integers	$(x) + (x + 2)$
The sum of two consecutive odd integers	$(x) + (x + 2)$

Step 6 *Immediately retranslate the equation, as you now have it, back into English.* The translation will not sound like a normal English phrase, but the meaning should be the same as the original problem. If the meaning is not the same, the equation is incorrect and needs to be rewritten. Rewrite the equation until it means the same as the English phrase.

Step 7 *Review the equation to see if it is similar to equations from your homework and if it makes sense.* Some formulas dealing with specific word problems may need to be rewritten. Distance problems, for example, may need to be written solving for each of the other variables in the formula. Distance = rate x time; therefore, time = distance/rate, and rate = distance/time. Usually, a distance problem will identify the specific variable to be solved.

Step 8 *Solve the equation using the rules of algebra.*

Remember: Whatever is done to one side of the equation must be done to the other side of the equation. The unknown must end up on one side of the equation, by itself. If you have more than one unknown, then use the substitution or elimination method to solve the equations.

Step 9 *Look at your answer to see if it makes common sense.*

Example: If tax was added to an item, it should cost more; if a discount was applied to an item, it should cost less. Is there more than one answer? Does your answer match the original question? Does your answer have the correct units?

Step 10 *Put your answer back into the original equation to see if it is correct.* If one side of the equation equals the other side of the equation, then you have the correct answer. If you do not have the correct answer, go back to step 5.

The most difficult part of solving word problems is translating part of a sentence into algebraic symbols and then into algebraic expressions. Review Figure 6 (Translating English Terms into Algebraic Symbols) and Figure 7 (Translating English Words into Algebraic Expressions), both on the previous page.

How to Recall What You Have Learned

After completing your homework problems, a good visual learning technique is to make note cards. Note cards are 3" by 5" index cards on which you place information that is difficult to learn or material you think will be on the test.

On the front of the note cards write a math problem or information that you need to know. Color-code the important information in red or blue. On the back of the note card, write how to work the problem or explain important information.

Example: If you are having difficulty remembering the rules for multiplying positive and negative numbers, you would write some examples on the front of the note card with the answers on the back.

Make note cards on important information you might forget. Every time you have five spare minutes, pull out your note cards and review them. You can glance at the front of the card, repeat to yourself the answer, and check yourself with the back of the card. If you are correct and know the information on a card, do not put it back in the deck. Mix up the cards you do not know and pick another card to test yourself on the information. Keep doing this until there are no cards left that you do not know.

If you are an auditory learner, then use the tape recorder like the note cards. Record the important information as you would on the front of the note card. Then leave a blank space on the recording before recording the answer. Play the tape back. When you hear the silence, put the tape on pause. Then say the answer out loud to yourself. Take the

tape player off pause and see if you were correct.

Review What You Have Learned

After finishing your homework, close the textbook and try to remember what you have learned. Ask yourself these questions: "What major concepts did I learn tonight?" "What test questions might the instructor ask on this material?"

Recall for about three to four minutes the major points of the assignment, especially the areas you had difficulty understanding. Write down questions for the instructor or tutor. Since most forgetting occurs right after learning the material, this short review will help you retain the new material.



1. List the ten steps to solving word problems.

Step 1: _____

_____

_____

Step 2: _____

_____

_____

Step 3: _____

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Step 4: _____

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Step 5: _____

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Step 6: _____

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_____

Step 7: _____

_____

_____

Step 8: _____

_____

_____

Step 9: _____

_____

_____

Step 10: _____

_____

_____

2. What is your best way to recall what you have learned?

_____

_____

_____

_____

3. How does reviewing right after doing your homework help you remember?

_____

_____

_____

_____

How to work with a study buddy

You need to have a study buddy when you miss class or when you do your homework. A study buddy is a friend or classmate who is taking the same course. You can find a study buddy by talking to your classmates or making friends in the math lab.

Try to find a study buddy who knows more about math than you do. Tell the class instructor that you are trying to find a study buddy and ask which students make the best grades. Meet with your study buddy several times a week to work on problems and to discuss math. If you miss class, get the notes from

your study buddy so you will not get behind.

Call your study buddy when you get stuck on your homework. You can solve math problems over the phone. Do not sit for half an hour or an hour trying to work one problem; that will destroy your confidence, waste valuable time, and possibly alienate your study buddy. Think how much you could have learned by trying the problem for 15 minutes and then calling your study buddy for help. Spend, at the maximum, 15 minutes on one problem before going on to the next problem or calling your study buddy.

> **Remember:** A study buddy can improve your learning while helping you complete the homework assignment. Just do not overuse your study buddy or expect that person to do your homework for you.



1. How can you select your study buddy?

   _____
   _____
   _____

2. How can a study buddy help you learn math?

   _____
   _____
   _____
   _____
   _____

The benefits of study breaks

Psychologists have discovered that learning decreases if you do not take study breaks. Therefore, take a break after studying math for 45 minutes to one hour.

If you have studied for only 15 or 20 minutes and feel you are not retaining the information or your mind is wandering, take a break. If you continue to force yourself to study, you will not learn the material. After taking a break, return to studying.

If you still cannot study after taking a break, review your purpose for studying and your educational goals. Think about what is required to graduate. It will probably come down to the fact that you will have to pass math. Think about how studying math today will help you pass the next test; this will increase your chances of passing the course and of graduating.

Write on an index card three positive statements about yourself and three positive statements about studying. Look at this index card every time you have a study problem. Use every opportunity available to reinforce your study habits.



1. How can study breaks improve your learning?

   _____
   _____
   _____
   _____
   _____

Name: _____ Date: _____



Chapter 5 Review



1. Reading one page of a math text might take you _____ _____ _____.

2. Checking your homework answers can improve your _____ and help prepare you for a _____.

Matching

(for students in pre-algebra class)

3. _____	Product	A. +
4. _____	Increased	B. -
5. _____	Sum	C. x
6. _____	Per	D. /
7. _____	Product	E. =
8. _____	Is	
9. _____	Reduced	
10. _____	Remainder	

(for students taking elementary algebra or higher-level math course)

3. _____	Difference between x and 3	A. $10x$
4. _____	Quotient of x and 3	B. $x/3$
5. _____	Five times the sum of x and 2	C. $5(x-4)$
		D. $5(x+2)$
6. _____	A number added to 5	E. $5=x+3$
7. _____	Five less than 10	F. $5+x$
8. _____	Ten times x	G. 10 -5
9. _____	Five is three more than a number	H. $x-3$
10. _____	Five times the difference of a number and four	

11. Make sure you finish your homework by working problems you _____ do.

12. Using _____ _____ is a visual way to recall what you learned in math.

13. Using a _____ _____ is an auditory way to recall what you learned in math.

14. Most _____ occurs right after learning; a short review will help you _____ the new material.

15. A _____ _____ can improve your learning while helping you complete the homework assignment.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

How can you immediately use it?

_____
_____
_____
_____
_____
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_____
_____
_____

Chapter 6

How to Remember What You Have Learned

To understand the learning process, you must understand how your memory works. You learn by conditioning and thinking. But memorization is different from learning. For memorization, your brain must perform several tasks, including receiving the information, storing the information, and recalling the information.

By understanding how your memory works, you will be better able to learn at which point your memory is failing you. Most students usually experience memory trouble between the time their brain receives the information and the time the information is stored.

There are many techniques for learning information that can help you receive and store information without losing it in the process. Some of these techniques may be more successful than others, depending on how you learn best.

In this chapter, you will discover

- how you learn,
- how short-term memory affects what you remember,
- how working memory affects what you remember,
- how long-term memory/reasoning affects what you remember,
- how to use memory techniques,
- how to develop practice tests, and
- how to use number sense.

How you learn

Educators tell us that learning is the process of "achieving competency." More simply put, it is how you become good at something. The three ways of learning are by conditioning, by thinking, and by a combination of conditioning and thinking.

Learning by Conditioning and Thinking

Conditioning is learning things with a maximum of physical and emotional reaction and a minimum of thinking.

> **Examples:** Repeating the word *pi* to yourself and practicing where the symbol is found on a calculator are two forms of conditioned learning. You are learning using your voice and your eye-hand coordination (physical activities), and you are doing very little thinking.

Thinking is defined as learning with a maximum of thought and a minimum of emotional and physical reaction.

> **Example:** Learning about *pi* by thinking is different from learning about it by conditioning. To learn "pi" by thinking, you would have to do the calculations necessary to result in the numeric value that the word *pi* represents. You are learning *using your mind* (thought activities), and you are using very little emotional or physical energy to learn *pi* in this way.

The most successful way to learn is to combine thinking and conditioning. The best learning combination is to learn by thinking first and conditioning second.

Learning by thinking means that you learn by

- *observing*,
- *processing*, and
- *understanding* the information.

How Your Memory Works

Memory is different from learning; it requires reception, storage, and retrieval of information. You *receive* information through your five senses (*sensory input*): what you see, feel, hear, smell, and taste.

Examples: In math class, you will use your sense of *vision* both to watch the instructor demonstrate problems on the chalkboard and to read printed materials. You will use your sense of *hearing* to listen to the instructor and other students discuss the problems. Your sense of *touch* will be used to operate your calculator and to appreciate geometric shapes. In chemistry and other classes, however, you may additionally use your senses of *smell* and *taste* to identify substances.

The sensory register briefly holds an exact image or sound of each sensory experience until it can be processed. If the information is not processed immediately, it is forgotten. The sensory register helps us go from one situation to the next without cluttering up our minds with trivial information.

Processing the information involves placing it in either short-term memory, working memory, or long-term memory.

How short-term memory affects what you remember

Information that passes through the sensory register is stored in short-term memory. Remembering something for a short time is not hard to do for most students. By conscious effort, you can remember math laws, facts, and formulas received by the sensory register (your five senses) and put them in short-term memory. You can recognize them and register them in your mind as something to remember for a short time.

Example: When you are studying math, you can tell yourself the distributive property is illustrated by $a(b + c) = ab + ac$. By deliberately telling yourself to remember that fact (by using conditioning – repeating or writing it again and again), you can remember it, at least for a while, because you have put it in short-term memory.

Psychologists have found that short-term memory cannot hold an unlimited amount of information. You may be able to use short-term memory to remember one phone number or a few formulas but not five phone numbers or ten formulas.

Items placed into short-term memory usually fade fast, as the name suggests.

Examples: Looking up a telephone number in the directory, remembering it long enough to dial, then forgetting it immediately; learning the name of a person at a large party or in a class but forgetting it completely within a few seconds: cramming for a test and forgetting most of it before taking the test.

Short-term memory involves the ability to recall information immediately after it is given (without any interruptions). It is useful in helping you to concentrate on a few concepts at a time, but it is not the best way to learn. This is especially true of students who have attention problems or problems with

short-term memory.

Students with short-term memory problems forget a math step as soon as the instructor explains it. To remember more facts or ideas and keep them in memory a longer period of time – especially at test time – use of a better system than short-term memory is required.

Changes in the way math is taught include the idea that conditioning (the most often used teaching form) leads only to short-term memory. Conditioning is often used for short-term recall of concept skills. The information learned in this way can only be stored for longer periods of time by thinking. Thinking takes place when applying the skills to working the problems.

How working memory affects what you remember

Working memory (or long-term retrieval) is that part of the brain that can work on problems for a longer period of time than can short-term memory. Working memory, then, offers an increase in the amount of *time* that information is held in memory. (An increase in the *volume* of information that can be held requires long-term memory.)

Working memory involves the ability to recall information after learning has been consistently interrupted over a period of several minutes. Students with working memory problems may listen to a math lecture and understand each step as it is explained. When the instructor goes back to a previous step discussed several minutes prior, however, the student has difficulty explaining or remembering the reasons for the steps. These students have difficulty remembering series steps long enough to understand the concept.

Working memory can be compared to a mental work space or an internal chalkboard. Just like a chalkboard, working memory has limited space, which can cause a "bottleneck" in learning.

Working memory uses the information (such as multiplication tables) recalled from long-term memory. Overlearning math facts – by storing them in long-term memory – can free up more working memory to solve problems. Working memory is like how much RAM you have in your computer.

> **Example:** In solving 26 x 32, you would put the intermediate products 52 (*from 2* x *26*) and 780 (*from 30* x *26* – remember, 3 is in the 10s place, so make it 30) into working memory and add them together. The more automatic the multiplication, the less working memory you use. If you cannot remember your multiplication, you use up working memory trying to solve the multiplication problem. This leaves you with less working memory to solve the resulting addition problem.

How long-term memory/reasoning affects what you remember

Long-term memory is a storehouse of material that is retained for long periods of time. Long-term memory is recalled into working memory to solve problems. It is *not* a matter of trying harder and harder to remember more and more unrelated facts or ideas; it is a matter of organizing your short-term memories and working memories into meaningful information.

Reasoning is thinking about memories, comprehending their meanings, and understanding their concepts. Without rehearsing the information, it will not be processed into long-term memory/reasoning.

The main problem students face is converting learned material from short-term memory or working memory into long-term memory – and understanding it. To place information into long-term memory, you need to understand it and effectively concentrate on it.

> **Remember:** Securing math information into long-term memory is not accomplished by just doing the homework – you must also understand it.

The Role of "Memory Output" in Testing

Memory output is what educators call a retrieving process. It is necessary for verbal or written examinations. It is the method by which you put information stored in your long-term memory onto paper or into words. Memory output can be blocked by three things:

1. insufficient processing of information into long-term memory,
2. test anxiety, or
3. poor test-taking skills.

If you did not completely place all of the information you learned into long-term memory, you may not be able to give complete answers on tests.

Test anxiety can decrease your ability to recall important information or it can cause you to block out information totally. Ways to decrease test anxiety were discussed in Chapter 3. Students who work on their test-taking skills can improve their memory output. Techniques to improve both your memory output and your test-taking skills will be explained in Chapter 7.

Understanding the Stages of Memory

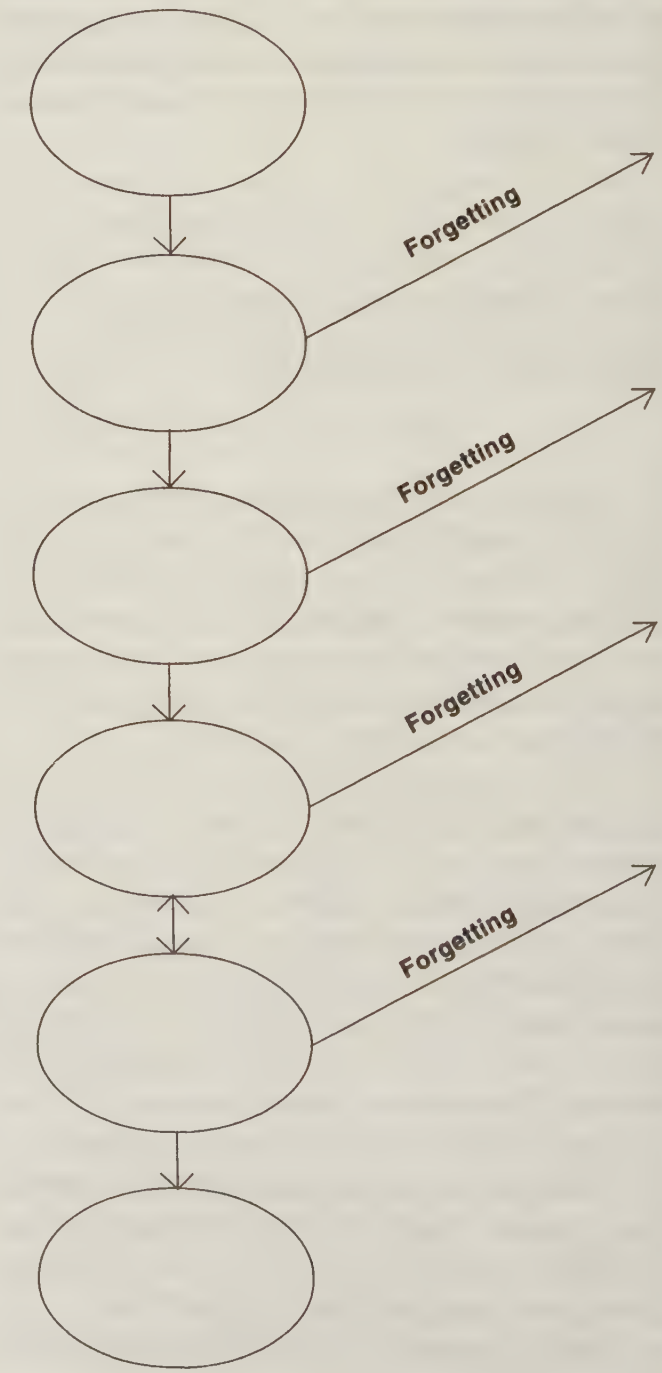
Understanding the stages of memory will help you answer this common question about learning math: "How can I understand the procedures to solve a math problem one day and forget how to solve a similar problem two days later?"

There are three good answers to this question. First, after initially learning how to solve the problem, you did not *rehearse* the solving process enough for it to enter your long-term memory. Second, you did get the information into long-term memory, but the information was not *reviewed* frequently enough and was therefore forgotten. Third, you memorized how to work the problem but did not *understand* the concept.

Remember: Locating where your memory breaks down and compensating for those weaknesses will improve your math learning.

– Section Review –

1. Label the ovals below with stages of memory and list the reasons for forgetting under the slanted arrows.



2. Describe the stages of memory by explaining and giving examples of the following terms:

Sensory input _____

_____

_____

Example _____

_____

Sensory register _____

_____

_____

Example _____

_____

Short-term memory _____

_____

_____

Example _____

_____

Working memory _____

_____

_____

Example _____

_____

Long-term memory _____

_____

_____

Example _____

_____

Memory output _____

_____

_____

Example _____

_____

3. What are three conditions that can block your memory output?

Condition 1: _____

_____

_____

Condition 2: _____

_____

_____

Condition 3: _____

_____

_____

4. Review the stages of memory and list the stages at which your memory breaks down. For each stage of breakdown, say how you can prevent it.

Stage _____

_____

_____

Prevention _____

_____

_____

_____

Stage _____

_____

_____

Prevention _____

_____

_____

_____

Stage _____

_____

_____

Prevention _____

_____

_____

_____

How to use memory techniques

There are many different techniques that can help you store information in your long-term memory. Having a positive attitude about studying, using your best learning sense(s), decreasing distractions, and other techniques can improve your long-term memory.

Have a Good Study/Math Attitude

Having a positive attitude about studying will help you concentrate and improve your retention. This means you need to have at least a neutral math attitude (you neither like nor dislike it), and you should reserve the right to actually learn to *like* math. View studying as an opportunity to learn rather than as an unpleasant task. Tell yourself that you *can* learn the material and that *learning it will help you pass the course and graduate.*

Use Your Best "Learning Sense"

Using your best *learning sense* (what educators call your "predominant learning modality") can improve how well you learn and enhance the transfer of knowledge into long-term memory/reasoning. Your learning senses are vision, hearing, touching, and so on. Ask yourself if you learn best by watching (vision), listening (hearing), or touching (feeling).

Visual (Watching) Learner

Knowing that you are a *visual math learner* can help you select the memory technique that will work best for you. Repeatedly reading and writing math materials being studied is the best way for a visual learner to study.

A visual way to decrease distractions is using the "my mind is full" concept. Imagine that your mind is completely filled with thoughts of learning math, and other distracting thoughts cannot enter. Your mind has one-way input and output, which only responds to thinking about math when you are doing homework or studying.

Auditory (Hearing) Learner

If you are an *auditory learner* (one who learns best by hearing the information), then learning formulas may be best accomplished by repeating them back to yourself or recording them on a tape recorder and listening to them. Reading out loud is one of the best auditory ways to get important information into long-term memory. Saying facts and ideas out loud improves your ability to think and remember. If you cannot recite out loud, recite the material to yourself, emphasizing the key words.

An auditory way to improve your concentration is by becoming aware of your distractions and telling yourself, out loud, to concentrate. If you are in a location where talking out loud will cause a disturbance, mouth the words "start concentrating" as you say them in your mind. Your concentration periods should increase.

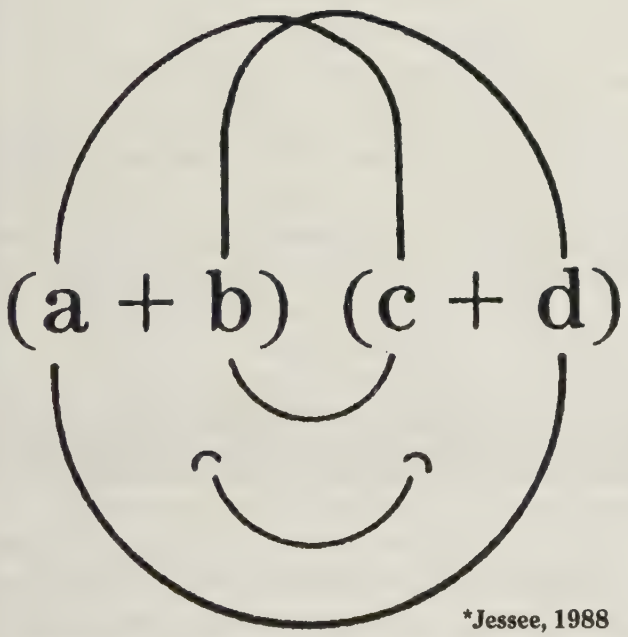
Tactile/Concrete (Touching) Learner

A tactile/concrete learner needs to feel and touch the material to learn it. Unfortunately, this learning sense is not used by most math instructors. As a result, students who depend heavily on feeling and touching for learning will have the most difficulty developing effective learning techniques. For example, if you wanted to learn the FOIL method (see Figure 8, on the facing page, and the acronym example on page 83), you would take your fingers and trace the "face" to remember the steps.

A tactile/concrete way to improve your study concentration is by counting the number of distractions for each study session. Place a sheet of paper by your book when doing homework. When you catch yourself not concentrating, put the letter C on the sheet of paper. This will remind you to concentrate and get back to work. After each study period, count up the number of C's and watch the number decrease.

Figure 8
The FOIL Method

F (a) (c)
O (a) (d)
I (b) (c)
L (b) (d)



*Jessee, 1988

FOIL is used to remember the procedure to *multiply two binomials.* The letters in FOIL stand for **F**irst, **O**utside, **I**nside, and **L**ast. To use FOIL, multiply

— the **F**irst terms, (*a*) (*c*),
— the **O**utside terms, (*a*) (*d*),
— the **I**nside terms, (*b*) (*c*),
— the **L**ast terms, (*b*) (*d*).

To learn FOIL, trace your finger along the FOIL route.

Using Multiple Senses

If you have difficulty learning material from one sense, you might want to try learning through two or three senses. Involving two or more senses in learning improves your learning and remembering.

If your primary sense is visual and your secondary sense is auditory, you may want to write down equations while saying them out loud to yourself. Writing and reciting the material at the same time combines visual and auditory and some tactile/concrete styles of learning and is therefore an excellent way to improve learning

Studying with a pen or highlighter is a visual as well as a tactile/concrete way to improve your concentration. Placing the pen or highlighter in your hand and using it will force you to concentrate more on what you are reading. After you correctly write and recite the material back to yourself one time, do it five or ten more times to overlearn it.

Be a Selective Learner

Being selective in your math learning will improve your memory. Prioritize the materials you are studying; decide which facts you need to know and which ones you can ignore. Narrow down information into laws and principles that can be generalized. Learn the laws and principles 100 percent.

Example: If you have been given a list of math principles and laws to learn for a test, put each one on an index card. As you go through them, create two piles: a "I already know this" pile and a "I don't know this" pile. Then study *only* the "I don't know this" pile. Study the "I don't know this" pile until it is completely memorized and understood.

Become an Organizer

Organizing your math material into idea/fact clusters or groups will help you learn and memorize it. Grouping similar material in a problem log or

calculator log is an example of categorizing information. Do not learn isolated facts; always try to connect them to other similar material.

Use Visual Imagery

Using mental pictures or diagrams to help you learn the material is especially helpful for the visual learners and those who are right-hemisphere dominant (who tend to learn best by visual and spatial methods). Mental pictures and actual diagrams involve 100 percent of your brain power. Picture the steps to solve difficult math problems in your mind.

Example: Use the FOIL method (see Figure 8, page 81) to visually learn how to multiply binomials. Memorize the face until you can sketch it from memory. If you need to use it during a test, you can then sketch the face onto your scratch paper and refer to it.

Make Associations

Association learning can help you remember better. Find a link between new facts and some well-established old facts and study them together. The recalling of old facts will help you remember the new facts and strengthen a mental connection between the two. Make up your own associations to remember math properties and laws.

Remember: The more ridiculous the association, the more likely you are to remember it.

Examples: When learning the *commutative property*, remember that the word *commutative* sounds like the word *community*. A community is made up of different types of people who could be labeled *as* an a group and a b group. However, in a community of a people and b people, it does not matter if we count the a people first or the b people first; we still have the same total number of people in the community. Thus, $a + b = b + a$.

When learning the *distributive law of multiplication over addition*, such as $a(b + c)$, remember that "distributive" sounds like "distributor," which is associated with giving out a product. The distributor a is giving its products to b and c.

Use Mnemonic Devices

The use of mnemonic devices is another way to help you remember. Mnemonic devices are easily remembered words, phrases, or rhymes associated with difficult-to-remember principles or facts.

Example: Many students become confused with the *order of operations*. These students mix up the order of the steps in solving a problem, such as dividing before adding the numbers in the parentheses. A mnemonic device to remember the order of operations is "Please Excuse My Dear Aunt Sally." The first letter in each of the words represents the math function to be completed from the first to the last. Thus, the order of operations is **P**arentheses (***Please***), **E**xponents (***Excuse***), **M**ultiplication (***My***), **D**ivision (***Dear***), **A**ddition (***Aunt***), and **S**ubtraction (***Sally***). Remember to multiply and/or divide whatever comes first, from left to right. Also, add or subtract whatever comes first, from left to right.

Use Acronyms

Acronyms are another memory device to help you learn math. Acronyms are word forms created from the first letters of a series of words.

Example: FOIL is one of the most common math acronyms. FOIL is used to remember the procedure for multiplying two binomials. Each letter in the word *FOIL* represents a math operation. **FOIL** stands for **F**irst, **O**utside, **I**nside, and **L**ast, as it applies to multiplying two binomials such as $(2x + 3)(x + 7)$. The **F**irst quantities are $2x$ (in the first expression) and x (in the second expression). The **O**utside quantities are $2x$ (in the first expression) and 7 (in the second expression). The **I**nside quantities are 3 (in the first expression) and x (in the second expression). The **L**ast quantities are 3 (in the first expression) and 7 (in the second expression). This results in **F** $(2x)(x)$ + **O** $(2x)(7)$ + **I** $(3)(x)$ + **L** $(3)(7)$.

Do the multiplication to get $2x^2 + 14x + 3x + 21$, which adds up to $2x^2 + 17x + 21$. See Figure 8 (The FOIL Method), page 81.

– Section Review –

1. How can visual learners improve their memory?

2. How can auditory learners improve their memory?

3. How can tactile/concrete learners improve their memory?

4. How can you use multiple senses to improve your memory?

5. Give an example of being a selective learner in your math class.

6. From your current math lessons, make up and explain one association remembering device that is not in this workbook.

7. For your next major math test, make up and explain one mnemonic device that is not in this workbook.

8. For your next major math test, make up and explain one acronym that is not in this workbook.

How to develop practice tests

Developing a practice test is one of the best ways to evaluate your memory and math skills before taking a real test. You want to find out what you do not know *before* the real test instead of *during* the test. Practice tests should be as real as possible and should include time constraints.

You can create a practice test by reworking all the problems since your last test that you have recorded in your problem log. Another practice test can be developed using every other problem in the textbook chapter tests. Further, you can use the solutions manual to generate other problems with which to test yourself. You can also use old exams from the previous semester. Check to see if the math lab/LRC or library has tests on file from previous semesters, or ask your instructor for other tests. For some students, a better way to prepare for a test is the group method.

Example: Hold a group study session several days before the test. Have each student prepare a test with ten questions. On the back of the test, have the answers listed, worked out step by step. Have each member of the study group exchange his or her test with another member of the group. Once all the tests have been completed, have the author of each test discuss with the group the procedures used to solve those problems.

If group work improves your learning, you may want to hold a group study session at least once a week. Make sure the individual or group test is completed at least three days before the real test.

Completing practice math tests will help you increase your testing skills. It will also reveal your test problem weaknesses in enough time for you to learn how to solve the problem before the real test. If you have difficulty with any of the problems during class or after taking the practice test, be sure to see your tutor or instructor.

After taking the practice test(s), you should know what parts you do not understand (and need to study) and what is likely to be on the test. Put this valuable information on one sheet of paper. This information needs to be understood and memorized. It may include formulas, rules, or steps to solving a problem.

Use the learning strategies discussed in this chapter to remember this information. A good example of how this information should look is what students might call a mental "cheat sheet." Obviously, you cannot use the written form of this sheet during the real test.

If you cannot take a practice test, put down on your mental cheat sheet the valuable information you will need for the test. Work to understand and memorize your mental cheat sheet. Chapter 7, "How to Improve Your Math Test-Taking Skills," will discuss how to use the information on the mental cheat sheet – *without cheating.*



1. List three different ways you can make up a practice test.

   Way 1: _____

   _____

   _____

   _____

   Way 2: _____

   _____

   _____

   _____

   Way 3: _____

   _____

   _____

   _____

2. List ten different types of problems you would put

on your practice test for the next exam.

Problem 1: _____

_____

Problem 2: _____

_____

Problem 3: _____

_____

Problem 4. _____

_____

Problem 5: _____

_____

Problem 6: _____

_____

Problem 7: _____

_____

Problem 8: _____

_____

Problem 9: _____

_____

Problem 10: _____

_____

3. Give these ten problems to your study buddy and get ten problems from him or her. Compare your answers with those of your study buddy.

   How many did you get right? _____
   How can you correct the wrong answers? _____
   _____
   _____
   _____

How to use number sense

When taking your practice tests, you should use "number sense," or estimations, to make sure your answer is reasonable. Number sense is like common sense but it applies to math. Number sense is the ability to see if your answer makes sense without using algorithms. (Algorithms are the sequential math steps used to solve problems.) These following examples demonstrate solving a math problem (from a national math test given to high school students) using algorithms and number sense.

Example: Solve 3.04 x 5.3. Students use algorithms to solve this problem by multiplying the number 3.04 by 5.3, in sequence. Seventy-two percent of the students answered the problem correctly using algorithms.

Example: Estimate the product 3.04 x 5.3, and choose from the following answers.

A)	1.6	C) 160
B)	16	D) 1600

Only 15 percent of the students chose B, which is the correct answer. Twenty-eight percent of the students chose A. Using *estimating* to solve the answer, a whopping 85 percent of the students got the problem wrong.

These students were incorrectly using their "mental chalkboard" instead of using number sense. In using number sense to answer, you would multiply the numbers to the left of the decimal in each number to get an estimate of the answer. To estimate the answer, then, you would multiply 3 (the number to the left of the decimal in 3.04) by 5 (the number to the left of the decimal in 5.3) and expect the answer to be a little larger than 15.

It appears that the students' procedural processing (the use of algorithms) was good, but when asked to solve a non-routine problem using estimating (which is easier than using algorithms), the results were disappointing.

Another example of using number sense, or estimating, is in "rounding off."

Taking the time to estimate the answer to a math problem is a good way to check your answer. Another way to check your answer is to see if it is reasonable. Many students forget this important step and get the answer wrong. This is especially true of word or story problems.

Example: Solve 48 + 48 by rounding off. Rounding off means mentally changing the number (up or down) to make it more manageable to you, without using algorithms. By rounding off, 48 becomes 50 (easier to work with). 50 + 50 = 100. If the choices for answers were 104, 100, 98 and 96, you would then subtract 4 from the 100 (since each number was rounded up by 2) and you would get 96.

Also, when dealing with an equation, make sure to put the answer back into the equation to see if one side of the equation equals the other. If the two sides are not equal, you have the wrong answer. If you have extra time left over after you have completed a test, you should check answers using this method.

Examples: When solving a rate-and-distance problem, use your common sense to realize that one car cannot go 500 miles per hour to catch the other car. However, the car can go 50 miles per hour.

The same common sense rule applies to age-word problems, where the age of a person cannot be 150 years, but it can be 15 years.

Further, in solving equations, x is *usually* a number that is less than 20. When you solve a problem for x and get 50, then, this isn't reasonable, and you should recheck your calculations.

Example: In solving the equation $x + 3 = 9$, you calculated that $x = 5$. To check your answer, substitute x with 5 and see if the problem works out correctly. 5 + 3 does not equal 9, so you know you have made a mistake and need to recalculate the problem. The correct answer, by the way, is $x = 6$.

Remember: Number sense is a way to get more math problems correct by estimating your answer to determine if it is reasonable.

– Section Review –

1. Give an explanation of number sense. _____

2. How does number sense compare to common sense? _____

3. Give an example of a recent math problem you missed due to poor number sense. _____

4. List two ways you can improve your number sense.

   First Way _____

   Second Way _____



Chapter 6 Review



1. _____ is learning with a maximum of physical and _____ reaction and a minimum of thinking.

2. _____ is defined as learning with a maximum of thought and a minimum of emotional and _____ reaction.

3. The way you receive information is through your five senses, which are _____, _____, _____, _____, and _____.

4. The _____ _____ briefly holds an exact image or sound of each sensory experience until it can be processed or forgotten.

5. Short-term memory involves the ability to _____ information _____ after it is given without any interruptions.

6. Working memory involves the ability to recall information after learning has been _____ over a period of several _____.

7. Working memory can also be considered your internal _____ that recalls information from _____ _____.

8. The main problem most students have is converting learned material into _____ _____ and _____ it.

9. Repeatedly reading and writing math material is one of the best ways for a _____ learner to study.

10. Reading _____ _____ is one of the best auditory ways to learn material.

11. A _____/_____ learner needs to feel and touch material to learn it.

12. Being a selective learner means _____ the material to study and learning the laws and principles _____ percent.

13. Mnemonic devices are easy-to-remember _____, _____, or _____ associated with difficult-to-remember principles or facts.

14. _____ are word forms created from the first letters of a series of words.

15. The reason to develop a practice test is to find out what you do not know _____ the test instead of _____ the test.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
_____
_____
_____

How can you immediately use it?

_____
_____
_____
_____
_____
_____
_____
_____
_____

Chapter 7

How to Improve Your Math Test-Taking Skills

In this chapter, you will learn

- why attending class and doing your homework may not be enough to pass,
- the general pretest rules,
- the ten steps to better test taking,
- the six types of test-taking errors, and
- how to prepare for the final exam.

Taking a math test is different from taking tests in other subjects. Math tests require not only that you to recall the information, but that you apply the information. Multiple-choice tests, for example, usually test you on recall, and if you do not know the answer, you can guess.

Math tests build on each other, whereas history tests often do not test you on previous material. Most math tests are speed tests, where the faster your are, the better grade you can receive; most social science tests are designed for everyone to finish.

Math test preparation and test-taking skills are different from preparation and skills needed for other tests. You need to have a test-taking plan and a test analysis plan to demonstrate your total knowledge on math tests. Students with these plans make better grades compared to students without them. Math instructors want to measure your math knowledge, not your poor test-taking skills.

Why attending class and doing your homework may not be enough to pass

Most students and some instructors believe that attending class and doing all the homework ensures an A or B on tests. This is far from true. Doing all the homework and getting the correct answers is very different in many ways from taking tests:

1. While doing homework, there is little anxiety. A test situation is just the opposite.

2. You are not under a time constraint while doing your homework; you may have to complete a test in 55 minutes or less.

3. If you get stuck on a homework problem, your textbook and notes are there to assist you. This is not true for most tests.

4. Once you learn how to do several problems in a homework assignment, the rest are similar. In a test, the problems are in random order.

5. In doing homework, you have the answers to at least half the problems in the back of the text and answers to all the problems in the solutions guide. This is not true for tests.

6. While doing homework, you have time to figure out how to correctly use your calculator.

During the test, you can waste valuable time figuring out how to use your calculator.

7. When doing homework, you can call your study buddy or ask the tutor for help, something you cannot do on the test.

Do not develop a false sense of security by believing you can make an A or B by doing just your homework. Tests measure more than just your math knowledge.

The general pretest rules

General rules are important when taking any type of test:

1. *Get a good night's sleep before taking a test.* This is true for the ACT, the SAT, and your math tests. If you imagine you are going to cram all night and perform well on your test with three to four hours of sleep, you are wrong. It would be better to get seven or eight hours of sleep and be fresh enough to use your memory to recall information needed to answer the questions.

2. *Start studying for the test at least three days ahead of time.* Make sure you take a practice test to find out, before the test, what you do not know. Review your problem log and work the problems. Review the concept errors you made on the last test. (How to identify and correct your concept errors will be discussed later in this chapter.) Meet with the instructor or tutor for help on those questions you cannot solve.

3. *Review only already learned material the night before a test.*

4. *Make sure you know all the information on your mental cheat sheet.* Review your notebook and glossary to make sure you understand the concepts. Work a few problems and recall the information on your mental cheat sheet right before you go to bed. Go directly to bed; do not watch television, listen to the radio, or party. While you are asleep, your mind will work on and remember the last thing you did before going to sleep.

5. *Get up in the morning at your usual time and review your notes and problem log.* Do not do any new problems, but make sure your calculator is working.



1. List five reasons why only attending class and doing your homework may not be enough to pass your math course.

Reason 1: _____
_____
_____

Reason 2: _____
_____
_____

Reason 3: _____
_____
_____

Reason 4: _____
_____
_____

Reason 5: _____
_____
_____

2. List and explain three general pretest rules that best apply to you.

Rule 1: _____
_____
_____

Rule 2: _____
_____
_____

Rule 3: _____
_____
_____

The ten steps to better test taking

Once you begin a test, follow the ten steps to better test taking:

Step 1 *Use a memory data dump.* Upon receiving your test, turn it over and write down the information that you put on your mental cheat sheet. Your mental cheat sheet has now turned into a mental list, and writing down this information is not cheating. Do not put your name on the test, do not skim it, just turn it over and write down those facts, figures, and formulas from your mental cheat sheet or other information you might not remember during the test. This is called your *first memory data dump.* The data dump provides memory cues for test questions.

Example: It might take you a while to remember how to do a coin problem. However, if you had immediately turned your test over and written down different ways of solving coin problems, it would be easier to solve the coin problem.

Step 2 *Preview the test.* Put your name on the test and start previewing. Previewing the test requires you to look through the entire test to find different types of problems and their point values. Put a mark by the questions that you can do without thinking. These are the questions that you will solve first.

Step 3 *Do a second memory data dump.* The secondary data dump is for writing down material that was jarred from your memory while previewing the test. Write this information on the back of the test.

Step 4 *Develop a test progress schedule.* When you begin setting up a test schedule, determine the point value for each question. Some test questions might be worth more points than others.

In some tests, word problems are worth 5 points and other questions might be worth 2 or 3 points. You must decide the best way to get the most points in the least amount of time. This might mean working the questions worth 2 or 3 points first and leaving the more difficult word problems for last.

Decide how many problems should be completed halfway though the test. You should have more than half the problems completed by that time.

Step 5 *Answer the easiest problems first.* Solve, in order, the problems you marked while previewing the test. Then review the answers to see if they make sense. Start working through the test as fast as you can while being accurate. Answers should be reasonable.

Example: The answer to a problem asking you to find the area of a rectangle cannot be negative, and the answer to a land-rate-distance problem cannot be 1000 miles per hour.

Clearly write down each step to get partial credit. Even if you end up missing the problem,

you might get some credit. In most math tests, the easier problems are near the beginning of the first page; you need to answer them efficiently and quickly. This will give you both more time for the harder problems and time to review.

Step 6 *Skip difficult problems.* If you find a problem that you do not know how to work, read it twice and automatically skip it. Reading it twice will help you understand the problem and put it into your working memory. While you are solving other problems, your mind is still working on that problem. Difficult problems could be the type of problem you have never seen before or a problem in which you get stuck on the second or third step. In either case, skip the problem and go on to the next one.

Step 7 *Review the skipped problems.* When working the skipped problems, think how you have solved other, similar problems as a cue to solving the skipped ones. Also try to remember how the instructor solved that type of problem on the board.

While reviewing skipped problems, or at any other time, you may have the "Aha!" response. The "Aha!" response is your remembering how to do a skipped problem. Do not wait to finish your current problem. Go to the problem on which you had the "Aha!" and finish that problem. If you wait to finish your current problem, your "Aha!" response could turn into an "Oh, no!" response.

Step 8 *Guess at the remaining problems.* Do as much work as you can on each problem, even if it is just writing down the first step. If you cannot write down the first step, rewrite the problem. Sometimes rewriting the problem can jar your memory enough to do the first step or the entire problem. If you leave the problem blank, you will get a zero. Do not waste too much time on guessing or trying to work the problems you cannot do.

Step 9 *Review the test.* Look for careless errors or other errors you may have made. Students usually lose 2 to 5 test points on errors that could have been caught in review. Do not talk yourself out of an answer just because it may not look right. This often happens when an answer does not come out even. It is possible for the answer to be a fraction or decimal.

Remember: Answers in math do not have "dress codes." Research reveals that the odds of changing a right answer to a wrong answer are greater than the odds of changing a wrong answer to a right one.

Step 10 *Use all the allowed test time.* Review each problem by substituting the answer back into the equation or doing the opposite function required to answer the question. If you cannot check the problem by the two ways mentioned, rework the problem on a separate sheet of paper and compare the answers. Do not leave the test room unless you have reviewed each problem two times or until the bell rings.

Remember: There is no prize for handing your test in first, and students who turn their papers in last do make A's.

Stapling your scratch paper to the math test when handing it in has several advantages:

- If you miscopied the answer from the scratch paper, you will probably get credit for the answer.
- If you get the answer incorrect due to a careless error, your work on the scratch paper could give you a few points.
- If you do get the problem wrong, it will be easier to locate the errors when the instructor reviews the test. This will prevent you from making the same mistakes on the next math test.

Remember: Handing in your scratch paper may get you extra points or improve your next test score.

– Section Review –

1. List and explain the ten steps to test taking.

Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

Step 5: _____

Step 6: _____

Step 7: _____

Step 8: _____

Step 9: _____

Step 10: _____

2. After trying the ten steps to test taking, develop your own personalized test-taking steps.

Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

Step 5: _____

Step 6: _____

Step 7: _____

Step 8: _____

Additional Steps _____

The six types of test-taking errors

To improve future test scores, you must conduct a test analysis of previous tests. In analyzing your tests, you should look for the following kinds of errors:

1. misread-directions errors
2. careless errors
3. concept errors
4. application errors
5. test-taking errors
6. study errors

Students who conduct math test analyses will improve their total test scores.

Misread-directions errors occur when you skip directions or misunderstand directions, but do the problem anyway.

Examples: Suppose you have this type of problem to solve: $(x + 1)(x + 1)$. Some students will try to solve for x, but the problem only calls for multiplication. You would solve for x only if you have an equation such as $(x + 1)(x + 1) = 0$.
Another common mistake is not reading the directions before doing several word problems or statistical problems. All too often, when a test is returned, you find only three out of the five problems had to be completed. Even if you did get all five of them correct, it cost you valuable time that could have been used obtaining additional test points.

To avoid misread-directions errors, read all the directions. If you do not understand them, ask the instructor for clarification.

Careless errors are mistakes that you can catch automatically upon reviewing the test. Both good and poor math students make careless errors. Such errors can cost a student a higher letter grade on a test.

Examples: *Dropping the sign:* $-3(2x) = 6x$, instead of $-6x$, which is the correct answer.
Not simplifying your answer: Leaving $(3x - 12)/3$ as your answer instead of simplifying it to $x - 4$.
Adding fractions: $1/2 + 1/3 = 2/5$, instead of $5/6$, which is the correct answer.
Word problems: $x = 15$ instead of "The student had 15 tickets."

However, many students want all their errors to be careless errors. This means that the students *did* know the math, but simply made silly mistakes. In such cases, I ask the student to solve the problem immediately, while I watch.

If the student can solve the problem or point out his or her mistake in a few seconds, it is a careless error. If the student cannot solve the problem immediately, it is not a careless error and is probably a concept error.

When working with students who make careless errors, I ask them two questions: First, "How many points did you lose due to careless errors?" Then I follow with, "How much time was left in the class period when you handed in your test?" Students who lose test points to careless errors are giving away points if they hand in their test papers before the test period ends.

To reduce careless errors, you must realize the types of careless errors made and recognize them when reviewing your test. If you cannot solve the missed problem immediately, it is not a careless error. If your major error is not simplifying the answer, review each answer as if it were a new problem and try to reduce it.

Concept errors are mistakes made when you do not understand the properties or principles required to work the problem. Concept errors, if not corrected, will follow you from test to test, causing a loss of test points.

Examples: Some common concept errors are not knowing:

$(-)(-)x = x$, *not* $-x$

$-1(2) > x(-1) = 2 < x$, *not* $2 > x$

$5/0$ is undefined, *not* 0

$(a + x) / x$ is *not* reduced to a

the order of operations

Concept errors must be corrected to improve your next math test score. Students who have numerous concept test errors will fail the next test and the course if concepts are not understood. Just going back to rework the concept error problems is not good enough. You must go back to your textbook or notes and learn why you missed those types of problems, not just the one problem itself.

The best way to learn how to work those types of problems is to set up a concept problem error page in the back of your notebook. Label the first page "Test One Concept Errors." Write down all your concept errors and how to solve the problems. Then, work five more problems that use the same concept. Now, in your own words, write the concepts that you are using to solve these problems.

If you cannot write the concept in your own words, you do not understand it. Get assistance from your instructor if you need help finding similar problems using the same concept or cannot understand the concept. Do this for every test.

Application errors occur when you know the concept but cannot apply it to the problem. Application errors usually are found in word problems, deducing formulas (such as the quadratic equation), and graphing. Even some better students become frustrated with application errors; they understand the material but cannot apply it to the problem.

To reduce application errors, you must predict the type of application problems that will be on the test. You must then think through and practice solving those types of problems using the concepts.

Example: If you must derive the quadratic formula, you should practice doing that backward and forward while telling yourself the concept used to move from one step to the next.

Application errors are common with word problems. After completing the word problem, reread the question to make sure you have applied the answer to the intended question. Application errors can be avoided with appropriate practice and insight.

Test-taking errors apply to the specific way you take tests. Some students consistently make the same types of test-taking errors. Through recognition, these bad test-taking habits can be replaced by good test-taking habits. The result will be higher test scores. The list that follows includes the test-taking errors that can cause you to lose many points on an exam:

1. *Missing more questions in the first third, second third, or last third of a test* is considered a test-taking error.

   Missing more questions in the first third of a test can be due to carelessness when doing easy problems or due to test anxiety.

   Missing questions in the last part of the test can be due to the fact that the last problems are more difficult than the earlier questions or due to increasing your test speed to finish the test.

   If you consistently miss more questions in a certain part of the test, use your remaining test time to review that section of the test first. This means you may review the last part of your test first.

2. *Not completing a problem to its last step* is another test-taking error. If you have this bad habit, review the last step of the test problem first, before doing an in-depth test review.

3. *Changing test answers from correct ones to incorrect ones* is a problem for some students. Find out if you are a good or bad answer-changer

by comparing the number of answers changed to correct and to incorrect answers. If you are a bad answer-changer, write on your test, "Don't change answers." Change answers only if you can prove to yourself or the instructor that the changed answer is correct.

4. *Getting stuck on one problem and spending too much time on it* is another test-taking error. You need to set a time limit on each problem before moving to the next problem. Working too long on a problem without success will increase your test anxiety and waste valuable time that could be used in solving other problems or in reviewing your test.

5. *Rushing through the easiest part of the test and making careless errors* is a common test-taking error for the better student. If you have the bad habit of getting more points taken off for the easy problems than for the hard problems, first review the easy problems, and later review the hard problems.

6. *Miscopying an answer from your scratch work to the test* is an uncommon test-taking error, but it does cost some students points. To avoid these kinds of errors, systematically compare your last problem step on scratch paper with the answer written on the test. In addition, always hand in your scratch work with your test.

7. *Leaving answers blank* will get you zero points. If you look at a problem and cannot figure out how to solve it, do not leave it blank. Write down some information about the problem, rewrite the problem, or try to do at least the first step.

   Remember: Writing down the first step of a problem is the key to solving the problem and obtaining partial credit.

8. *Solving only the first step of a two-step problem* causes problems for some students. These students get so excited when answering the first step of the problem that they forget about the second part. This is especially true on two-step word problems. To correct this test-taking error, write "two" in the margin of the problem. That will remind you that their are two steps or two answers to this problem.

9. *Not understanding all the functions of your calculator* can cause major testing problems. Some students only barely learn how to use the critical calculator functions. They then forget or have to relearn how to use their calculator, which costs test points and test time. Do not wait to learn how to use your calculator on the test. Overlearn the use of your calculator *before* the test.

10. *Leaving the test early without checking all your answers is a costly habit.* Do not worry about the first person who finishes the test and leaves. Many students start to get nervous when students start to leave after finishing the test. This can lead to test anxiety, mental blocks, and loss of recall.

    According to research, the first students finishing the test do not always get the best grades. It sometimes is the exact opposite. Ignore the exiting students, and always use the full time allowed.

Make sure you follow the ten steps to better test taking (see page 91). Review your test-taking procedures for discrepancies in following the ten steps to better test taking. Deviating from these proven ten steps will cost you points.

Study errors, the last type of mistake to look for in test analysis, occur when you study the wrong type of material or do not spend enough time on pertinent material. Review your test to find out if you missed problems because you did not practice that type of problem or because you did practice it but forgot how to do it during the test. Study errors will take some time to track down. But correcting study errors will help you on future tests.

Most students, after analyzing one or several tests, will recognize at least one major, common test-taking error. Understanding the effects of this test-taking error should change your study techniques or test-taking strategy.

Example: If there are seven minutes left in the test, should you review for careless errors or try to answer those two problems you could not totally solve? This is a trick question. The real question is, Do you miss more points due to careless errors or concept errors, or are the missed points about even? The answer to this question should determine how you will spend the last minutes of the test. If you miss more points due to careless errors or miss about the same number of points due to careless/concept errors, review for careless errors.

Careless errors are easier to correct than concept errors. However, if you make very few or no careless errors, you should be working on those last two problems to get the greatest number of test points. Knowing your test-taking errors can add more points to your test by changing your test-taking procedure.

 **– Section Review –** 

1. List the six types of test-taking errors.

   Error 1: _____
   _____
   _____

   Error 2: _____
   _____
   _____

   Error 3: _____
   _____
   _____

   Error 4: _____
   _____
   _____

   Error 5: _____
   _____
   _____

   Error 6: _____
   _____
   _____

2. Which of these errors have you made in the past?

   _____
   _____
   _____
   _____
   _____
   _____

3. How can you avoid these types of errors on the next test?

   _____
   _____
   _____
   _____
   _____
   _____

4. Review your last test to list the type of errors, the points lost for each error, and examples of the errors. (Remember: Careless errors are problems you can solve immediately.)

   Error 1: _____
   Points Lost _____
   Error Example _____
   _____
   _____

Error 2: _____
Points Lost _____
Error Example _____
_____
_____

Error 3: _____
Points Lost _____
Error Example _____
_____
_____

Error 4: _____
Points Lost _____
Error Example _____
_____
_____

Error 5: _____
Points Lost _____
Error Example _____
_____
_____

Error 6: _____
Points Lost _____
Error Example _____
_____
_____

5. List any additional concept error problems that were not included in question 2.

Concept Error Problems _____
_____
_____
_____

Using the pages in the back of your math notebook, rework the concept error problems and do five math problems just like them. For each concept error, write why you could not solve the problem on the test and what new information you learned to solve the problem. Do this for every major test. Give an example of one of your concept error(s) and the reasons you can now solve it.

Concept Error Problem(s) _____
_____
_____
_____

Reasons I Can Now Solve It _____
_____
_____
_____
_____
_____
_____
_____
_____
_____

How to prepare for the final exam

The first day of class is when you start preparing for the final exam. Look at the syllabus or ask the instructor if the final exam is cumulative. A cumulative exam covers everything from the first chapter to the last chapter. Most math final exams are cumulative.

The second question you should ask is if the final exam is a departmental exam or if it is made up by your instructor. In most cases, departmental exams are more difficult and need a little different preparation. If you have a departmental final, you need to ask for last year's test and ask other students what their instructors say will be on the test.

The third question is, How much will the final exam count? Does it carry the same weight as a regular test or, as in some cases, will it count a third of your grade? If the latter is true, the final exam will usually make a letter grade difference on your final grade. The final exam could also determine if you

pass or fail the course. Knowing this information before the final exam will help you prepare.

Preparing for the final exam is similar to preparing for each chapter test. You must create a pretest to discover what you have forgotten. You can use questions from the textbook chapter tests or questions from your study group.

Review the concept errors that you recorded in the back of your notebook labeled "Test One," "Test Two," and so on. Review your problem log for questions you consistently miss. Even review material that you knew for the first and second test but which was not used on any other tests. Students forget how to work some of these problems.

Make sure to use the ten steps to better test taking (see page 91), and the information gained from your test analysis. Use all the time on the final exam, because you could improve your final grade by a full letter if you make an A on the final.



1. What are the three qustions you need to ask about your final exam?

   Question 1: _____
   _____
   _____

   Question 2: _____
   _____
   _____

   Question 3: _____
   _____
   _____

Lecture Notes

Name: _____ Date: _____



Chapter 7 Review



1. You should start studying for a test at least _____ days ahead of time.
2. You should be reviewing only already-_____ material the night before the test.
3. When previewing the test, put a _____ by the questions you can solve without thinking, and do those problems _____.
4. When taking a test, you must decide the best way to get the _____ points in the _____ amount of time.
5. Halfway through the test, you should have _____ than half of the problems completed.
6. If you cannot write down the first step of a problem, you should _____ the problem to help you remember how to solve it.
7. Do not leave the test room until you have reviewed each problem _____ times or until the bell rings.
8. In reviewing the test, if you cannot solve the problem immediately (within a few seconds), it is not a _____ error and is probably a _____ error.
9. Students who have numerous _____ errors will fail the next test and the course if the _____ are not understood.
10. The _____ day of class is when you start preparing for the final exam.

What is the most important information you learned from this chapter?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

How can you immediately use it?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

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About the Author

Over the past 15 years, Learning Specialist Dr. Paul Nolting has helped thousands of students improve their math learning and obtain better grades. Dr. Nolting is a national expert in assessing math learning problems – from study skills to learning disabilities – and developing effective learning strategies and testing accommodations.

Dr. Nolting is also a nationally recognized consultant and trainer of math study skills and of learning and testing accommodations for students with learning disabilities. He has conducted national training grant workshops on math learning for the Association on Higher Education and Disabilities and for a two-year University of Wyoming 1991 Training Grant of Basic Skills.

Dr. Nolting has conducted numerous national conference workshops on math learning for the National Developmental Education Association, the National Council of Educational Opportunity Association, and the American Mathematical Association of Two-Year Colleges. He is a consultant for the American College Test (ACT) and the Texas Higher Education Coordinating Board. He is also a consultant for Houghton Mifflin faculty development programs, conducting workshops on math study skills for math faculty. His text *Winning at Math: Your Guide to Learning Mathematics Through Successful Study Skills* is used throughout the United States, Canada, and the world as the definitive text for math study skills.

Dr. Nolting has consulted with numerous universities and colleges. Some of the universities with which Dr. Nolting has consulted are the University of Massachusetts, the University of Colorado-Boulder, Texas Tech. University, Black Hills State University, Tennessee Tech. University, and the University of Connecticut.

Some of the colleges with which he has consulted are San Antonio College, St. Louis Community College, J. Sargeant Reynolds College, Montgomery College, Broward Community College, Miami-Dade Community College, Northeast State Technical Community College, Landmark College, Denver Community College, Valencia Community College, and Pensacola Junior College. Dr. Nolting has consulted with over 75 colleges, universities, and high schools over the last 15 years.

Dr. Nolting holds a Ph.D. in education from the University of South Florida. His Ph.D. dissertation was "The Effects of Counseling and Study Skills Training on Mathematics Academic Achievement." He is an adjunct instructor for the University of South Florida, teaching assessment and appraisal courses at the graduate level.

His book *Winning at Math: Your Guide to Learning Mathematics Through Successful Study Skills* was selected Book of the Year by the National Association of Independent Publishers. "The strength of the book is the way the writer leads a reluctant student through a course from choosing a teacher to preparing for the final examination," says *Mathematics Teacher*, a publication of the National Council of Teachers of Mathematics. *Winning at Math* and Dr. Nolting's other titles are available at http://www.academicsuccess.com.

His two audiocassettes, *How to Reduce Test Anxiety* and *How to Ace Tests*, were also winners of awards in the National Association of Independent Publishers competition. "Dr. Nolting," says *Publisher's Report*, "is an innovative and outstanding educator and learning specialist."

A key speaker at numerous regional and national education conferences and conventions, Dr. Nolting has been widely acclaimed for his ability to communicate with faculty and students on the subject of improving math learning.

Books and Materials by Academic Success Press, Inc., and Dr. Paul Nolting

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For ordering information on *Winning at Math* and other books and learning materials listed on the previous pages, call toll free: 1-800-247-6553 or visit www.academicsuccess.com. Credit card, checks, and purchase orders are accepted. Shipping charges are $4.00 for the first item and $2.00 for any additional items. Academic Success Press can be contacted at PO Box 25002, Box 132, Bradenton, FL 34206. For a free catalog, call (941) 359-2819.

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